PHYS 270 – SUPPL. #1

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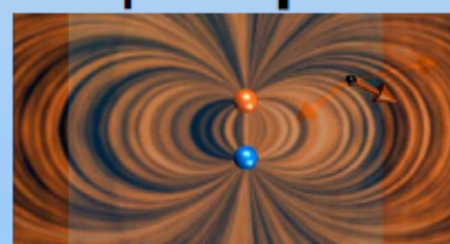
JAN. 25, 2011

GRAVITATIONAL – ELECTRIC FIELDS

	Mass m	Charge q (±)	
Create:	$\vec{\mathbf{g}} = -G\dfrac{m}{r^2}\hat{\mathbf{r}}$	$\vec{\mathbf{E}} = k_e\dfrac{q}{r^2}\hat{\mathbf{r}}$	FIELD
Feel:	$\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$	$\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}$	

Also saw…		Dipole **p**	FORCE

Create:

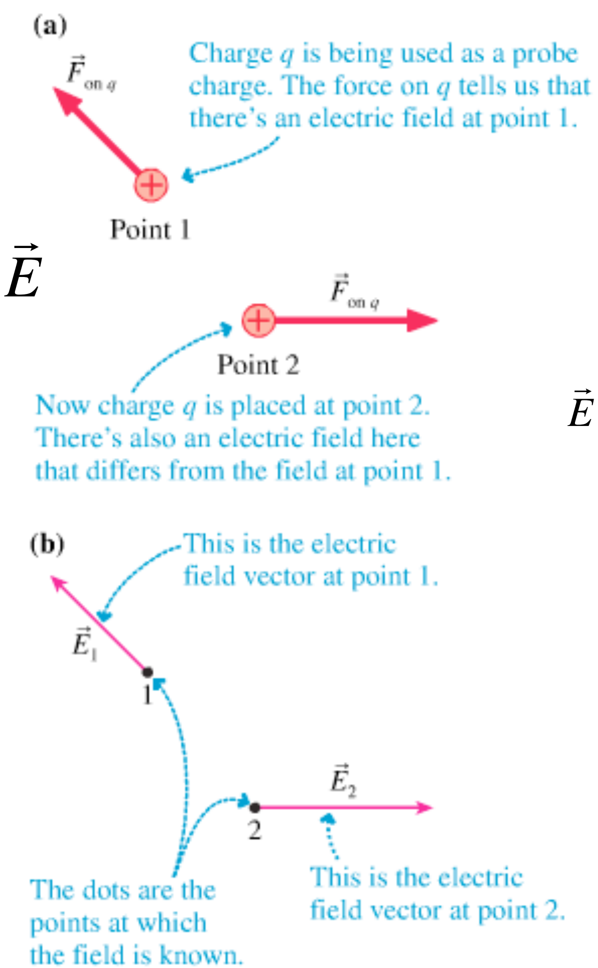


Feel:

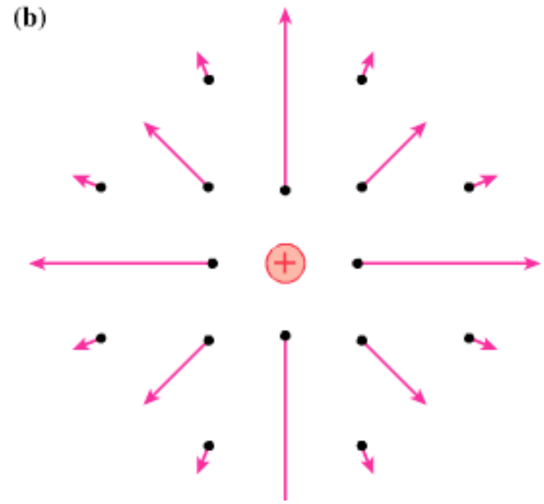
$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$$

ELECTRIC FIELD VECTORS

FIGURE 26.26 Charge q is a probe of the electric field.



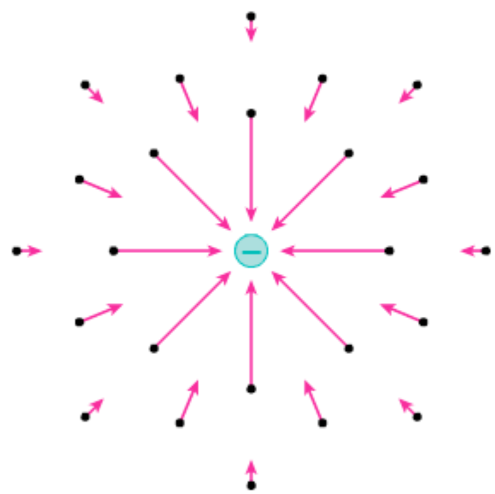
$$\vec{F} = q_T \vec{E}$$



E field of a positive charge

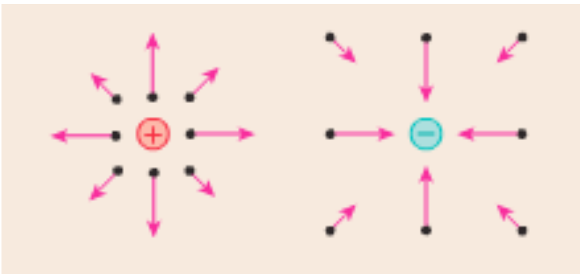
$$\vec{E} = \frac{1}{4\pi\varepsilon_O}\frac{Q}{r^2}\hat{r}$$

FIGURE 26.30 The electric field of a negative point charge.



ELECTRIC FIELD LINES

Gauss's law $\oiint \vec{E} \bullet d\vec{A} = \dfrac{q_{in}}{\varepsilon_o}$



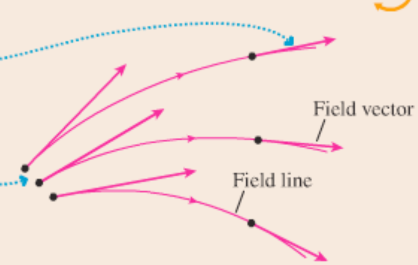
TACTICS BOX 27.1 **Drawing and using electric field lines**

❶ Electric field lines are continuous curves drawn tangent to the electric field vectors. Conversely, the electric field vector at any point is tangent to the field line at that point.

❷ Closely spaced field lines represent a larger field strength, with longer field vectors. Widely spaced lines indicate a smaller field strength.

❸ Electric field lines never cross.

❹ Electric field lines start from positive charges and end on negative charges.



(b)

DIPOLE

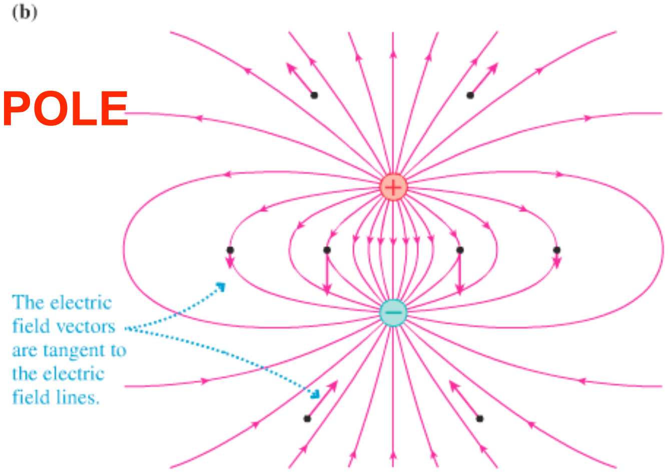
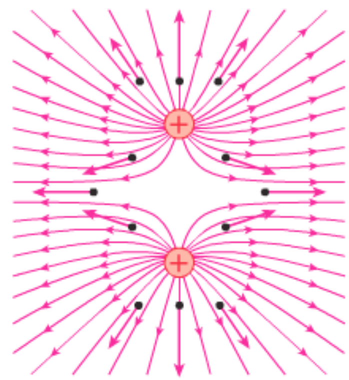
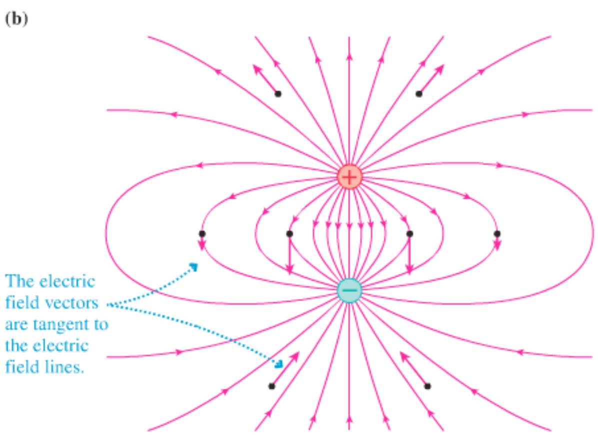


FIGURE 27.10 The electric field of two equal positive charges.



ELECTRIC DIPOLES



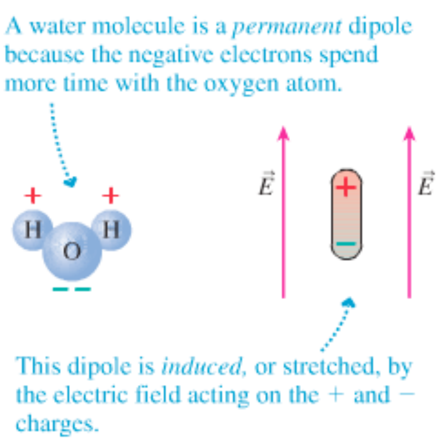
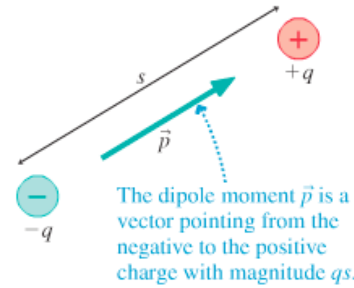


FIGURE 27.8 The dipole moment.



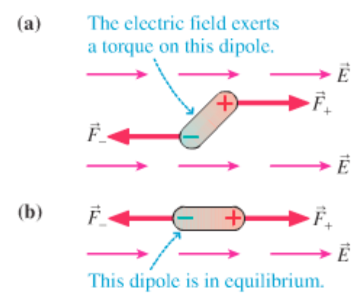
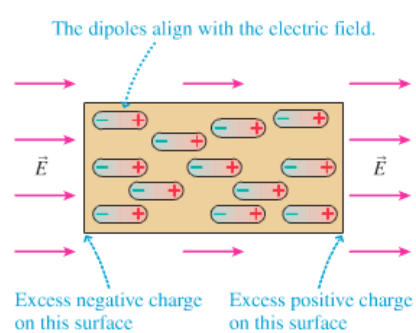


FIGURE 27.29 A sample of permanent dipoles is *polarized* in an electric field.



TORQUE – CROSS VECTOR PRODUCT

FIGURE 27.30 The torque on a dipole.

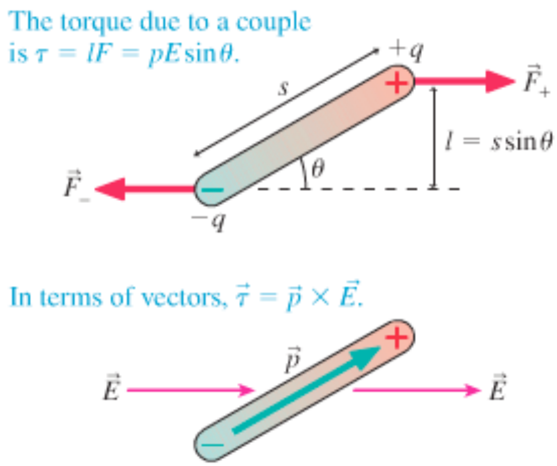
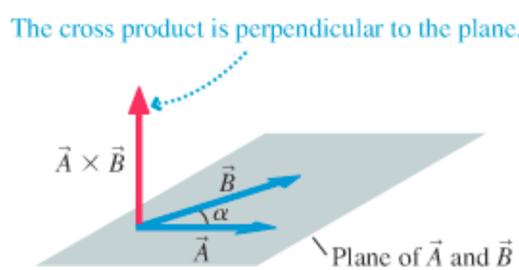


FIGURE 12.50 The cross product $\vec{A} \times \vec{B}$ is a vector perpendicular to the plane of vectors $\vec{A}$ and $\vec{B}$.

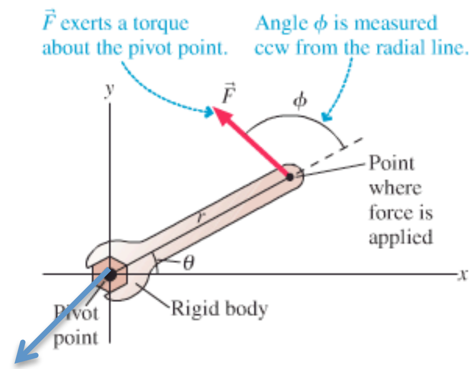


The ability of a force to cause a rotation depends on three factors:

1. The magnitude F of the force.
2. The distance r from the point of application to the pivot.
3. The angle at which the force is applied.

To make these ideas specific, **FIGURE 12.19** shows a force $\vec{F}$ applied at one point on a rigid body. For example, a string might be pulling on the object at that point, in which case the force would be a tension force. Figure 12.19 defines the distance r from the pivot to the point of application and the angle ϕ (Greek phi).

FIGURE 12.19 Force $\vec{F}$ exerts a torque about the pivot point.



NOTE ▶ Angle ϕ is measured *counterclockwise* from the dashed line that extends outward along the radial line. This is consistent with our sign convention for the angular position θ. ◀

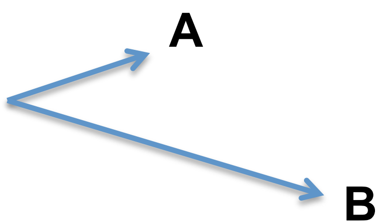
$$I\frac{d\vec{\omega}}{dt} = \vec{T} = \vec{r} \times \vec{F}$$

$$I\frac{d|\vec{\omega}|}{dt} = rF\sin\phi$$

$$\phi = 0 --> \omega = const$$

SCALAR AND VECTOR PRODUCT

Scalar product of two vectors
A and B -> C=AB cosα



VECTOR PRODUCT C=AxB

Make a loose fist with your *right* hand with your thumb extended outward. Orient your hand so that your thumb is perpendicular to the plane of $\vec{A}$ and $\vec{B}$ and your fingers are curling *from* the line of vector $\vec{A}$ *toward* the line of vector $\vec{B}$. Your thumb now points in the direction of $\vec{A} \times \vec{B}$.

Imagine using a screwdriver to turn the slot in the head of a screw from the direction of $\vec{A}$ to the direction of $\vec{B}$. The screw will move either "in" or "out." The direction in which the screw moves is the direction of $\vec{A} \times \vec{B}$.

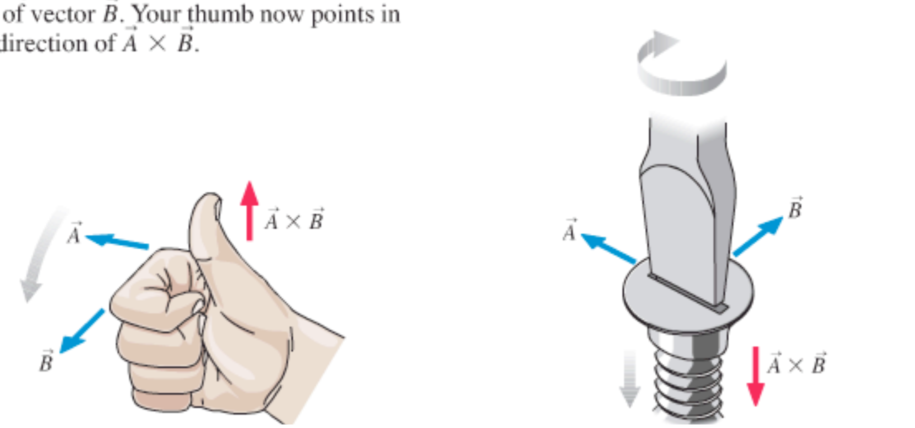
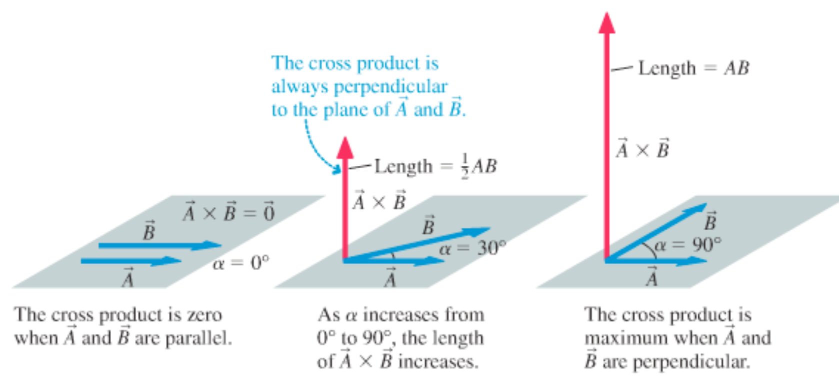
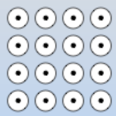


FIGURE 12.51 The magnitude of the cross-product vector increases from 0 to AB as α increases from $0°$ to $90°$.



Cross product – Right hand rule



OUT of page
"Arrow Head"

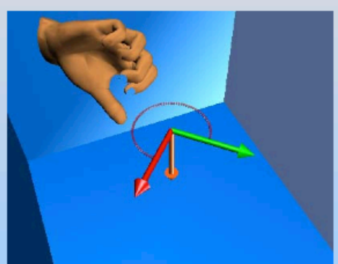
INTO page
"Arrow Tail"

Cross Product: Direction

Right Hand Rule #1: $\vec{\mathbf{C}} = \vec{\mathbf{A}} \mathrm{x} \vec{\mathbf{B}}$

1) Curl fingers of right hand in the direction that moves **A** (green vector) to **B** (red vector) through the smallest angle

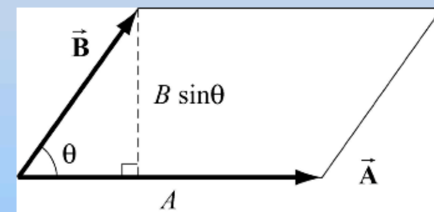
2) Thumb of right hand will point in direction of the cross product **C** (orange vector)



Amplitude

Computing magnitude of cross product A x B:

$$\vec{\mathbf{C}} = \vec{\mathbf{A}} \mathrm{x} \vec{\mathbf{B}} \qquad \left|\vec{\mathbf{C}}\right| = \left|\vec{\mathbf{A}}\right| \left|\vec{\mathbf{B}}\right| \sin\theta$$



$|\vec{\mathbf{C}}|$: area of parallelogram

Cross Product: Signs

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \qquad \hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}}$$

$$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}} \qquad \hat{\mathbf{k}} \times \hat{\mathbf{j}} = -\hat{\mathbf{i}}$$

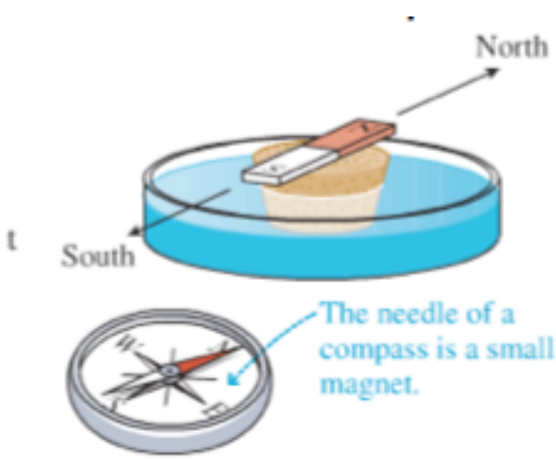
$$\hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \qquad \hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}}$$

Cross Product is Cyclic (left column)
Reversing **A** & **B** changes sign (right column)

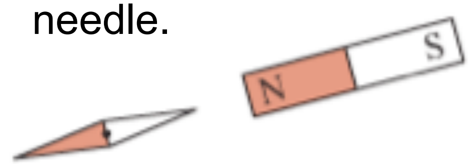
MAGNETISM - EXPERIMENTS

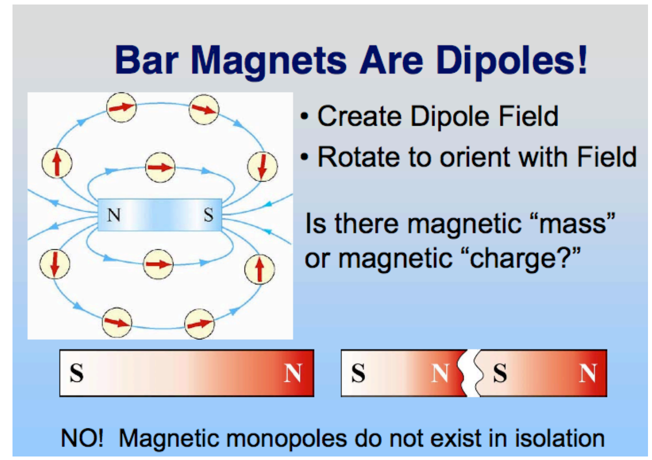
MAGNETIC PROBE → COMPASS → IS A PERMANENT MAGNET

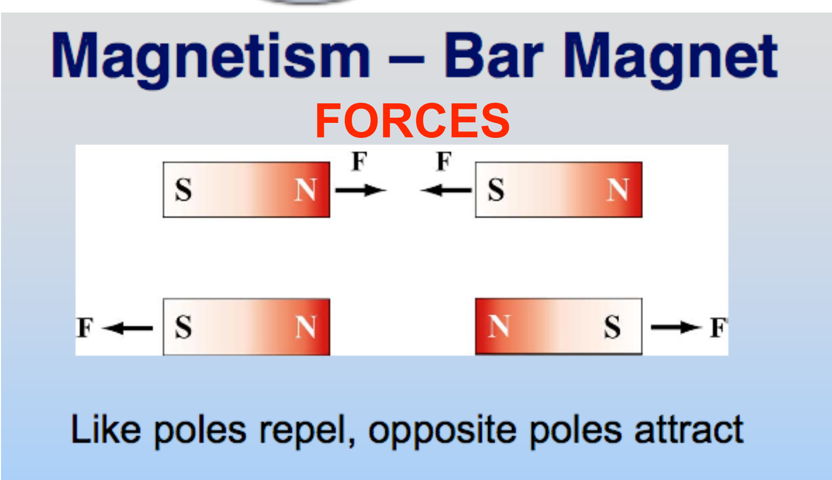
Definition of N and S poles

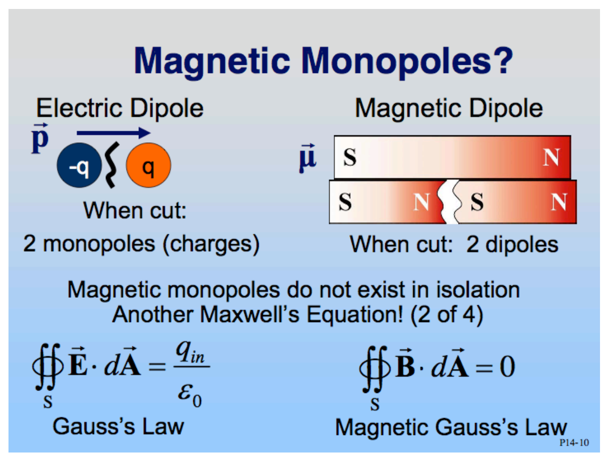


N attracts S of needle.









Long range force.
Acts only on magnetic materials.
Iron, Ni, Co

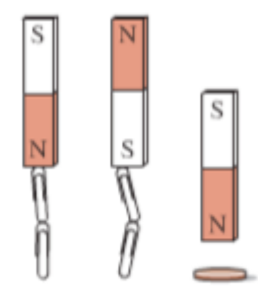
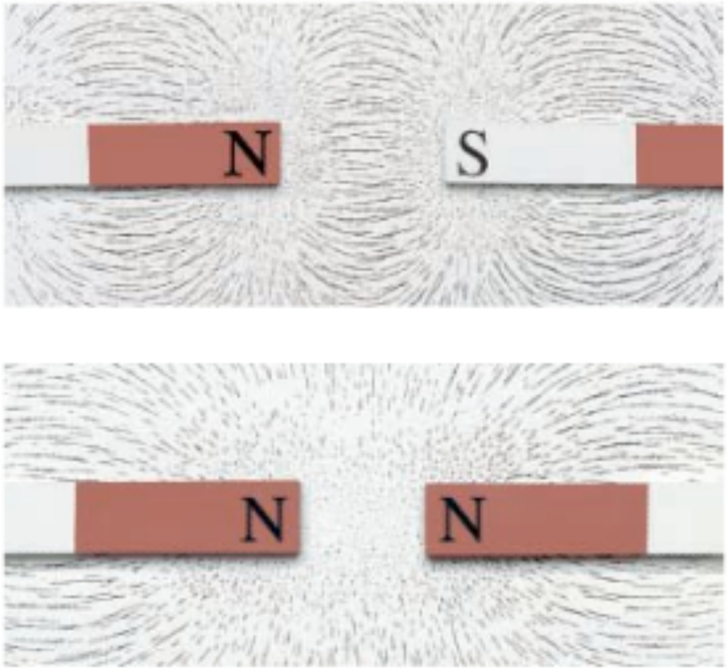
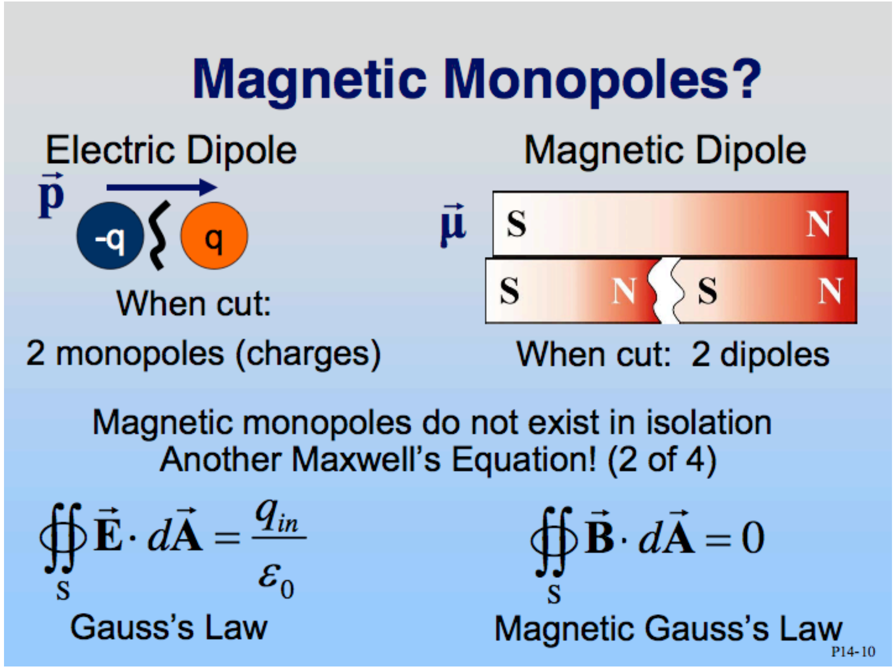


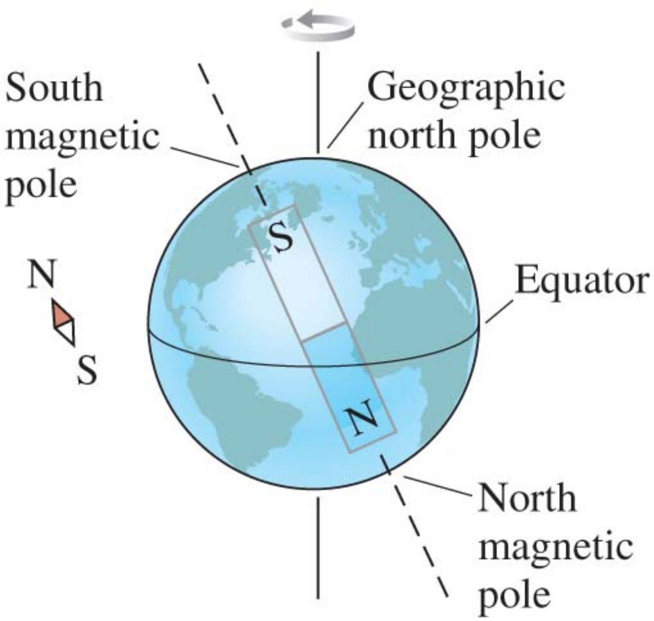
FIGURE 26.24 Iron filings sprinkled around the ends of a magnet suggest that the influence of the magnet extends into the space around it.

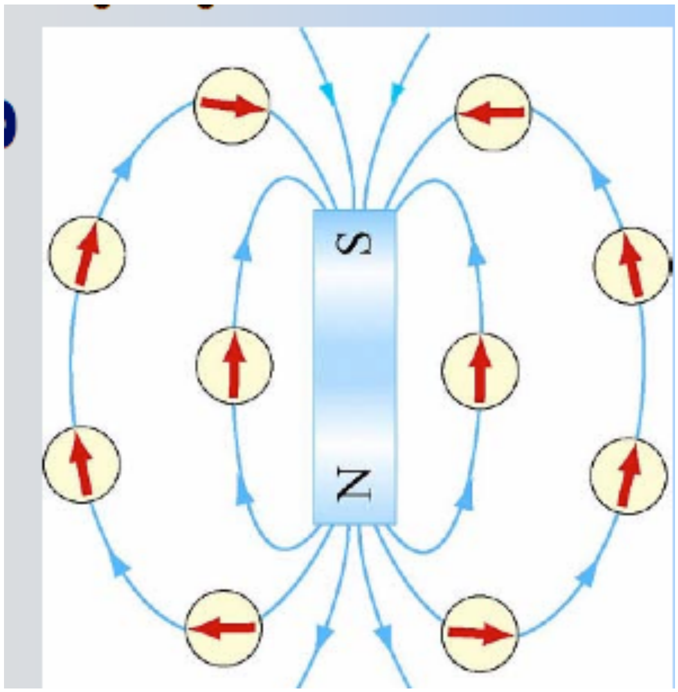




Earth's Magnetic Field

FIGURE 33.1 The earth is a large magnet.



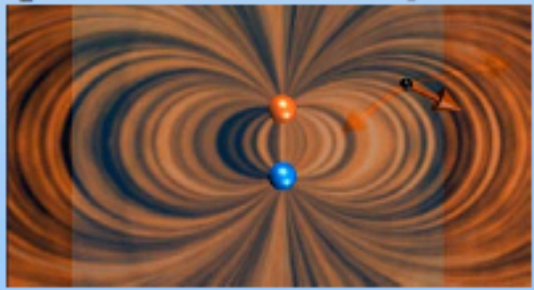


What creates the earth's magnetic field ?
Field reversal – last one

Fields: Grav., Electric, Magnetic

	Mass m	Charge q (±)	No Magnetic Monopoles!
Create:	$\vec{\mathbf{g}} = -G\dfrac{m}{r^2}\hat{\mathbf{r}}$	$\vec{\mathbf{E}} = k_e\dfrac{q}{r^2}\hat{\mathbf{r}}$	
Feel:	$\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$	$\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}$	

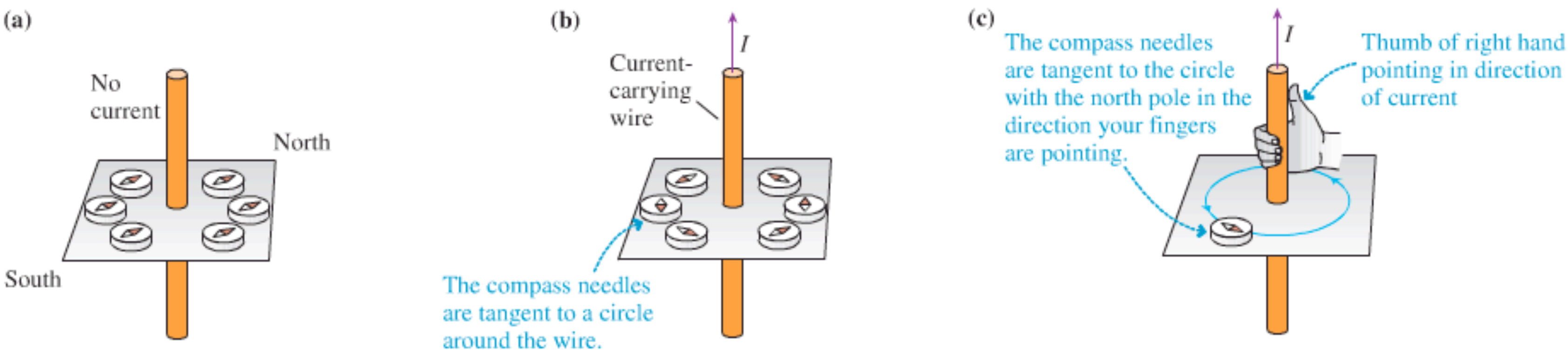
Also saw…		Dipole $\mathbf{p}$	Dipole μ
Create:		$\vec{\mathbf{E}} \rightarrow$	$\leftarrow \vec{\mathbf{B}}$
Feel:		$\vec{\boldsymbol{\tau}} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$	$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$



What Else Creates a Magnetic Field

Oersted -1819 -> Effect of an electric in a compass current in a compass → electric current creates a magnetic field

FIGURE 33.2 Response of compass needles to a current in a straight wire.



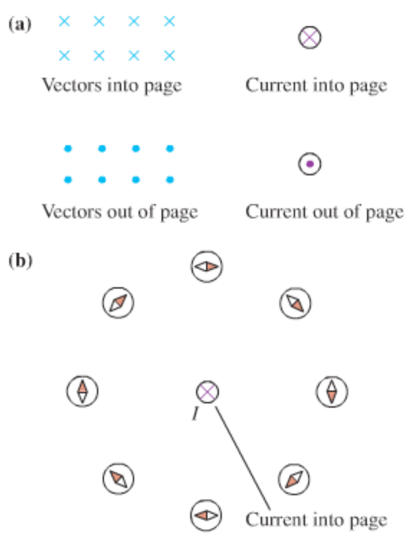
So what is a magnetic field ?

Nothing more than a way to visualize and calculate long range forces on objects in the vicinity of currents or permanent magnets.

Magnetic Field Properties:

- **B field created at all points in space surrounding a current or a magnet**
- **The B field at each point is a vector (magnitude + direction)**
- **B field exerts forces on magnetic poles. Force on N pole parallel to B. Force on S pole anti-parallel to B**

FIGURE 33.3 The notation for vectors and currents perpendicular to the page.



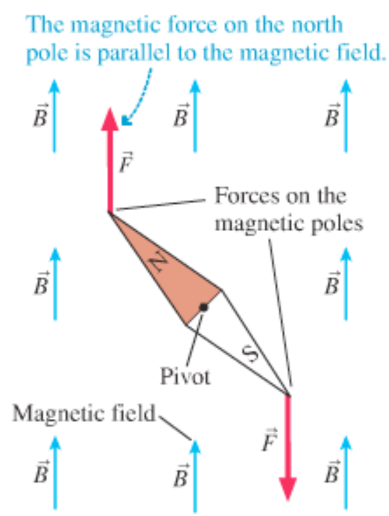
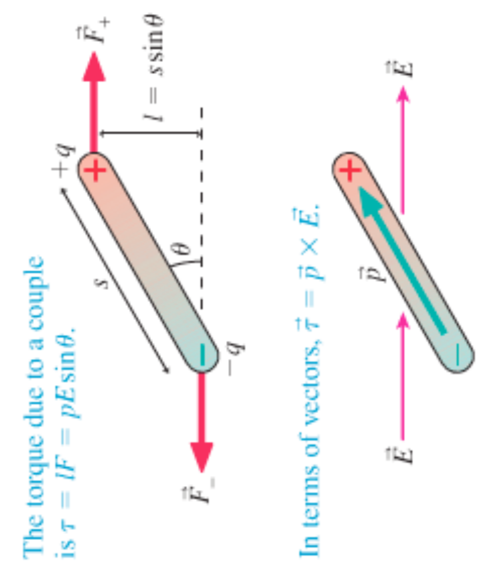
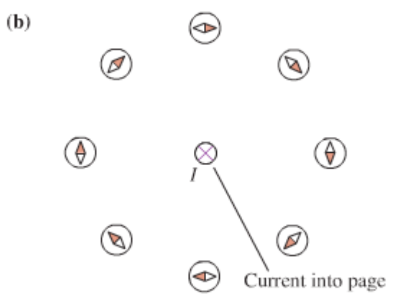


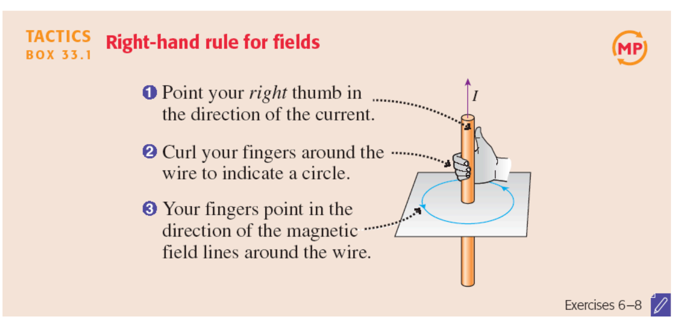
FIGURE 27.30 The torque on a dipole.

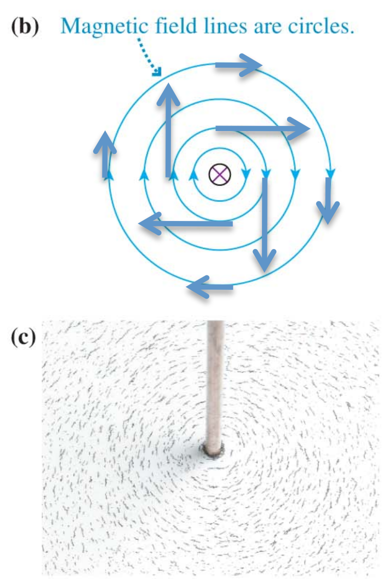


Magnetic Field Lines. Why ? What ?

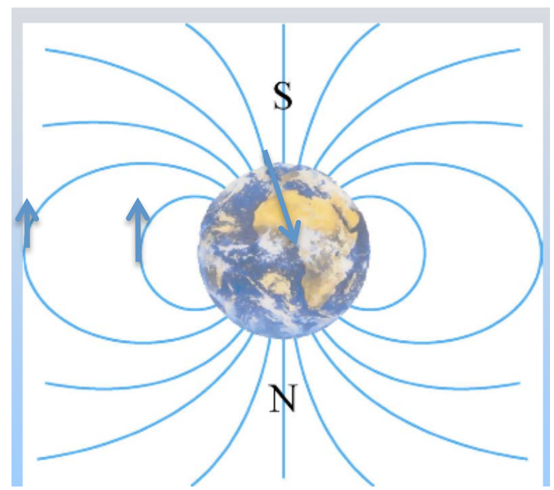
- **Nothing more than another convenient way to visualize magnetic forces**
- **Imaginary lines with the following properties:**
  - **The magnetic field is tangent to the lines**
  - **The field lines are closer together when the amplitude is larger**







The magnetic field is revealed by the pattern of iron filings around the current-carrying wire.



MAGNETIC FIELD DUE TO A CHARGE

FIGURE 33.6 The magnetic field of a moving point charge.

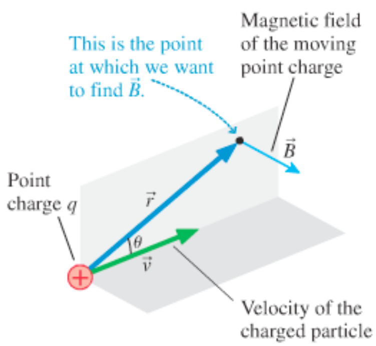
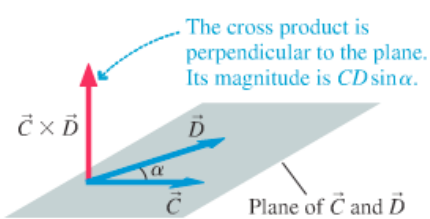


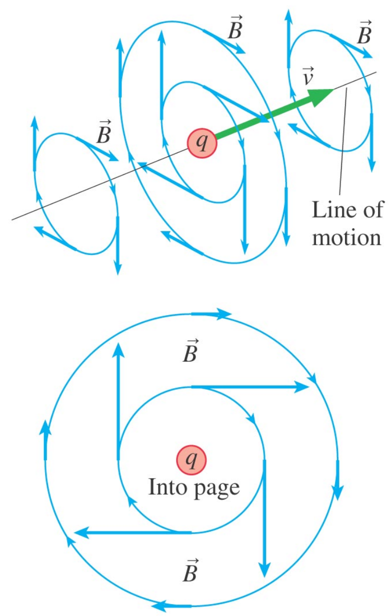
FIGURE 33.9 The cross product $\vec{C} \times \vec{D}$ is a vector perpendicular to the plane of vectors $\vec{C}$ and $\vec{D}$.



Biot –Savart law

$$\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2} \qquad \text{(magnetic field of a point charge)}$$

Direction by RH rule



Only moving charges create B

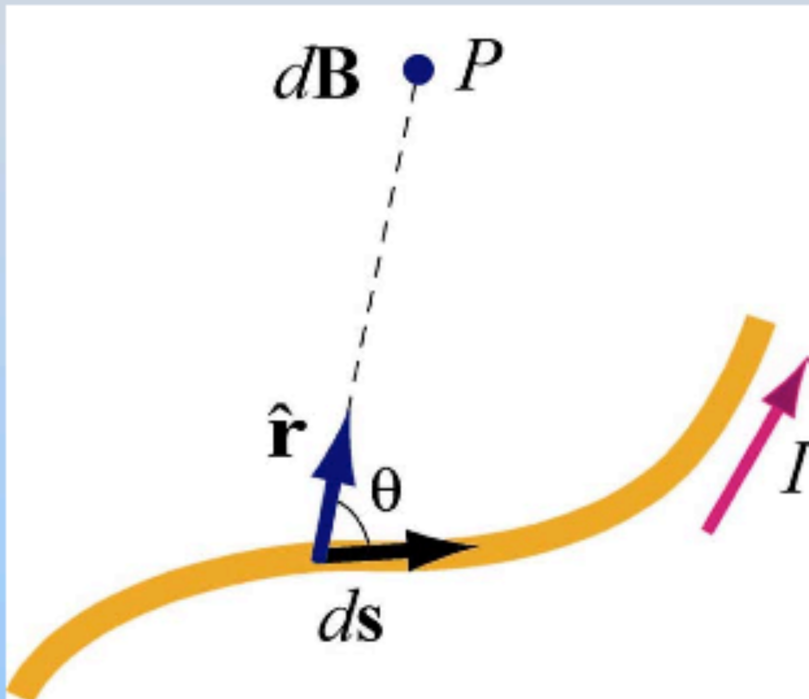
Units of B: Tesla
1 Tesla=1 T=1 N/Am

$\mu_o=4\pi \times 10^{-7}$ Tm/A

Magnetic Field due to a Current Element

Current element of length ds carrying current I produces a magnetic field:

Biot – Savart Law

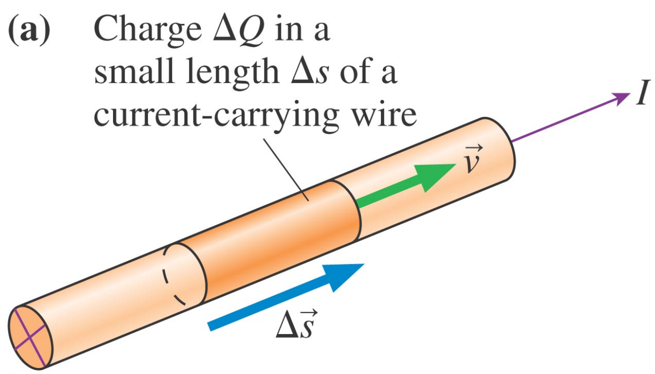


$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

How do we get this law ?

MAGNETIC FIELD DUE TO A CURRENT

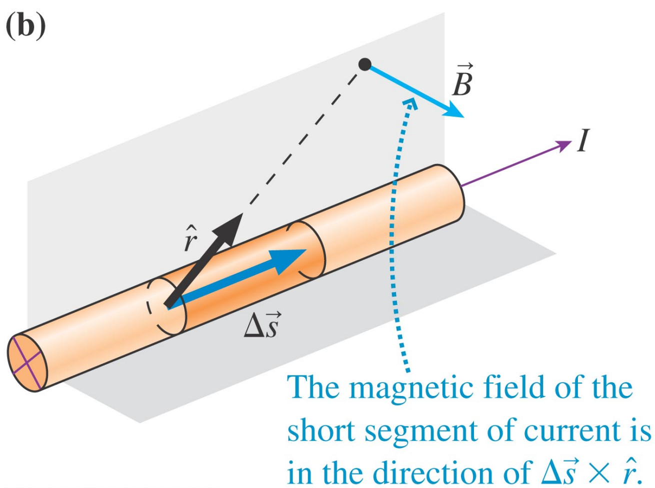
$$\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} \qquad \text{(magnetic field of a point charge)}$$



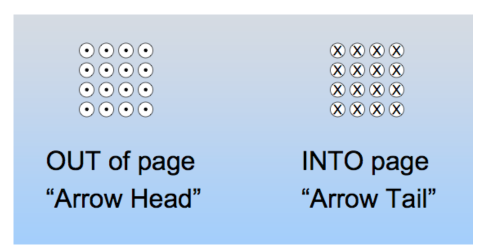
$$\Delta Q\vec{v} = \Delta Q\frac{\Delta\vec{s}}{\Delta t} = \frac{\Delta Q}{\Delta t}\Delta\vec{s} = I\Delta\vec{s}$$

$$\vec{B}_{\Delta s}(\vec{r}) = \frac{\mu_o}{4\pi}\frac{I\Delta\vec{s}\times\hat{r}}{r^2}$$

Biot-Savart

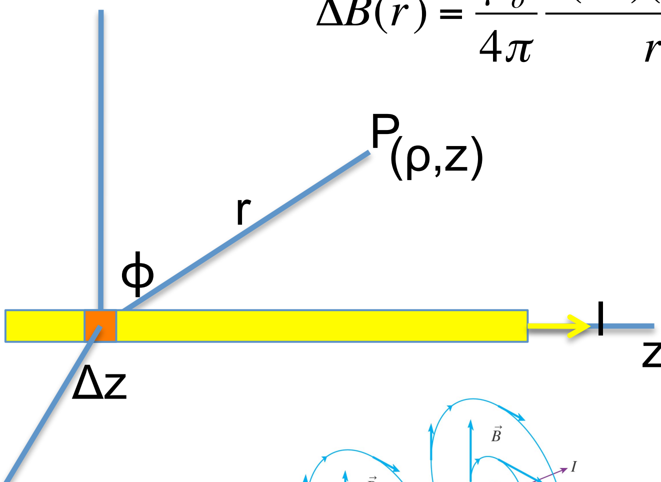


Convention



B Field of a long wire

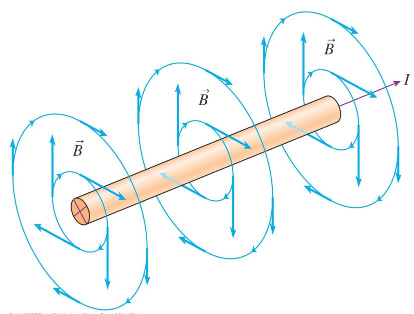
$$\Delta\vec{B}(\vec{r}) = \frac{\mu_o}{4\pi}\frac{I(\Delta z)(\hat{e}_z \times \hat{r})}{r^2} = \frac{\mu_o}{4\pi}\frac{I(\Delta z)[\hat{e}_z \times \hat{e}_\rho \sin\phi)]}{r^2} = \frac{\mu_o}{4\pi}\frac{I(\Delta z)\sin\phi\hat{e}_\theta}{r^2} \equiv \Delta B(\vec{r})\hat{e}_\theta$$



$$\Delta B(\vec{r}) = \frac{\mu_o}{4\pi}2\frac{I\Delta z\sin\phi}{\rho^2+z^2} = \frac{\mu_o}{4\pi}\frac{I\Delta z\sin\phi}{\rho^2+z^2}$$

$$\sin\phi = \rho/r = \rho/\sqrt{\rho^2+z^2}$$

$$\Delta B(\vec{r}) = \frac{\mu_o}{4\pi}\frac{I\rho\Delta z}{(\rho^2+z^2)^{3/2}}$$



$$B(\vec{r}) = \frac{\mu_o I\rho}{4\pi}\int_{z_1}^{z_2} dz/(\rho^2+z^2)^{3/2} = \frac{\mu_o I\rho}{4\pi}\frac{z}{\rho^2(z^2+\rho^2)^{1/2}}\Big]_{z_1}^{z_2}$$

Take $z_1 \rightarrow -\infty$ and $z_2 \rightarrow +\infty$ to find $B(\rho) = \frac{\mu_o}{2\pi}\frac{I}{\rho}$

Show that the above formula is valid as long as ρ<<l where l is the length of the wire

PROBLEM-SOLVING STRATEGY 33.1

The magnetic field of a current

MODEL Model the wire as a simple shape, such as a straight line or a loop.

VISUALIZE For the pictorial representation:

1. Draw a picture and establish a coordinate system.
2. Identify the point P at which you want to calculate the magnetic field.
3. Divide the current-carrying wire into segments for which you *already know* how to determine $\vec{B}$. This is usually, though not always, a division into very short segments of length Δs.
4. Draw the magnetic field vector for one or two segments. This will help you identify distances and angles that need to be calculated.
5. Look for symmetries that simplify the field. You may conclude that some components of $\vec{B}$ are zero.

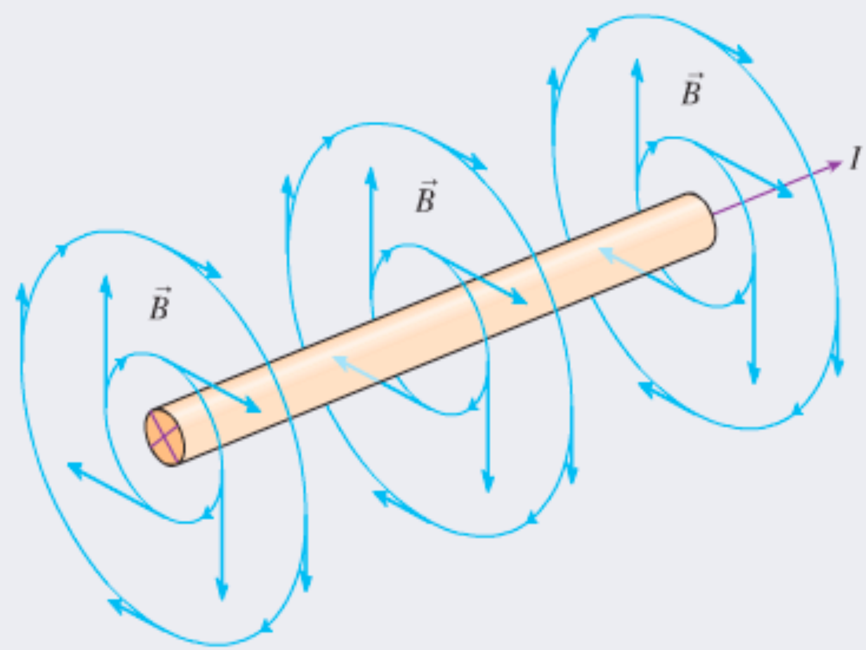
SOLVE The mathematical representation is $\vec{B}_{\text{net}} = \sum \vec{B}_i$.

- Use superposition to form an algebraic expression for *each* of the three components of $\vec{B}$ (unless you are sure one or more is zero) at point P.
- Let the (x, y, z)-coordinates of the point remain as variables.
- Express all angles and distances in terms of the coordinates.
- Let $\Delta s \to ds$ and the sum become an integral. Think carefully about the integration limits for this variable; they will depend on the boundaries of the wire and on the coordinate system you have chosen to use. Carry out the integration and simplify the results as much as possible.

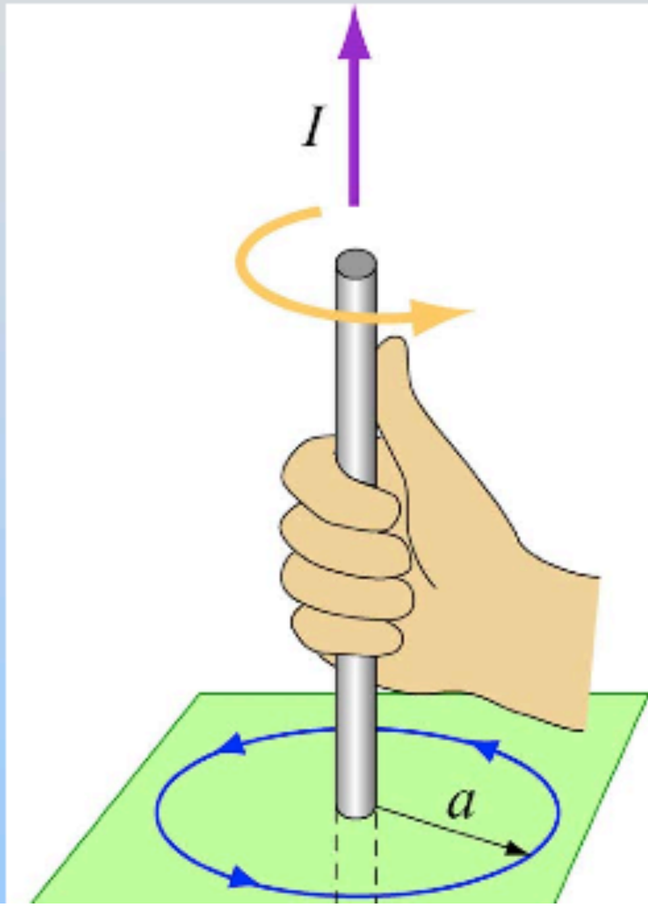
B FIELD DUE TO LONG WIRE

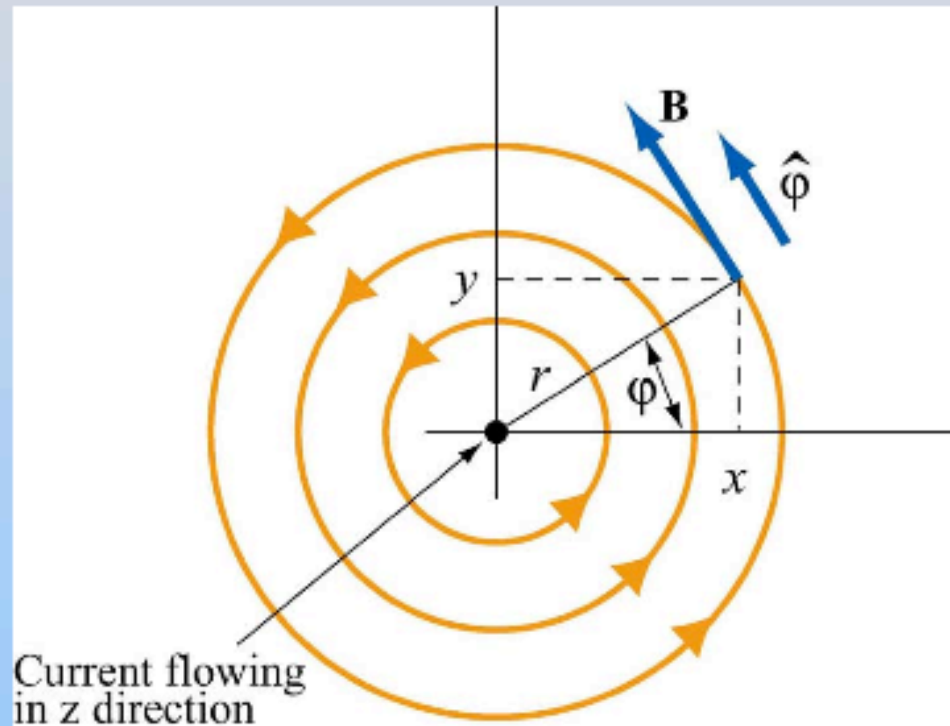
$$B_{\text{wire}} = \frac{\mu_0 I d}{4\pi} \frac{x}{d^2(x^2 + d^2)^{1/2}} \bigg|_{-\infty}^{\infty} = \frac{\mu_0}{2\pi} \frac{I}{d}$$

FIGURE 33.14 The magnetic field of a long, straight wire carrying current I.



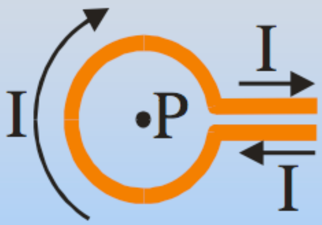
The Right-Hand Rule #2





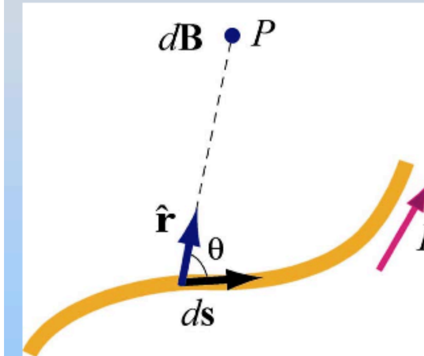
B field of a current loop

Consider a coil with radius *R* and current *I*



Find the magnetic field B at the center (P)

Current element of length ds carrying current I produces a magnetic field:

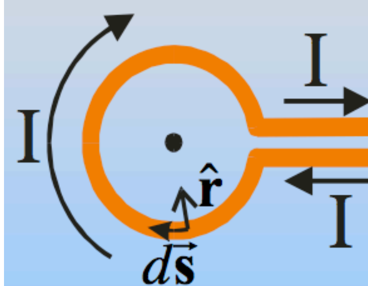


$$\boxed{d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}}$$

1) Think about it:
   - Legs contribute nothing
     I parallel to *r*
   - Ring makes field into page
2) Choose a ds
3) Pick your coordinates
4) Write Biot-Savart

In the circular part of the coil…

$$d\,\vec{\mathbf{s}} \perp \hat{\mathbf{r}} \quad \rightarrow \quad |\,d\,\vec{\mathbf{s}}\times\hat{\mathbf{r}}\,| = ds$$



Biot-Savart:

$$dB = \frac{\mu_0 I}{4\pi}\frac{\left|d\vec{\mathbf{s}}\times\hat{\mathbf{r}}\right|}{r^2} = \frac{\mu_0 I}{4\pi}\frac{ds}{r^2}$$

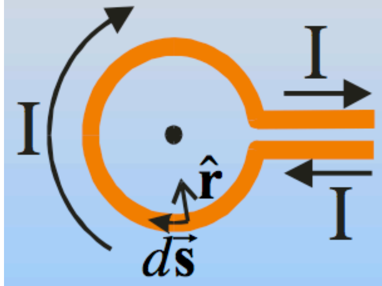
$$= \frac{\mu_0 I}{4\pi}\frac{R\,d\theta}{R^2}$$

$$= \frac{\mu_0 I}{4\pi}\frac{d\theta}{R}$$

B field of a current loop

In the circular part of the coil…

$$d\vec{\mathbf{s}} \perp \hat{\mathbf{r}} \quad \rightarrow \quad |d\vec{\mathbf{s}} \times \hat{\mathbf{r}}| = ds$$



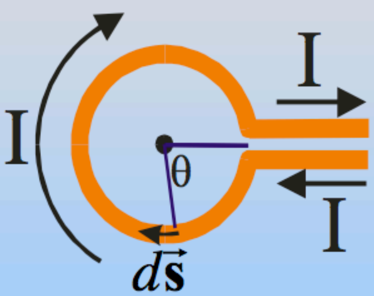
Biot-Savart:

$$dB = \frac{\mu_0 I}{4\pi}\frac{|d\vec{\mathbf{s}} \times \hat{\mathbf{r}}|}{r^2} = \frac{\mu_0 I}{4\pi}\frac{ds}{r^2}$$

$$= \frac{\mu_0 I}{4\pi}\frac{R\,d\theta}{R^2}$$

$$= \frac{\mu_0 I}{4\pi}\frac{d\theta}{R}$$

Consider a coil with radius *R* and current *I*



$$dB = \frac{\mu_0 I}{4\pi}\frac{d\theta}{R}$$

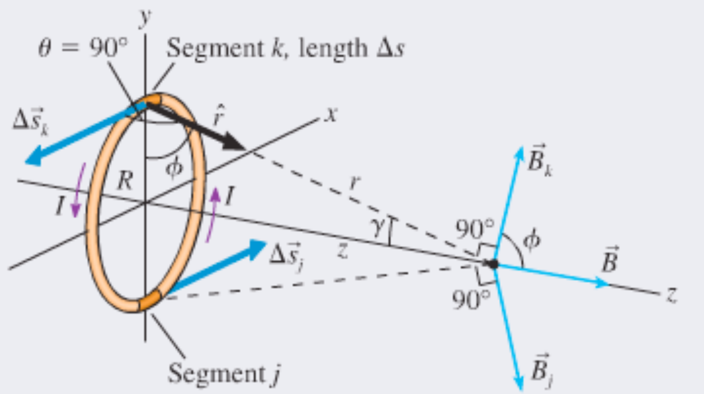
$$B = \int dB = \int_0^{2\pi} \frac{\mu_0 I}{4\pi}\frac{d\theta}{R}$$

$$= \frac{\mu_0 I}{4\pi R}\int_0^{2\pi} d\theta = \frac{\mu_0 I}{4\pi R}(2\pi)$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2R} \text{ into page}$$

Magnetic Dipole

FIGURE 33.16 Calculating the magnetic field of a current loop.



Compute on axis B for z>>R

For z=0 and on axis the fields produced by k and j are equal and add so that we get

$$\Delta B(\rho = 0, z = 0) = 2(\mu_o I / 4\pi)(R\Delta\theta / R^2)$$

Adding all segments results in replacing $\Delta\theta$ by π so that we get

$$B(\rho = 0, z = 0) = (\mu_0 / 2\pi)(I / R)$$

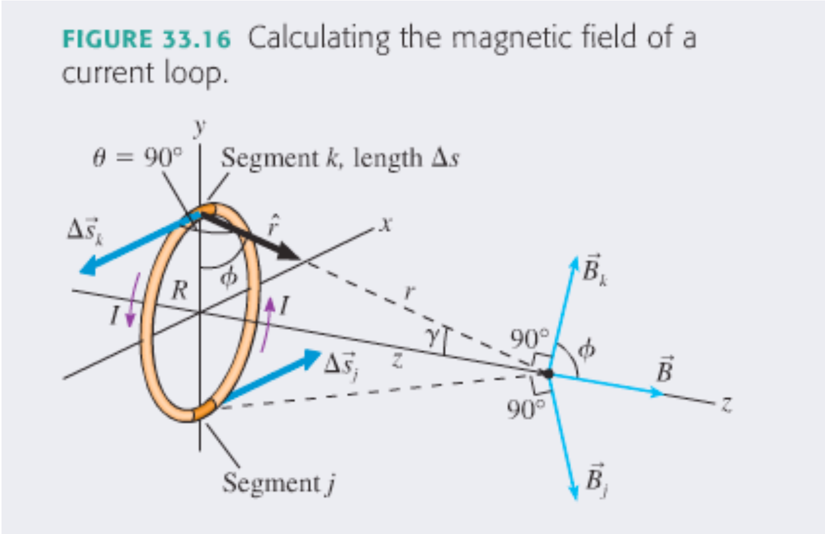
At a distance z≠0 each element creates a z and ρ component. The ρ components of two anti-diametrical arcs are antiprallel and cancel out leaving only the z-component that add. This modifies the previous result to

$$\Delta B_z(\rho = 0, z \neq 0) = 2(\mu_o I / 2\pi)[R\Delta\theta / (R^2 + z^2)^{1/2}]\cos\phi = 2(\mu_o I / 2\pi)[R\Delta\theta / (R^2 + z^2)][R / (R^2 + z^2)^{1/2}]$$

$$B_z(\rho = 0, z \neq 0) = (\mu_o / 2)\frac{IR^2}{(z^2 + R^2)^{3/2}} = (\mu_o / 4\pi)\frac{I\pi R^2}{(z^2 + R^2)^{3/2}}$$

$$z >> R \rightarrow B_z(\rho = 0, z) = (\mu_o / 4\pi)\frac{I\pi R^2}{z^3} = (\mu_o / 4\pi)\frac{IA}{z^3}$$

Magnetic Dipole



Compute on axis B for z>>R

B_k perpendicular to Δs_k and **r.**
The ρ-component of of $\mathbf{B}_k$ is cancelled by the ρ component of $\mathbf{B}_j$. Same for all anti-diametric segments. We are thus left by only a z-component. Need to compute only $(B_k)_z = B_k \cos\phi$.

$$\left|\Delta\vec{s}_k \times \hat{r}\right| = \Delta s$$

$$(B_k)_z = \frac{\mu_o}{4\pi}\frac{I\Delta s}{r^2}\cos\phi$$

$$\cos\phi = R / r$$

$$(B_k)_z = \frac{\mu_o}{4\pi}\frac{IR\Delta s}{(z^2 + R^2)^{3/2}}$$

For z>>R

$$(B_k)_z \approx \frac{\mu_o}{4\pi}\frac{IR\Delta s}{z^3}$$

$$B_{loop} \approx \frac{\mu_o}{4\pi}\frac{IR\Delta s}{z^3} = \frac{\mu_o}{2}\frac{IR^2}{z^3}$$

Magnetic Dipole moment

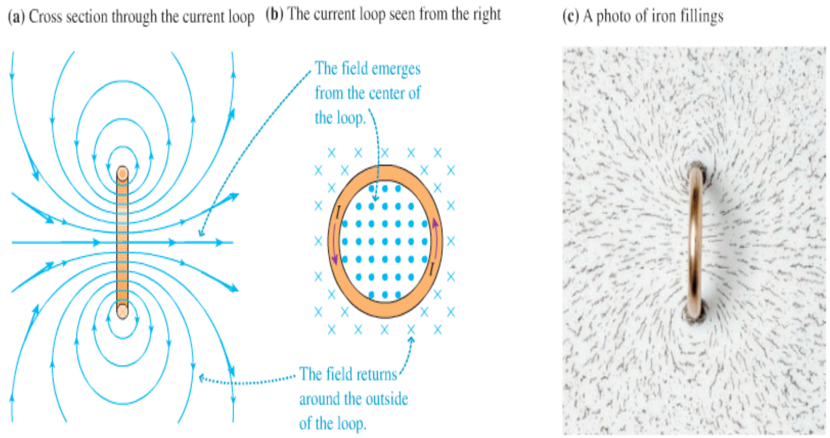
$$B_{loop} \approx \frac{\mu_o}{4\pi}\frac{2I\pi R^2}{z^3} = \frac{\mu_o}{4\pi}\frac{2IA}{z^3} = \frac{\mu_o}{4\pi}\frac{2I\mu}{z^3}$$

$$\vec{\mu} \equiv \vec{A}I$$

$$\vec{E}_{dipole} = \frac{1}{4\pi\varepsilon_o}\frac{2\vec{p}}{z^3}$$

MAGNETIC DIPOLE

FIGURE 33.18 The magnetic field of a current loop.



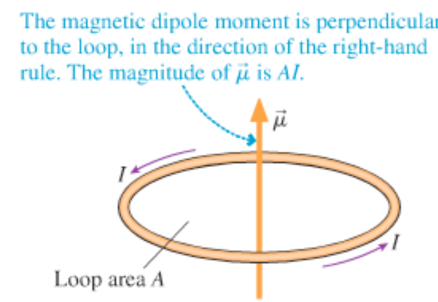


FIGURE 33.19 A current loop has magnetic poles and generates the same magnetic field as a flat permanent magnet.

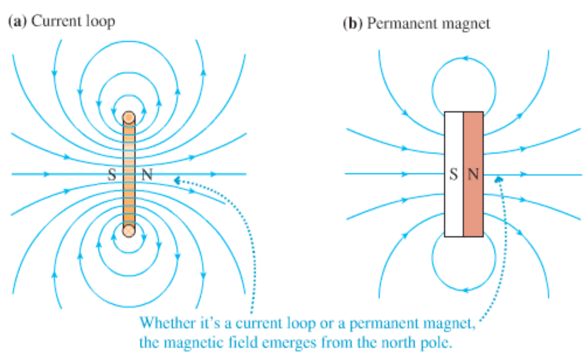
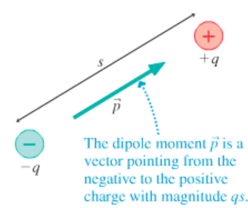


FIGURE 27.8 The dipole moment.



$$\vec{E}(0,0,z) = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{z^3},\ \vec{p} = e_z p, z >> l$$

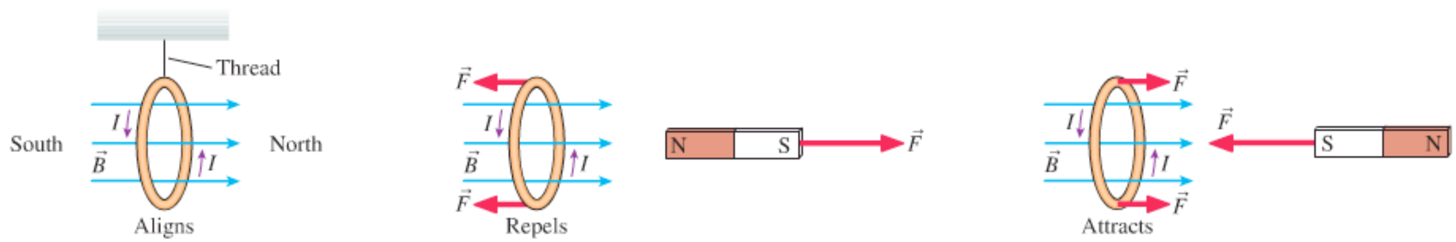
$$\vec{B}(0,0,z) = \frac{\mu_o}{4\pi}\frac{2\vec{\mu}}{z^3}, z >> R$$

$$\mu = I\pi R^2 = I(Area)$$

NOTE ▶ The magnetic field *inside* a permanent magnet differs from the magnetic field at the center of a current loop. Only the exterior field of a magnet matches the field of a current loop. ◀

LOOP – FLAT MAGNET EQUIVALENCE

Investigating current loops



A current loop hung by a thread aligns itself with the magnetic field pointing north.

The north pole of a permanent magnet repels the side of a current loop from which the magnetic field is emerging.

The south pole of a permanent magnet attracts the side of a current loop from which the magnetic field is emerging.

A FLAT PERMANENT PERMANENT MAGNET AND A CURRENT LOOP (ELECTROMAGNET) GENERATE THE SAME MAGNETIC FIELD
THE FIELD OF A DIPOLE