PHYS 270-SPRING 2011

LECTURE # 19
Dennis Papadopoulos

End of Classical Physics
Quantization – Bohr Atom

Chapters 38-39

April 14, 2011

HOW TO MEASURE SPECTRA

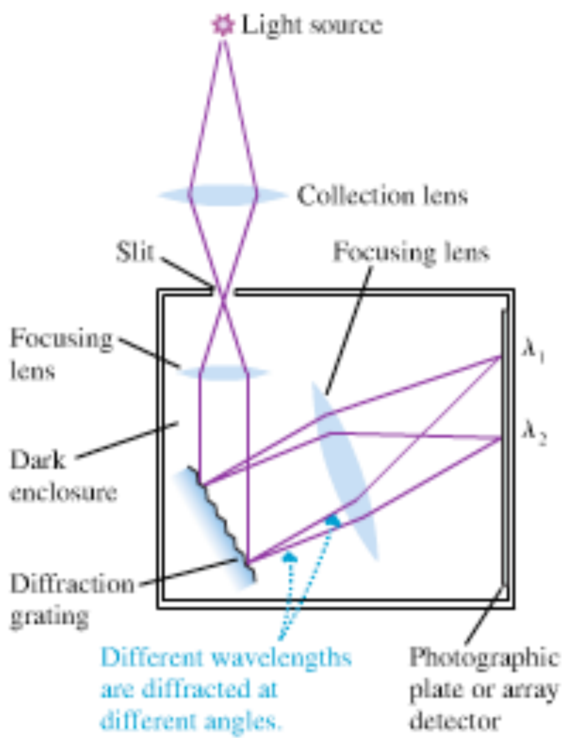
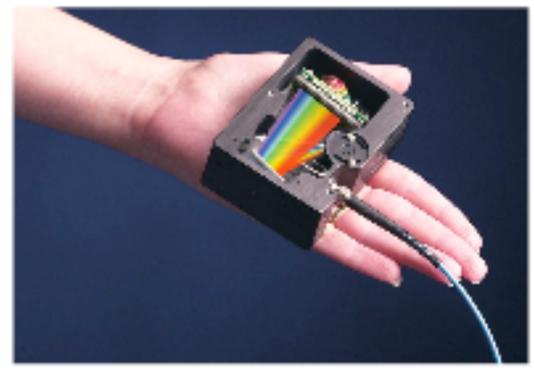


FIGURE 25.1 A diffraction spectrometer for the accurate measurement of wavelengths.



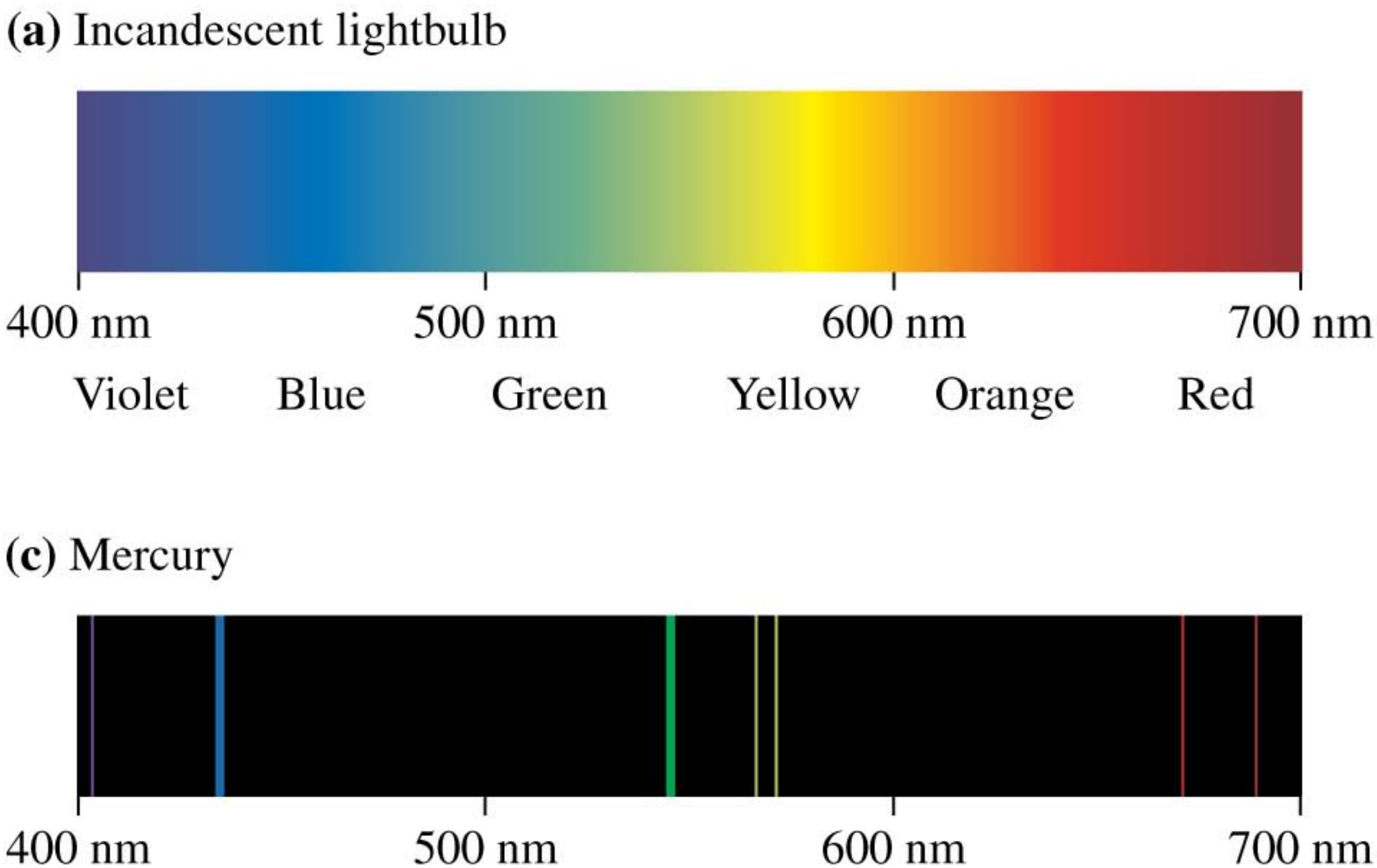
Some modern spectrometers are small enough to hold in your hand. (The rainbow has been superimposed to show how it works.)

Spectroscopy: Unlocking the Structure of Atoms

There are two types of spectra, continuous spectra and discrete spectra:

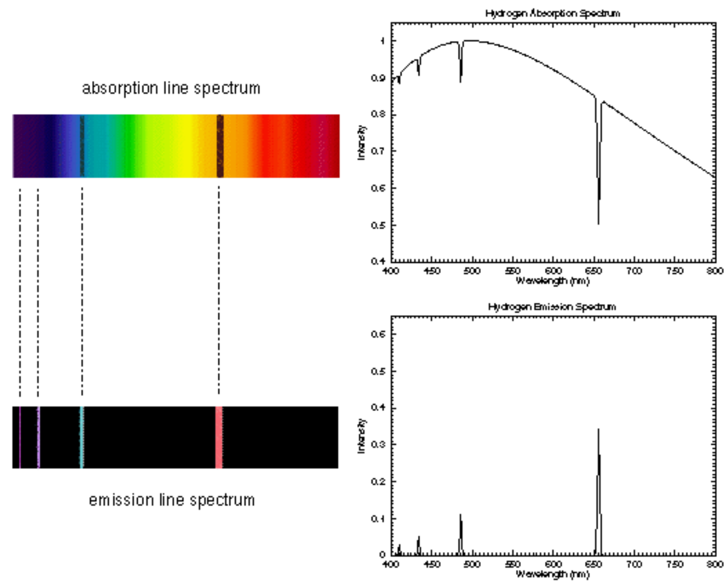
- Hot, self-luminous objects, such as the sun or an incandescent light bulb, emit a *continuous spectrum* in which a rainbow is formed by light being emitted at every possible wavelength.
- In contrast, the light emitted by a gas discharge tube (such as those used to make neon signs) contains only certain discrete, individual wavelengths. Such a spectrum is called a **discrete spectrum.**

FIGURE 25.2 Examples of spectra in the visible wavelength range 400–700 nm.



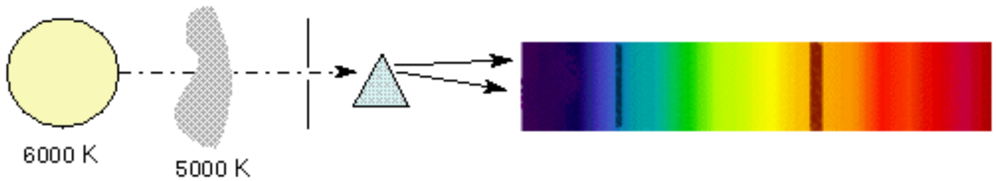
Absorption and Emission Lines

Close examination of the spectra from the Sun and other stars reveals that the rainbow of colors has many dark lines in it, called **absorption lines**. They are produced by the cooler thin gas in the upper layers of the stars absorbing certain colors of light produced by the hotter dense lower layers. You can also see them in the reflected light spectrum from planets. Some of the colors in the sunlight reflecting off the planets are absorbed by the molecules on the planet's surface or in its atmosphere. The spectra of hot, thin (low density) gas clouds are a series of bright lines called **emission lines**. In both of these types of spectra you see spectral features at certain, discrete wavelengths (or colors) and no where else.

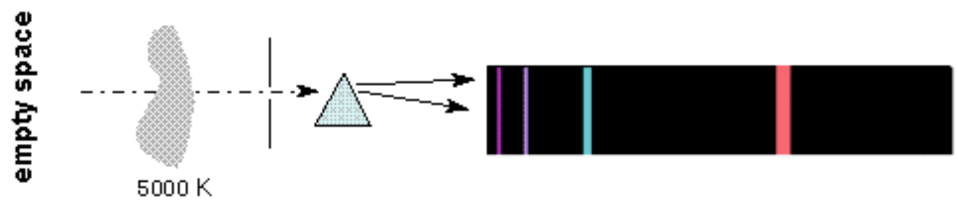


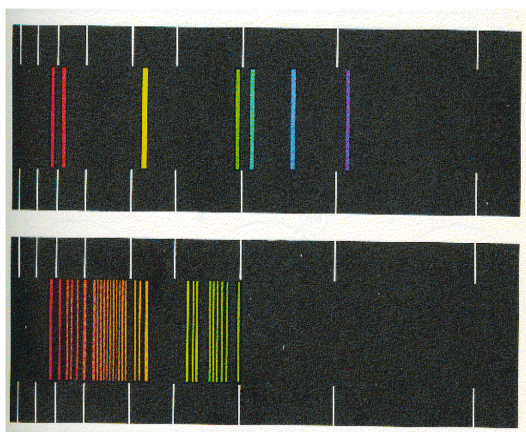
Two ways of showing the same spectra: on the **left** are pictures of the dispersed light and on the **right** are plots of the intensity vs. wavelength. Notice that the pattern of spectral lines in the absorption and emission line spectra are the **same** since the gas is the same.

See also http://www.learner.org/teacherslab/science/light/color/spectra/spectra_1.html



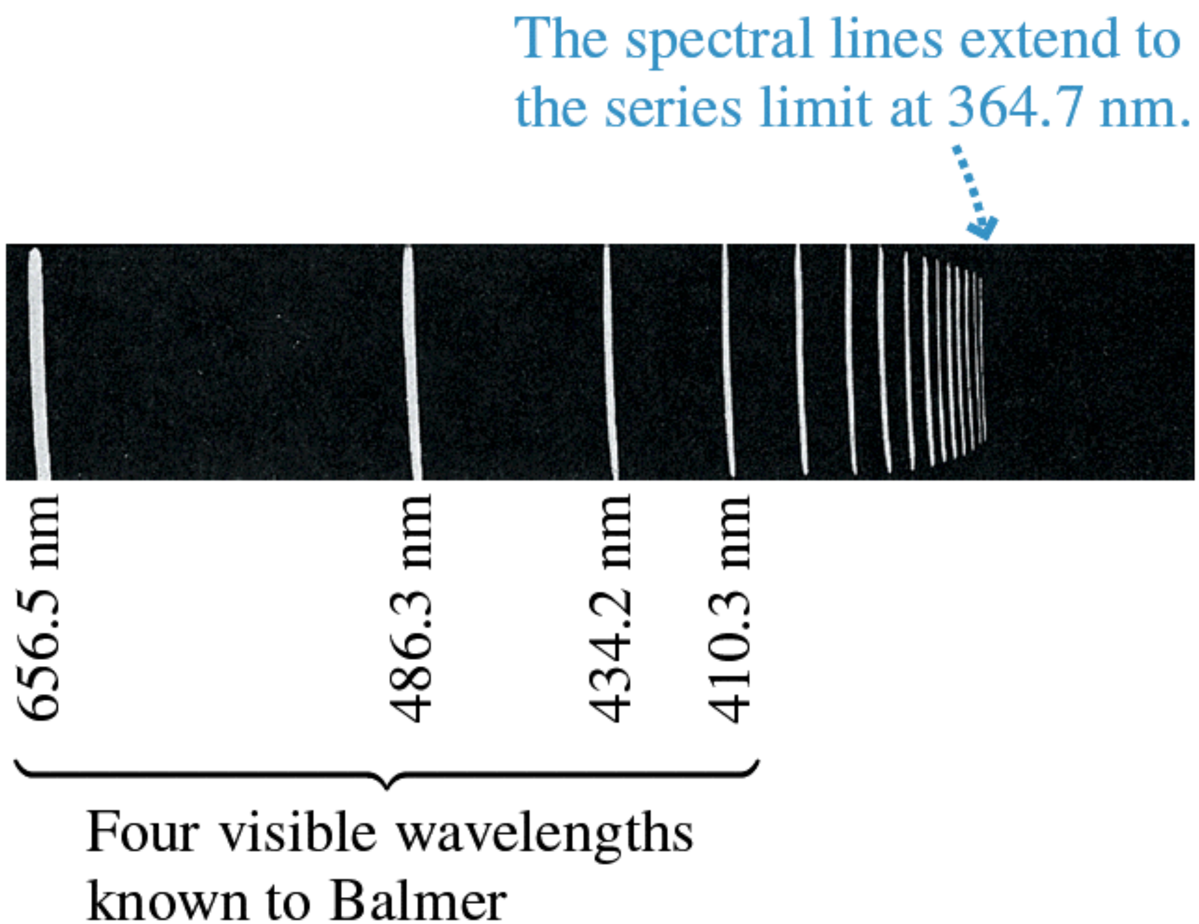
Type of spectrum seen depends on the temperature of the thin gas **relative to** the background. TOP: thin gas is *cooler* so **absorption lines** are seen. BOTTOM: thin gas is *hotter* so **emission lines** are seen.





When the light from luminous gases is dispersed by a glass prism it is found to produce a line spectrum. Above: helium spectrum. Below: neon spectrum.

ATOMIC HYDROGEN SPECTRUM



Balmer - Numerology

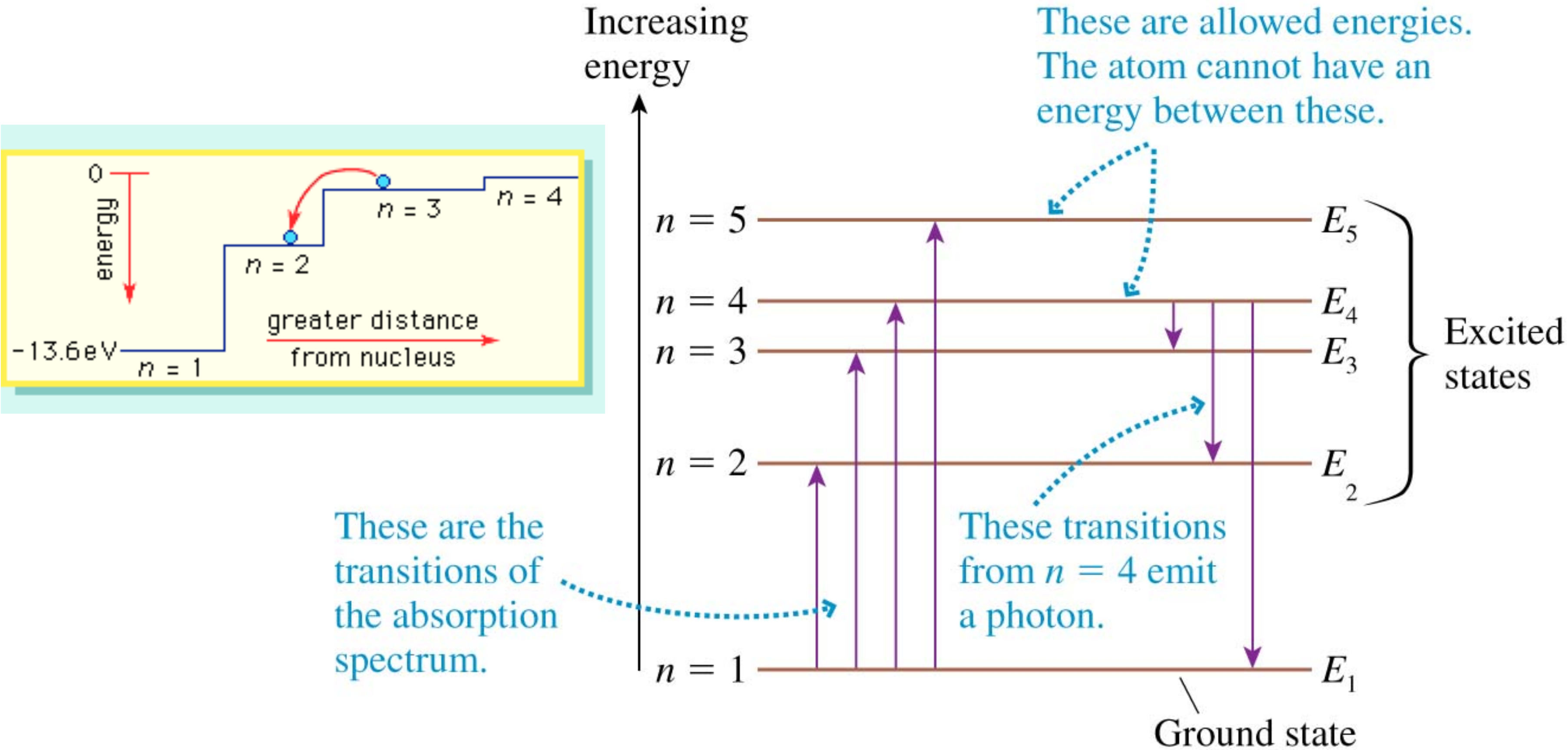
Hydrogen
Spectrum

Balmer math schoolteacher liked to amuse himself by taking four numbers and then finding an equation that described their relationship. While playing with the four numbers he found something very interesting about the number a=3.645 μm
They followed the progression

$9a/5,\ 16a/12,\ 25a/21,\ 36a/32 \longrightarrow \frac{3^2}{3^2-4}a, \frac{4^2}{4^2-4}a, \frac{5^2}{5^2-4}a, \frac{6^2}{6^2-4}a$

$$\lambda = \frac{n^2}{n^2-4}a, (n = 3, 4, 5)$$

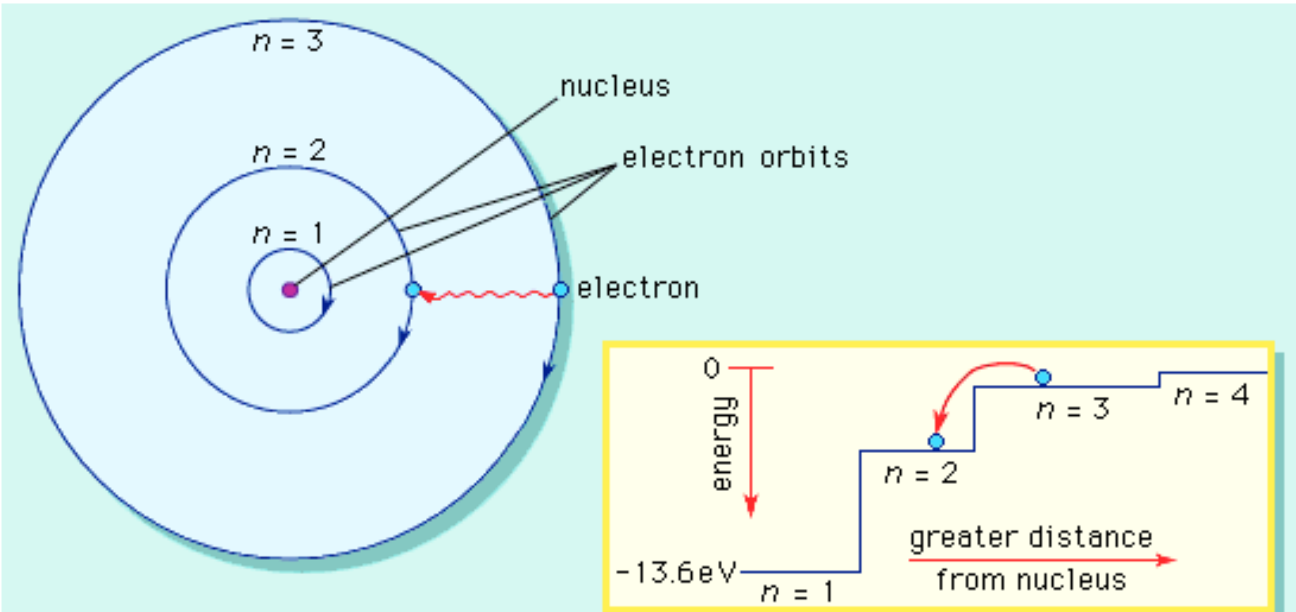
FIGURE 39.18 An energy-level diagram.



$$hf = hc/\lambda = (hc/a)\frac{n^2-4}{n^2} \longrightarrow \frac{3^2}{3^2-4}a, \frac{4^2}{4^2-4}a, \frac{5^2}{5^2-4}a, \frac{6^2}{6^2-4}a$$

$$E_1 = (hc/a)(5/9)$$

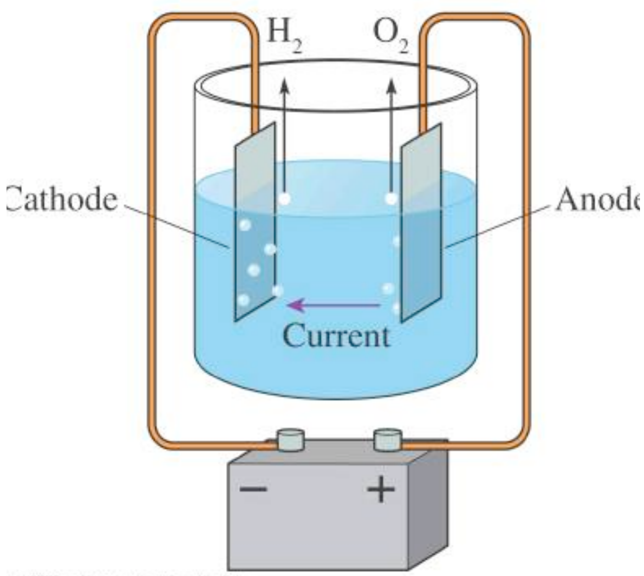
$9a/5$, $16a/12$, $25a/21$, $36a/32$

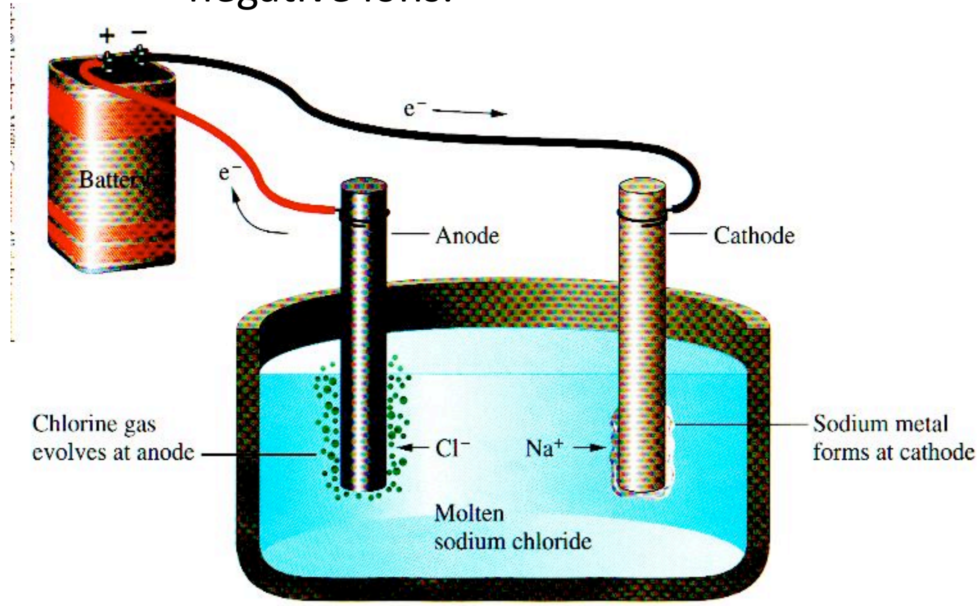

n = 3
n = 2
n = 1
nucleus
electron orbits
electron
0
energy
n = 3
n = 4
n = 2
greater distance
from nucleus
-13.6eV
n = 1

Matter is composed of atoms that are a combination of positive and negative ions

Current through the water decomposes it to Hydrogen and Oxygen –Bubbles of them come out near electrodes

Faraday: Electrolysis can be understood on the basis of atomic theory of matter- Charge associated with each atom or molecule in the solution. Positive and negative ions.

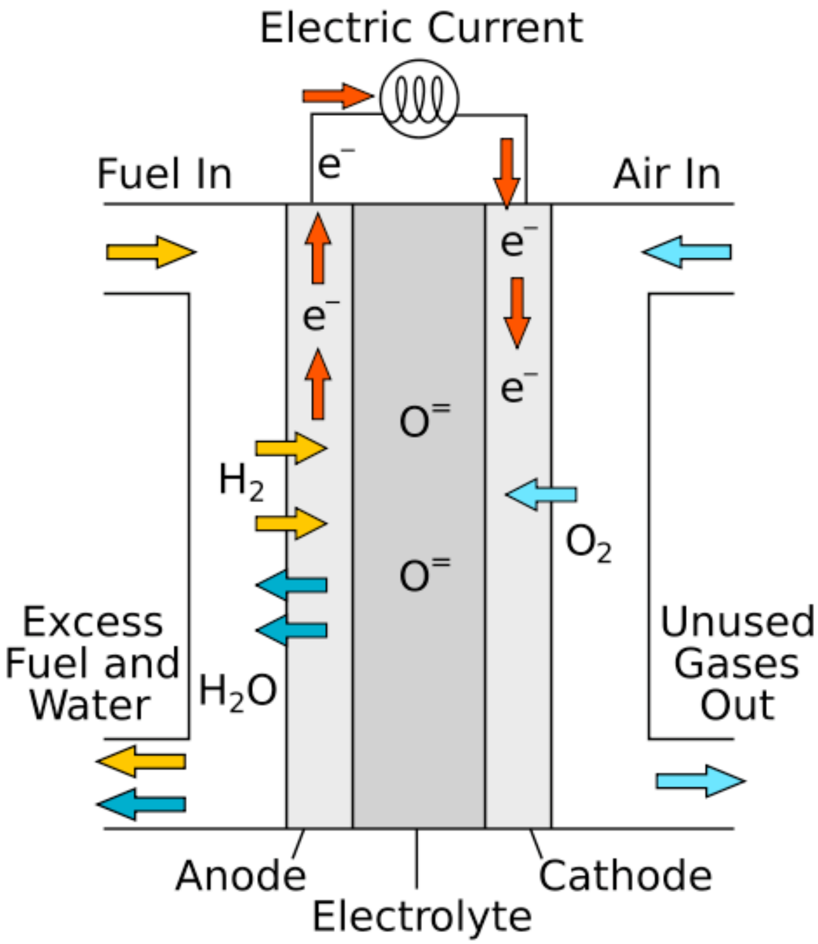


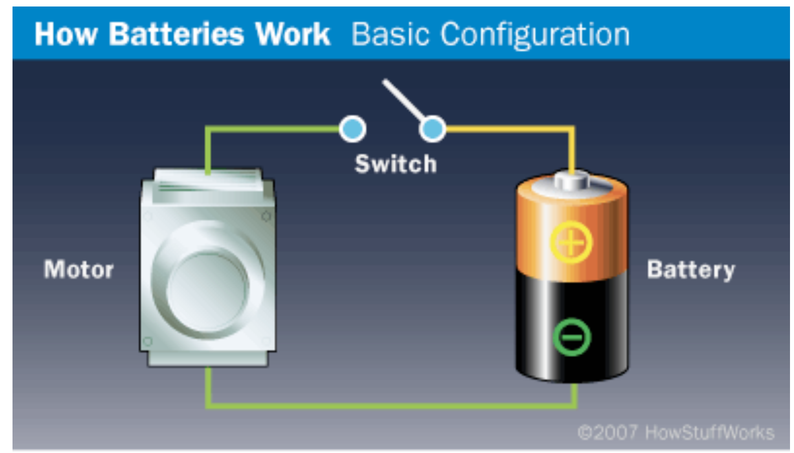


$\boxed{Cl^- \to Cl + e^-}$

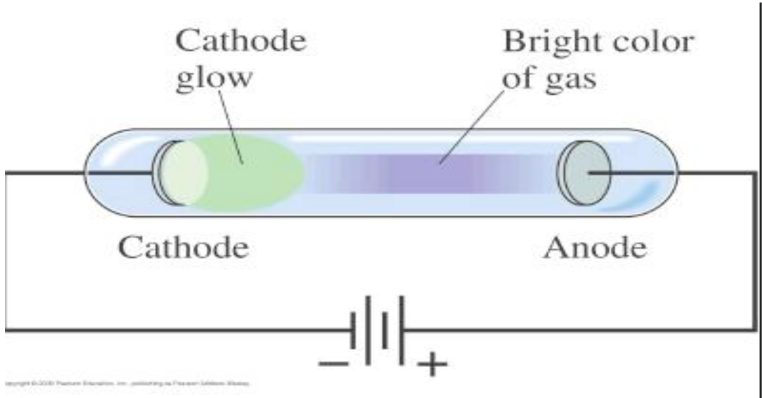
$Na^+ + e^- \to Na(l)$

Battery vs. Fuel Cell





Gaseous Discharges



1. Current flows driving discharge
2. Discharge color depends on gas (N_2, Neon)
3. Cathode glow – Independent of gas

Connection between color of light and type of atoms in the discharge

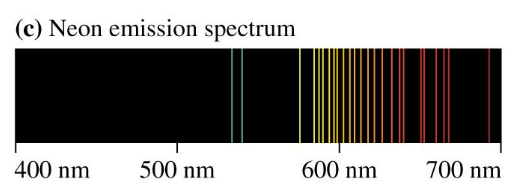
SPECTROSCOPY

Unification of matter-electricity-light

Reduce gas pressure (good pumps)

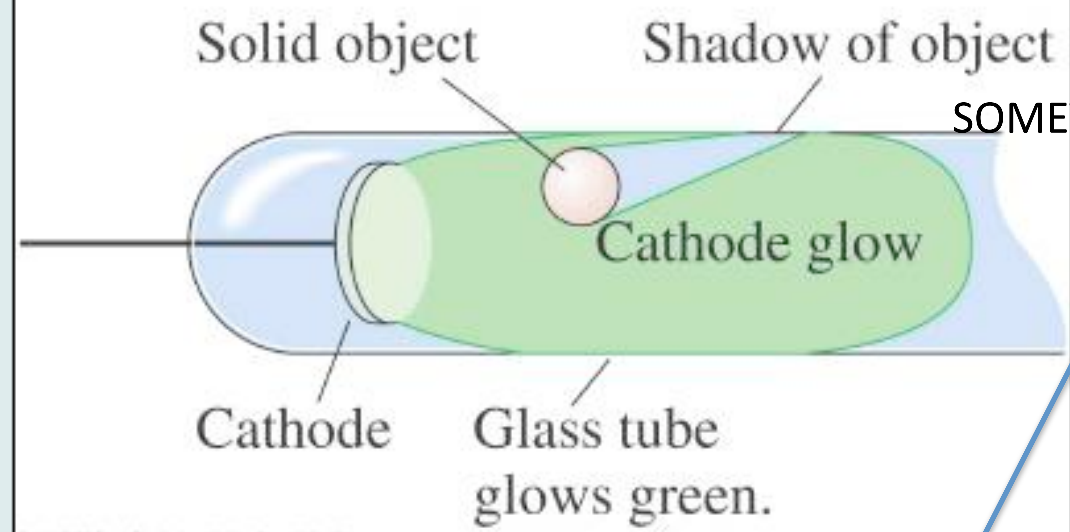
1. Bright colored glow reduces and becomes extinct
2. Cathode glow fills tube
3. Put a metal object creates shadow

NEON EXAMPLE

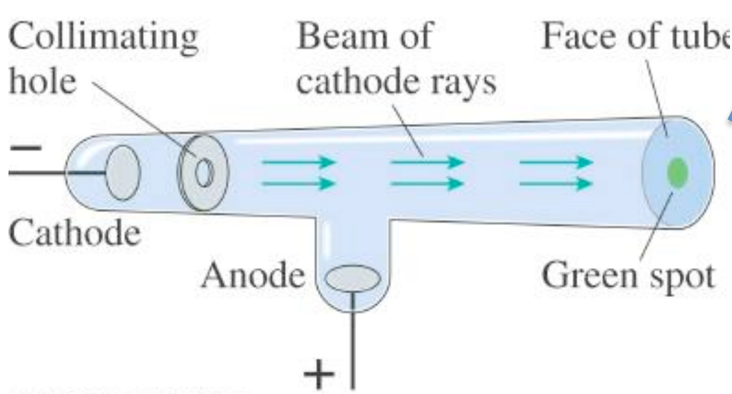


Cathode Rays

IMPROVED PUMPS->LOW PRESSURE-> CATODE GLOW DOMINATES+GLASS EMITS GREENISH GLOW (FLUORESCENCE)



SOMETHING BLOCKING FLOW



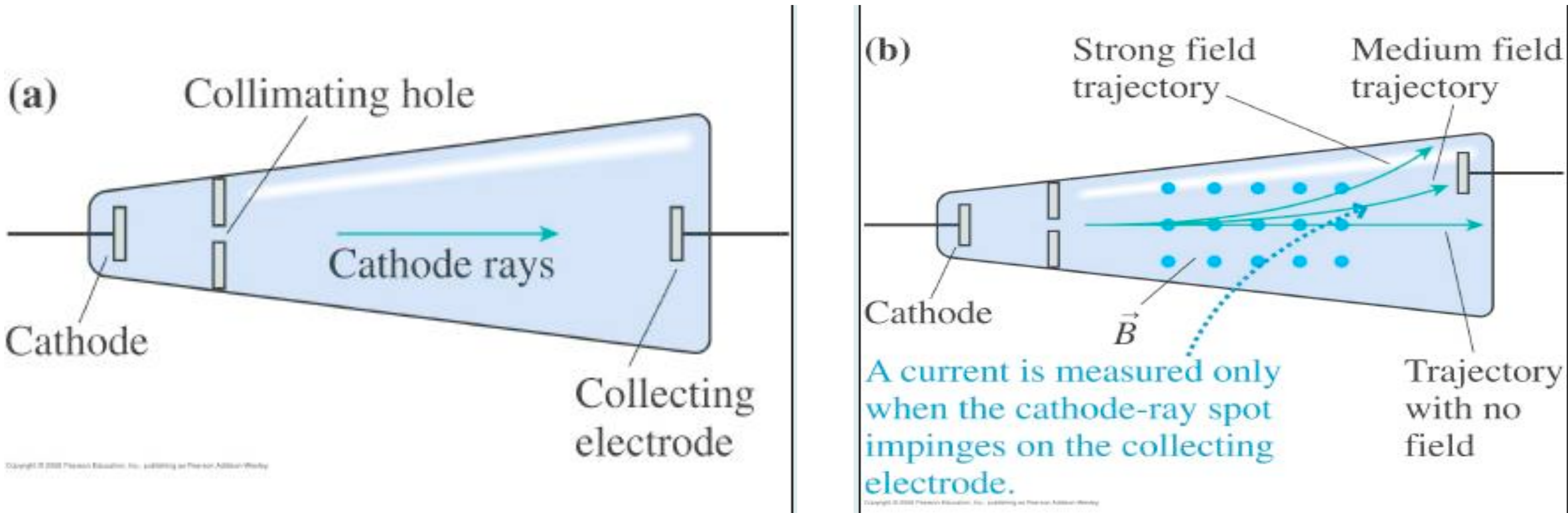
Crooke's Tubes led to

1. Electric current associated with cathode rays
2. The rays are deflected by a magnetic field like a negative charge
3. Cathodes of any metal produce cathode rays. Ray properties independent of metal type.
4. Rays can exert forces on objects. Thin foil gets hot and glows red.

Are Cathode Rays Charged Particles ?

Enter J.J.Thomson – OK for NaCl but what about He (monatomic gas)- Roentgen X-rays

Atoms should be composed of positive and negative parts



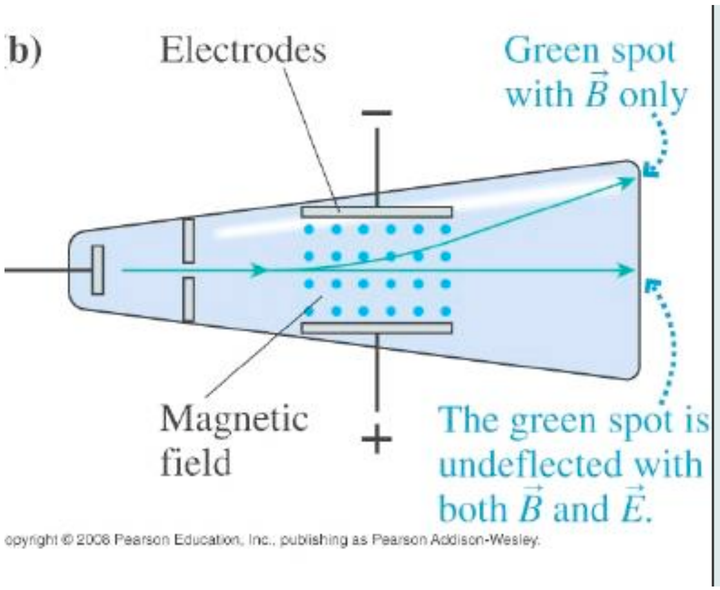
Deflection depends on q/m and v

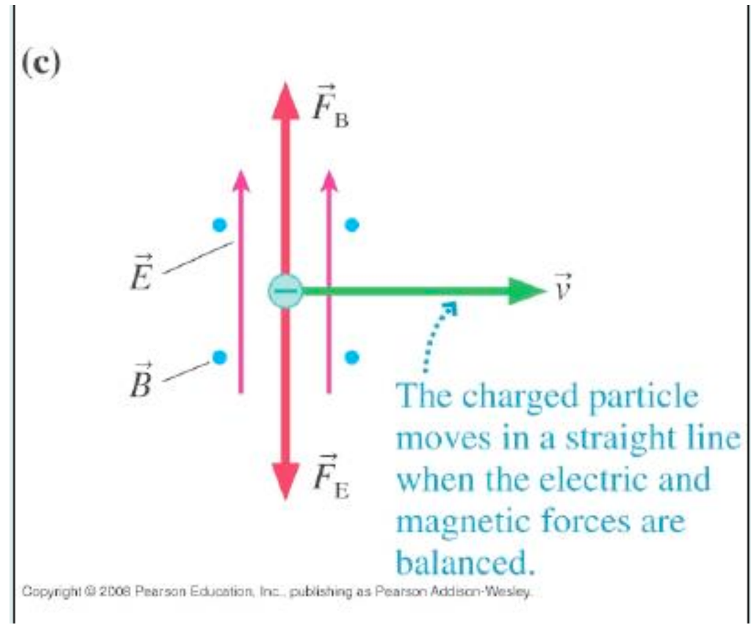
$$\frac{d\vec{v}}{dt} = \frac{q}{m}(\vec{v} \times \vec{B})$$

$$r = mv/qB$$

Confirmed that cathode rays were negatively charged particles but could not measure the q/m

Cross – Field Experiment





$$\frac{d\vec{v}}{dt} = \frac{q}{m}(\vec{E} + \vec{v} \times \vec{B})$$

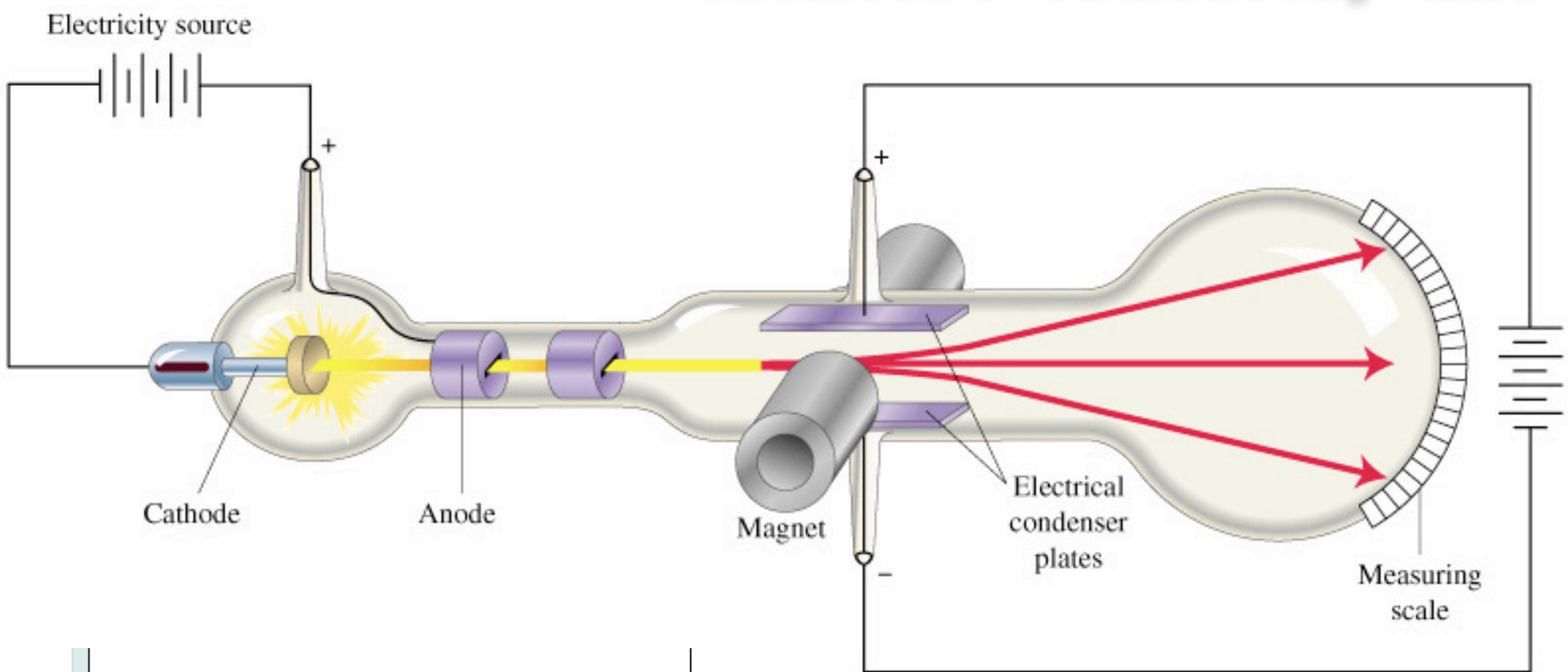
$$\frac{d\vec{v}}{dt} = 0$$

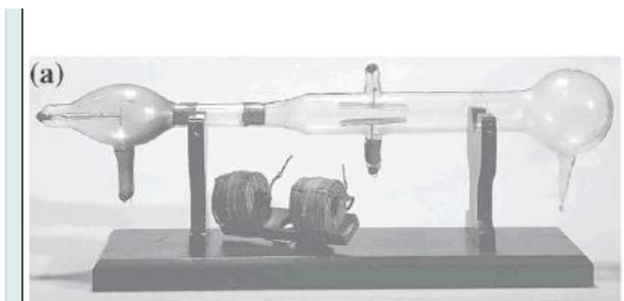
$$v_x = E / B$$

$$r = mv / qB = (m / q)(E / B^2)$$

Thomson found q/m=10^{11}Cb/kg
For Hydrogen q/m=10^8 Cb/kg
Cathode ray particle has either much larger charge or much smaller mass.

Thomson's "cathode ray" tube





FOUND THAT NEGATIVE PARTICLES EMITTED FROM METALS HAVE THE SAME q/m.

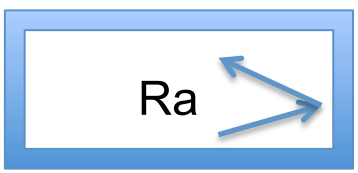
-> ELECTRONS ARE CONSTITUENT OF ALL MATTER

Enter Rutheford

Using Thomson's technique measured radioactive rays emitted from U samples

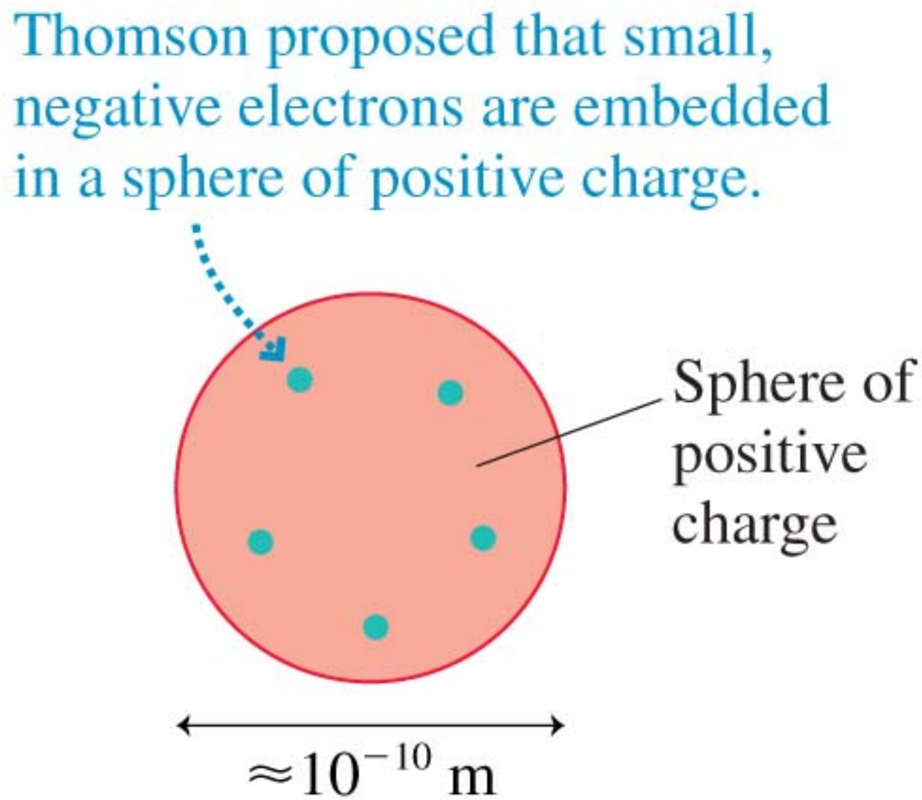
- as beta rays ($q/m = -e/m_e$ similar to cathode rays – electrons) penetrate .1 inch of metal
- as alpha rays with $q/m = e/2m_H$.

Alpha ray value indicates either singly charged H_2 ($q=e$ $m=2m_H$) or doubly ionized He atom ($q=2e$, $m=4m_H$



A few days later created a discharge and looked at the spectrum. He found He but not H

FIGURE 38.10 Thomson's raisin-cake model of the atom.

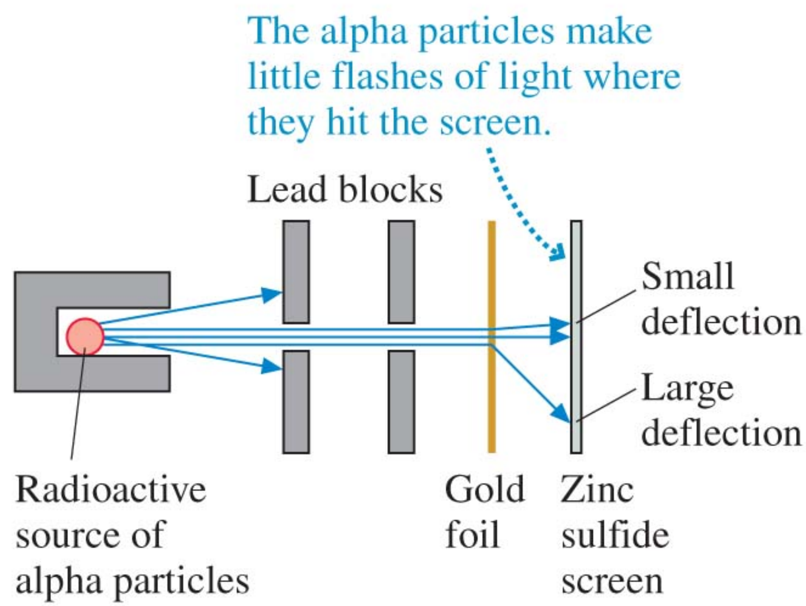


Alpha particles nuclei of He

Rutherford and the Discovery of the Nucleus

- In 1896 Rutherford's experiment was set up to see if any alpha particles were deflected from gold foil at *large* angles.
- Not only were alpha particles deflected at large angles, but a very few were reflected almost straight backward toward the source!

FIGURE 38.11 Rutherford's experiment to shoot high-speed alpha particles through a thin gold foil.



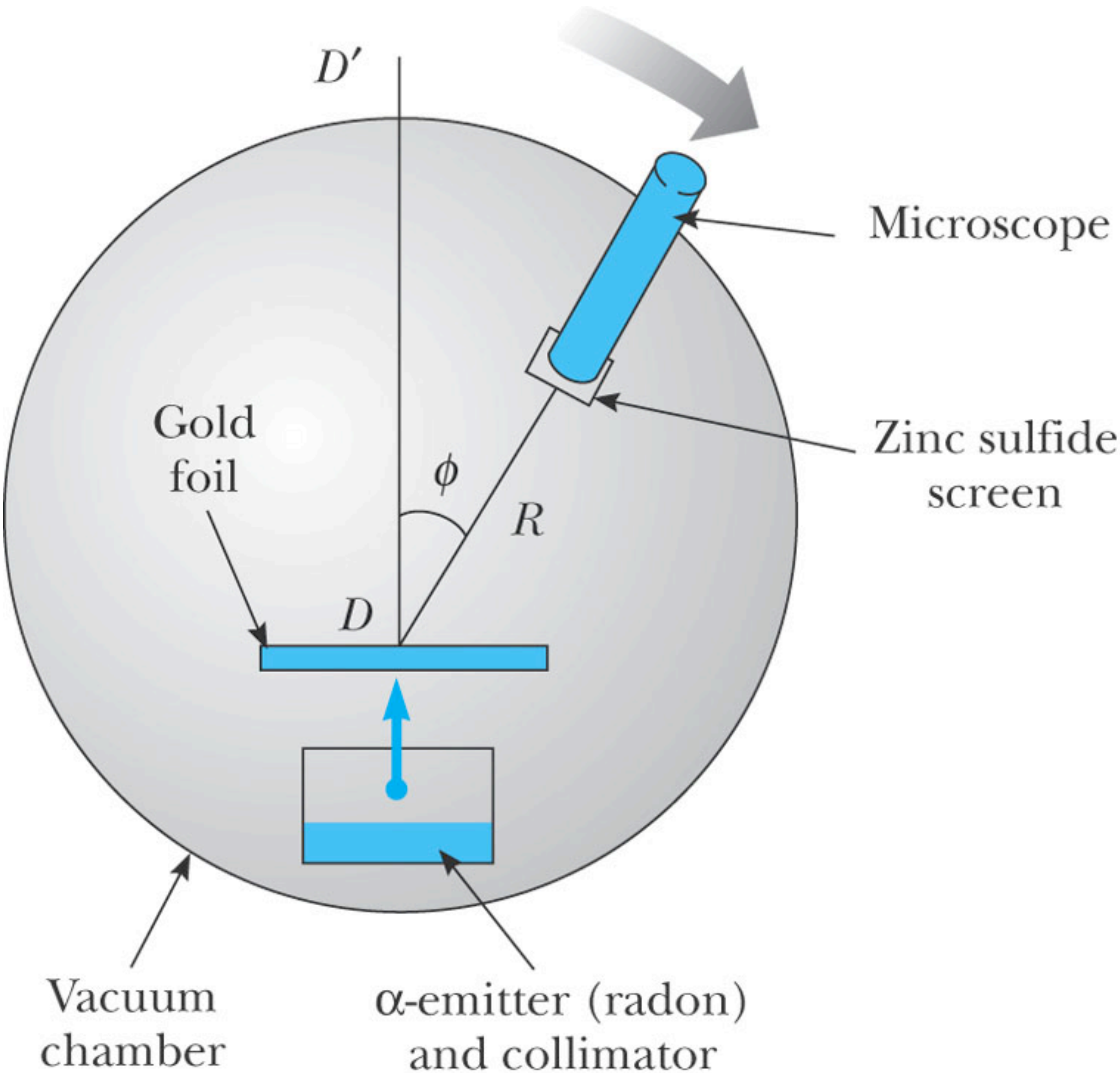
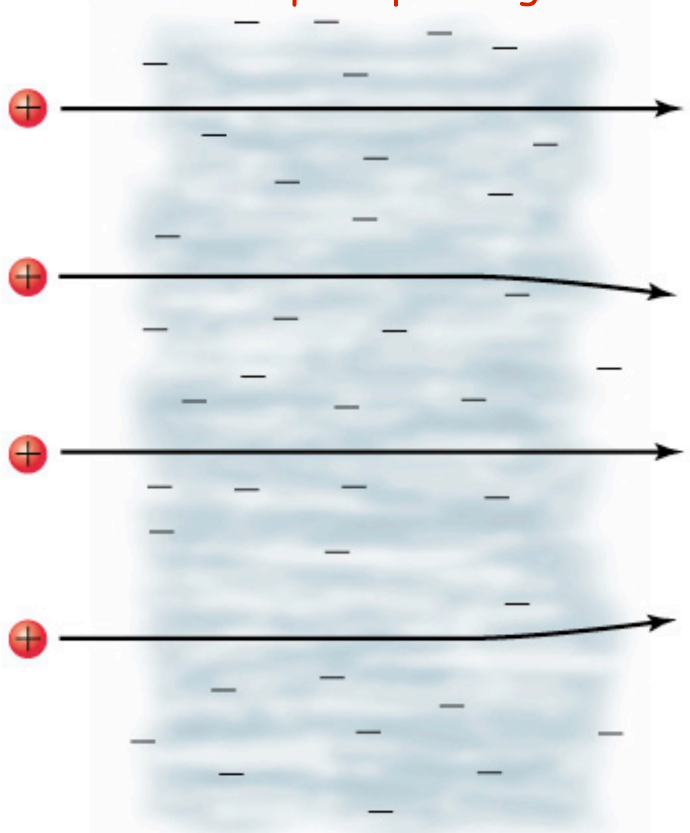

D′
Microscope
Gold
foil
ϕ
R
Zinc sulfide
screen
D
Vacuum
chamber
α-emitter (radon)
and collimator

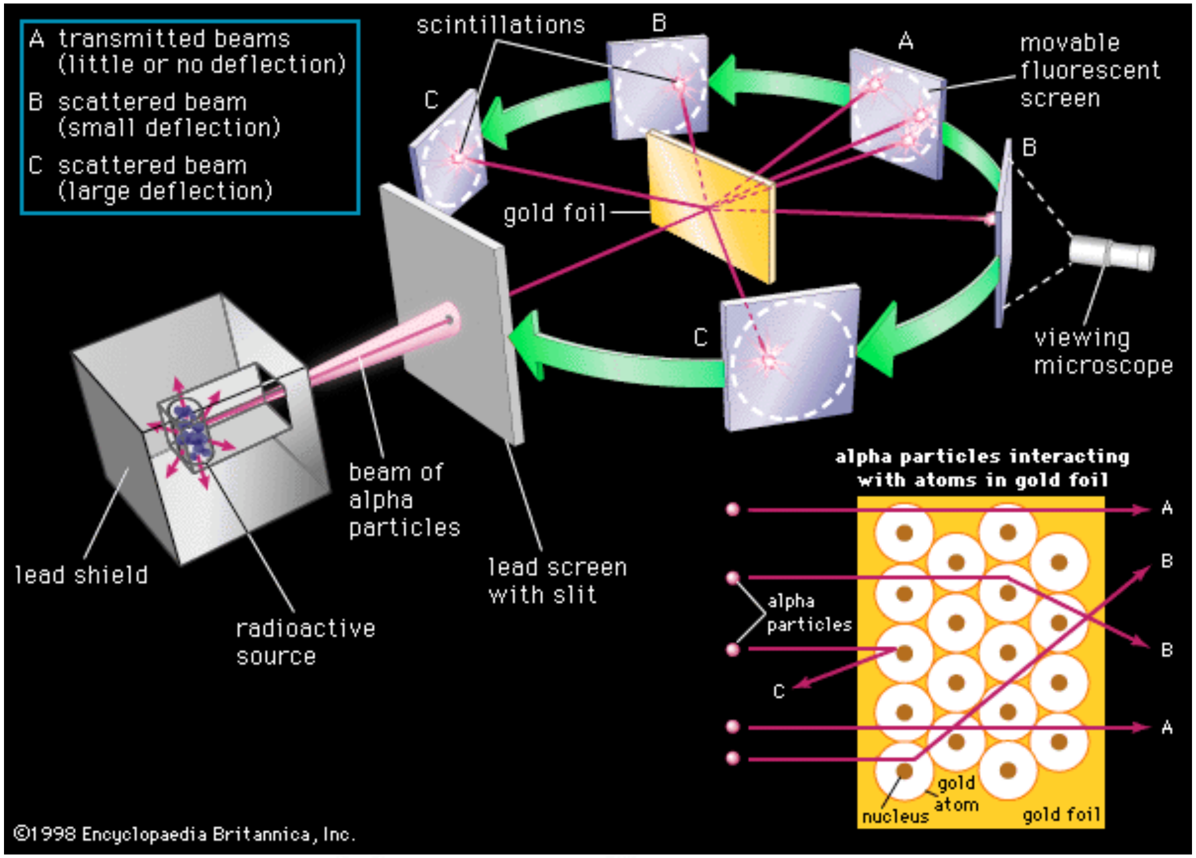
Fig. 4-10, p. 120

The discovery of the atomic nucleus: Rutherford Back Scattering

what Rutherford expected
from the "plum pudding" model



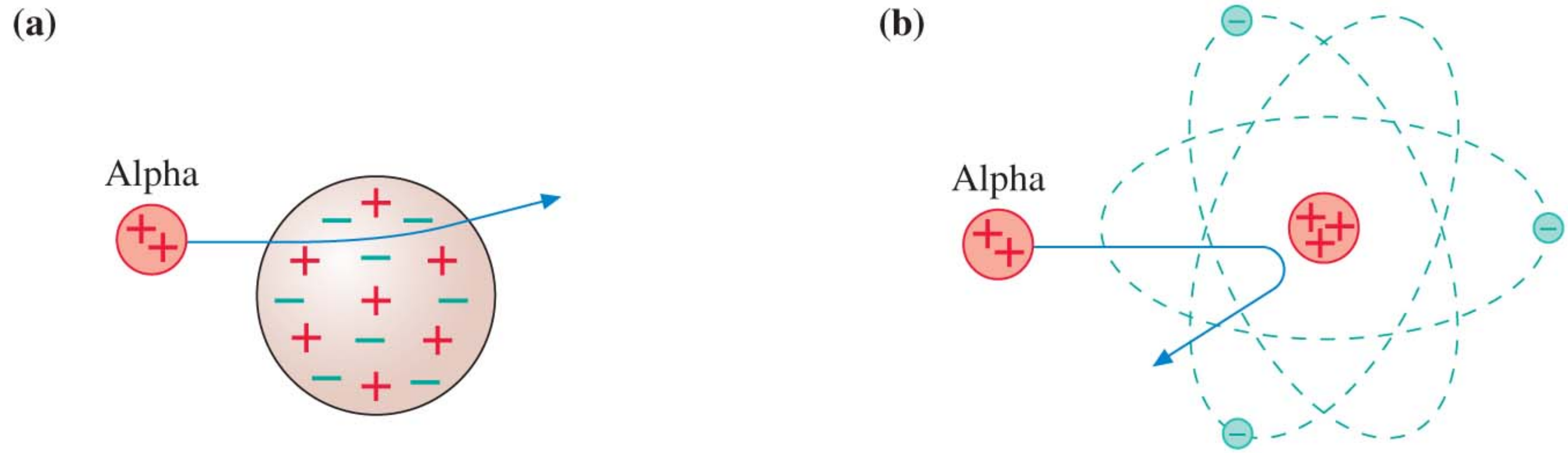
the clever experiment



the (surprise!) result

"It was almost as incredible as if you had fired a 15-inch shell at a piece of tissue paper and it came back and hit you."

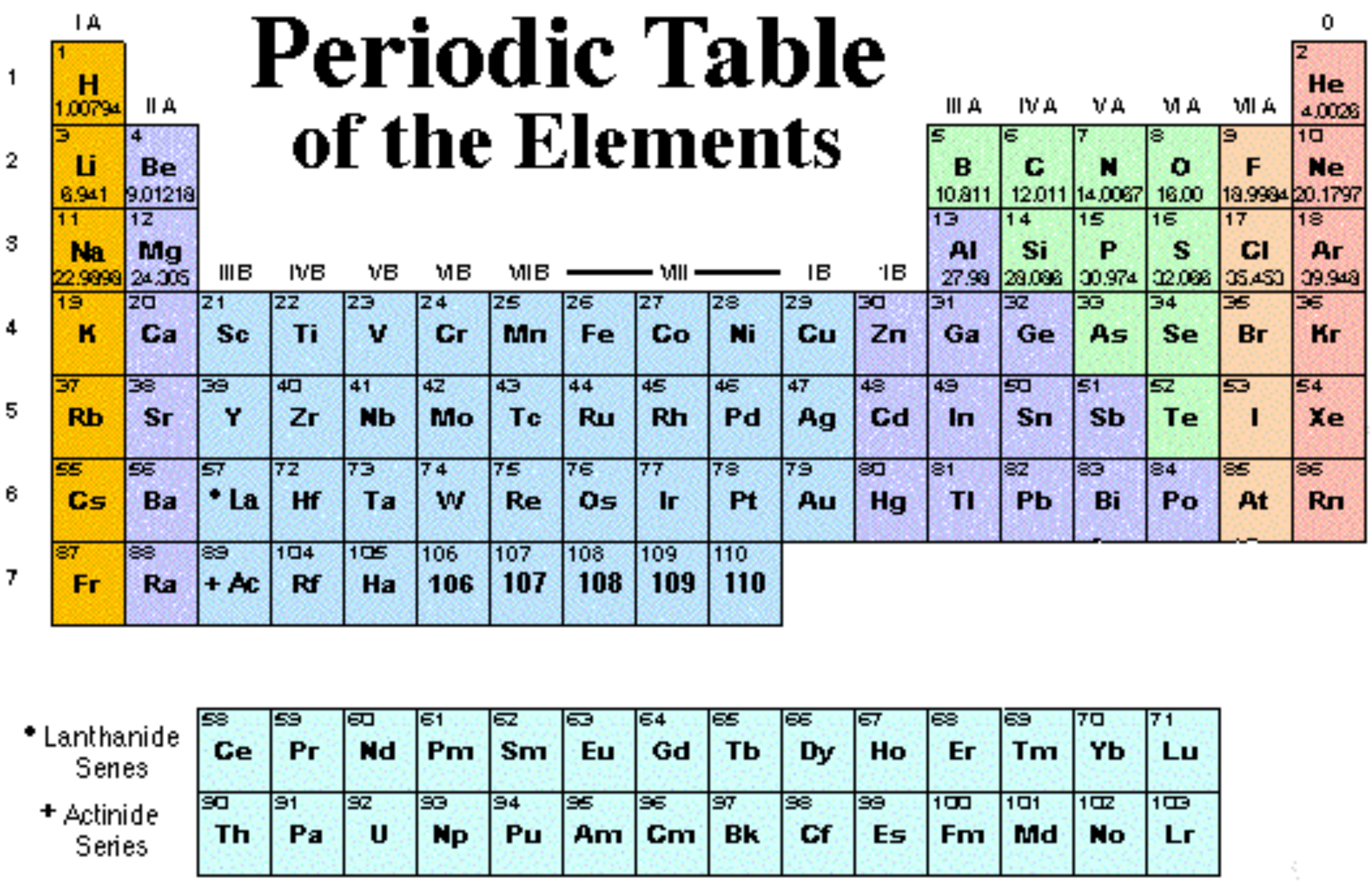
FIGURE 38.12 Alpha particles interact differently with a concentrated positive nucleus than they would with the spread-out charge in Thomson's model.



(a) The alpha particle is only slightly deflected by a Thomson atom because forces from the spread-out positive and negative charges nearly cancel.

(b) If the atom has a concentrated positive nucleus, some alpha particles will be able to come very close to the nucleus and thus feel a very strong repulsive force.

Atomic number Z number of electrons and number of positive charges nucleus

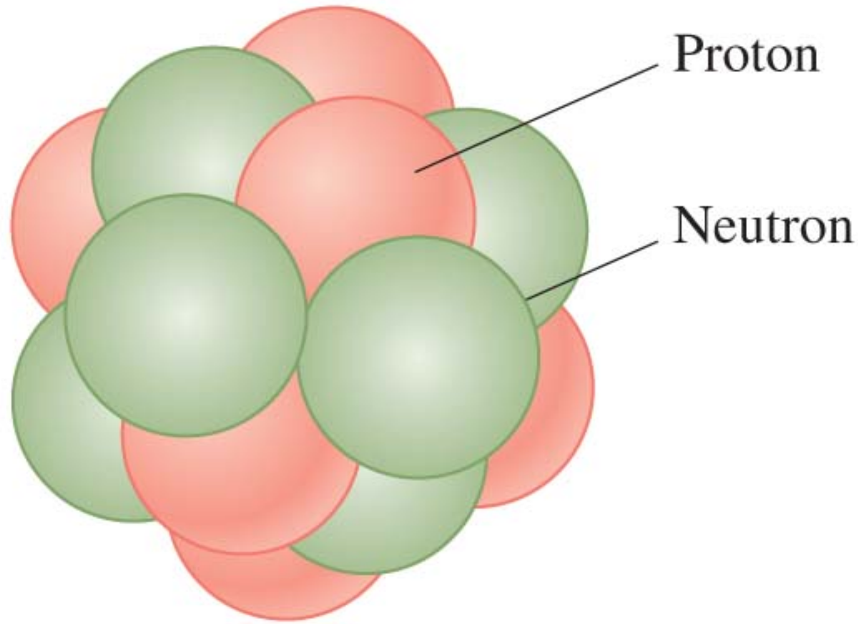


Puzzle atom mass more than Zm_p -> neutron

Into the Nucleus

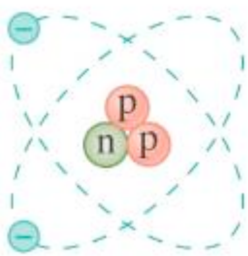
- The **atomic number** Z of an element describes the number of protons in the nucleus. Elements are listed in the periodic table by their atomic number.
- There are a *range* of neutron numbers N that happily form a nucleus with Z protons, creating a series of nuclei having the same Z-value but different masses. Such a series of nuclei are called **isotopes.**
- An atom's **mass number** A is defined to be $A = Z + N$. It is the total number of protons and neutrons in a nucleus.
- The notation used to label isotopes is ^{A}Z, where the mass number A is given as a *leading* superscript. The proton number Z is not specified by an actual number but, equivalently, by the chemical symbol for that element.

FIGURE 38.17 The nucleus of an atom contains protons and neutrons.

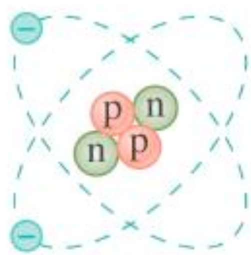


Chemical properties controlled by Z.

Isotopes



^{3}He
$Z = 2 \quad N = 1$
$A = 3$
0.0001% abundance



^{4}He
$Z = 2 \quad N = 2$
$A = 4$
99.9999% abundance

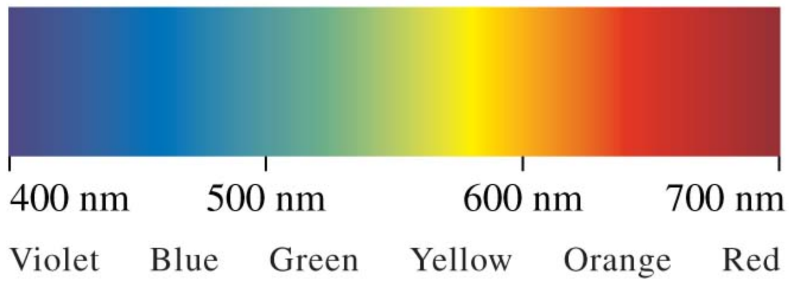
$${}^{4}_{2}He$$

The Emission of Light

Hot, self-luminous objects, such as the sun or an incandescent lightbulb, form a rainbow-like **continuous spectrum** in which light is emitted at every possible wavelength. The figure shows a continuous spectrum.

FIGURE 38.19 A grating spectrometer is used to study the emission of light.

(b) Incandescent lightbulb

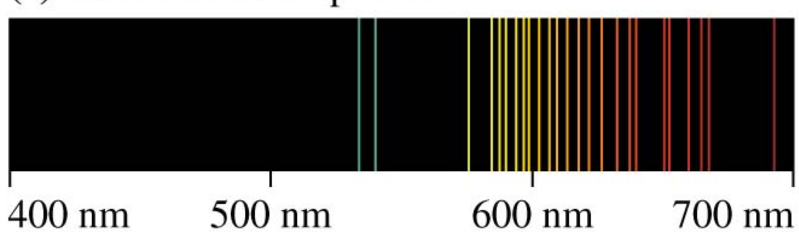


The Emission of Light

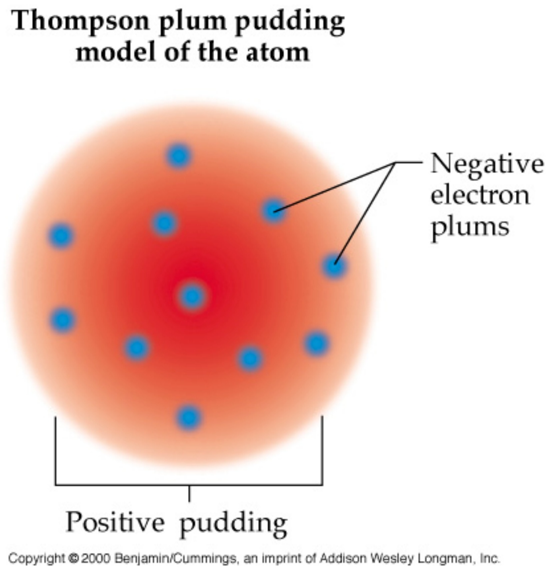
The light emitted by one of Faraday's gas discharge tubes contains only certain discrete, individual wavelengths. Such a spectrum is called a **discrete spectrum.** Each wavelength in a discrete spectrum is called a **spectral line** because of its appearance in photographs such as the one shown.

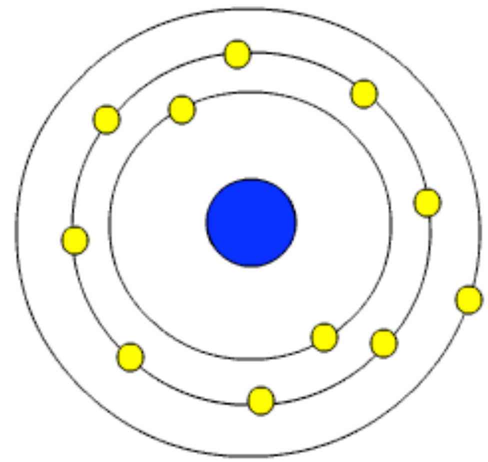
FIGURE 38.19 A grating spectrometer is used to study the emission of light.

(c) Neon emission spectrum



Building Blocks of the Atom

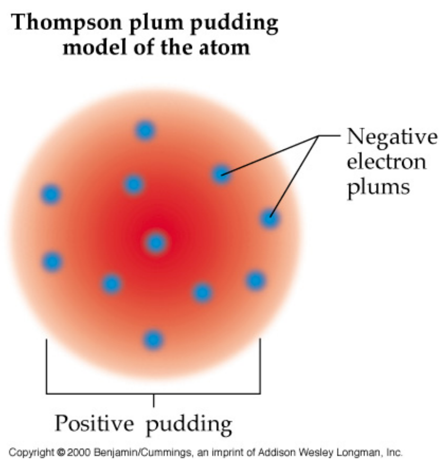


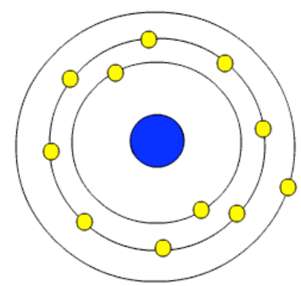


IF MATTER COMPOSED OF ATOMS WHAT ATOMS ARE COMPOSED OF?

- **FARADAY –ELECTROLYSIS**
- **THOMSON – ELECTRONS q/m**
- **MILLIKAN – ELECTRONIC q**
- **RUTHERFORD – NUCLEAR MODEL**

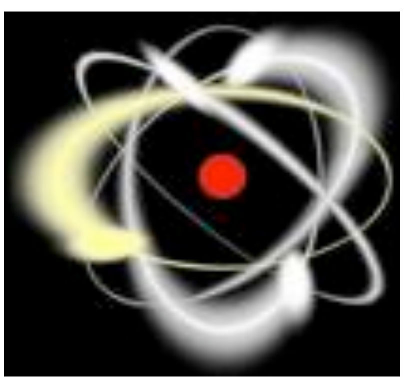
Evolution of our views of the atom

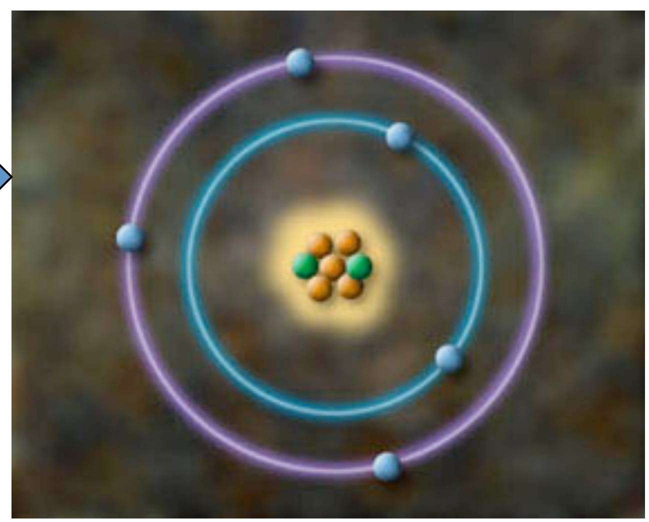


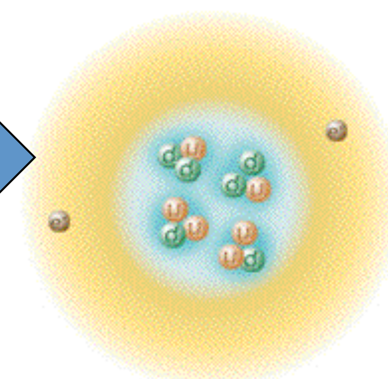


•positive charge is concentrated at the center of the atom in an area ~1/1000th the size of the atom

•the mass of the electron is very small compared to the mass of the atom (one thousand times less than the hydrogen atom



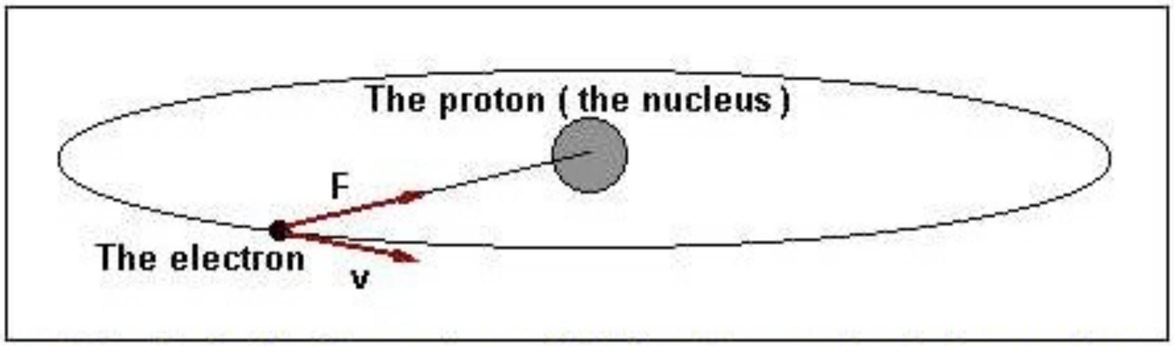




The Planetary Model

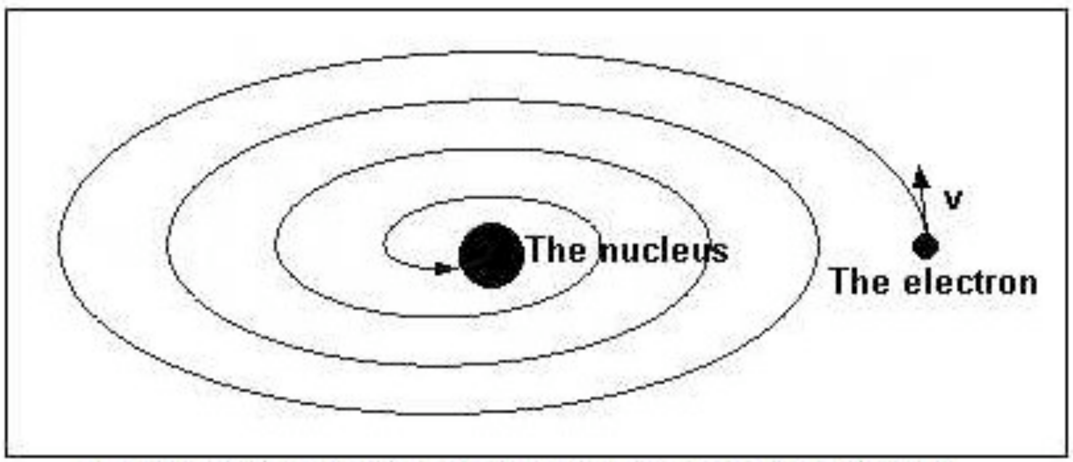
The attractive Coulomb force between the positive nucleus and the orbiting electron could provide the attractive force which keeps the electron in it's orbit, much as the planets orbit the sun with gravity providing the centripetal force.

$$F_{centripetal} = \frac{m_e v_0^2}{r} = m_e \omega_0^2 r$$



The Rutherford's atomic model. The electron circulating on the orbit around the nucleus with the velocity v is attracted by it with the force F

What's wrong with this picture?



In the planetary model of atom, the electron should emit energy and spirally fall on the nucleus.

Accelerating charges radiate. Could this electromagnetic radiation be the source of the spectral lines?

No. This radiation must come at the expense of the kinetic energy of the orbiting electron!

It will eventually spiral into the nucleus. The atom would be unstable!

Bohr Atom

$$\vec{F}_{cb} = k\frac{q_1 q_2}{r^2}\hat{r}$$

$$-k\frac{e^2}{r^2} + m\frac{v_\theta^2}{r} = 0$$

$$r = \frac{ke^2}{mv_\theta^2}$$

$$\hbar \equiv h/2\pi$$

$$mv_\theta r = n\hbar, n = 1,2,3,4\ldots$$

$$v_\theta = n\hbar/mr$$

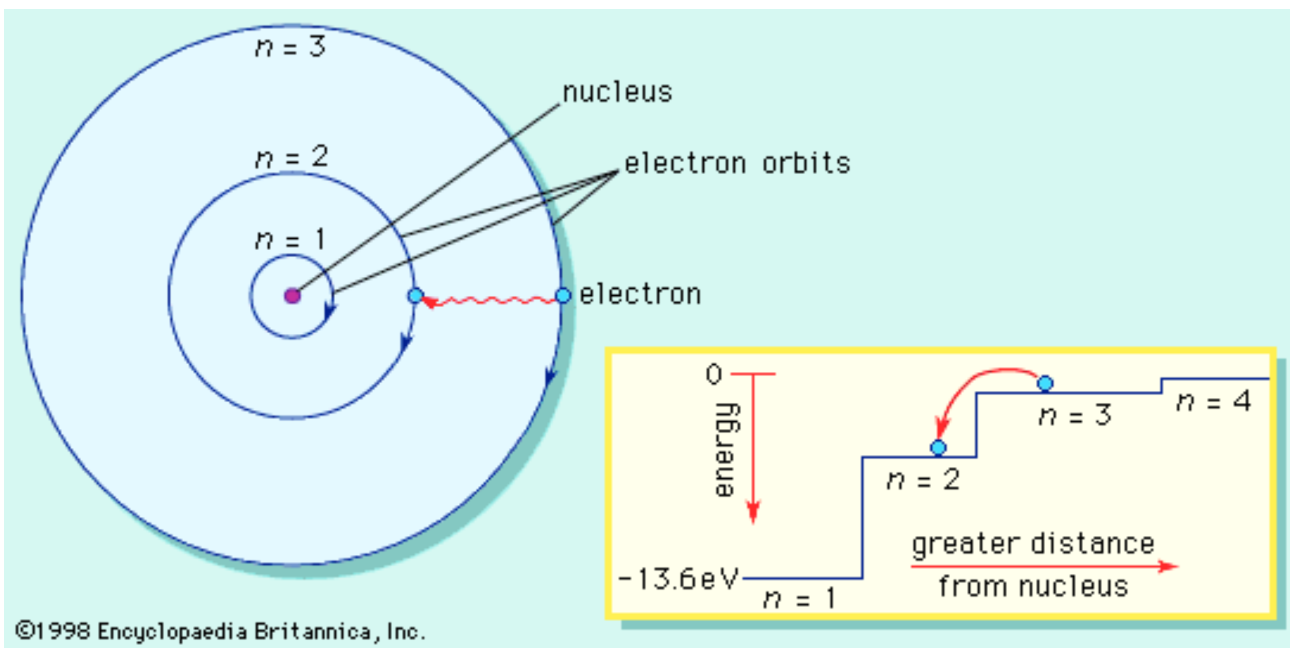
$$r = n^2(\hbar^2/kme^2)$$

$r=n^2a_B$,

$a_B= (h/2\pi)^2/(ke^2m)=.0529$ nm

$E_n=-E_1/n^2$

$E_1=13.6$ eV



The Spectrum of Hydrogen

- Hydrogen is the simplest atom, with one electron orbiting a proton, and it also has the simplest atomic spectrum.
- The emission lines have wavelengths which correspond to two integers, *m* and *n*.
- Every line in the hydrogen spectrum has a waveler

$$\lambda = \frac{91.18\ \text{nm}}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)} \qquad \begin{cases} m = 1 & \text{Lyman series} \\ m = 2 & \text{Balmer series} \\ m = 3 & \text{Paschen series} \\ \quad\vdots & \end{cases}$$

$$n = m + 1, m + 2, \ldots$$

Bohr's Model of Atomic Quantization

1. An atom consists of negative electrons orbiting a very small positive nucleus.
2. Atoms can exist only in certain **stationary states.** Each stationary state corresponds to a particular set of electron orbits around the nucleus. These states can be numbered 2, 3, 4, . . . , where n is the *quantum number.*
3. Each stationary state has an energy E_n. The stationary states of an atom are numbered in order of increasing energy: $E_1 < E_2 < E_3 < \ldots$
4. The lowest energy state of the atom E_1 is *stable* and can persist indefinitely. It is called the **ground state** of the atom. Other stationary states with energies E_2, E_3, E_4,. . . are called **excited states** of the atom.

Bohr's Model of Atomic Quantization

5. An atom can "jump" from one stationary state to another by emitting or absorbing a photon of frequency

$$f_{\text{photon}} = \frac{\Delta E_{\text{atom}}}{h}$$

where h is Planck's constant and and $\Delta E_{\text{atom}} = |E_f - E_i|$.

E_f and E_i are the energies of the initial and final states. Such a jump is called a **transition** or, sometimes, a **quantum jump.**

Bohr's Model of Atomic Quantization

6. An atom can move from a lower energy state to a higher energy state by absorbing energy $\Delta E_{\text{atom}} = E_f - E_i$ in an inelastic collision with an electron or another atom.

This process, called **collisional excitation,** is shown.

FIGURE 39.17 An atom can change stationary states by emitting or absorbing a photon or by undergoing a collision.

(b) Collisional excitation

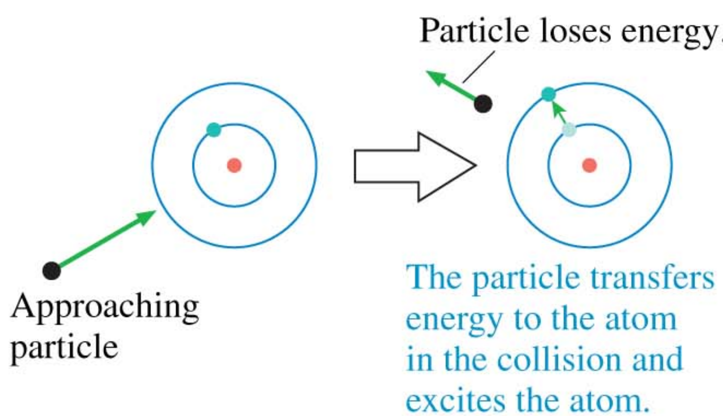
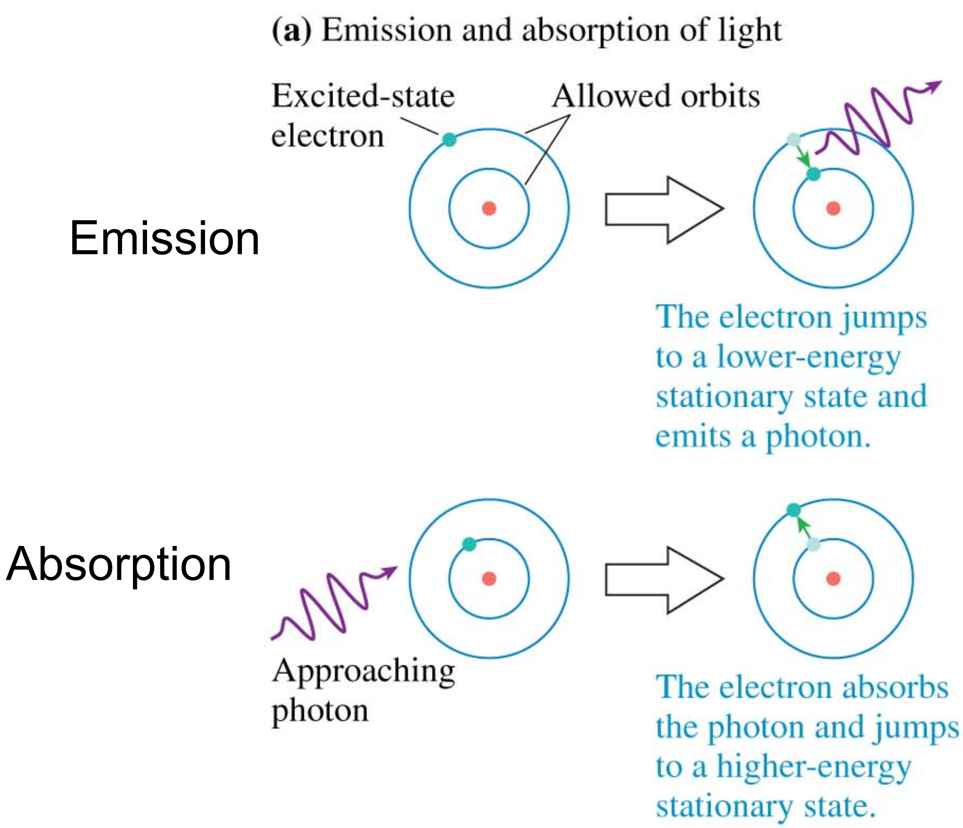
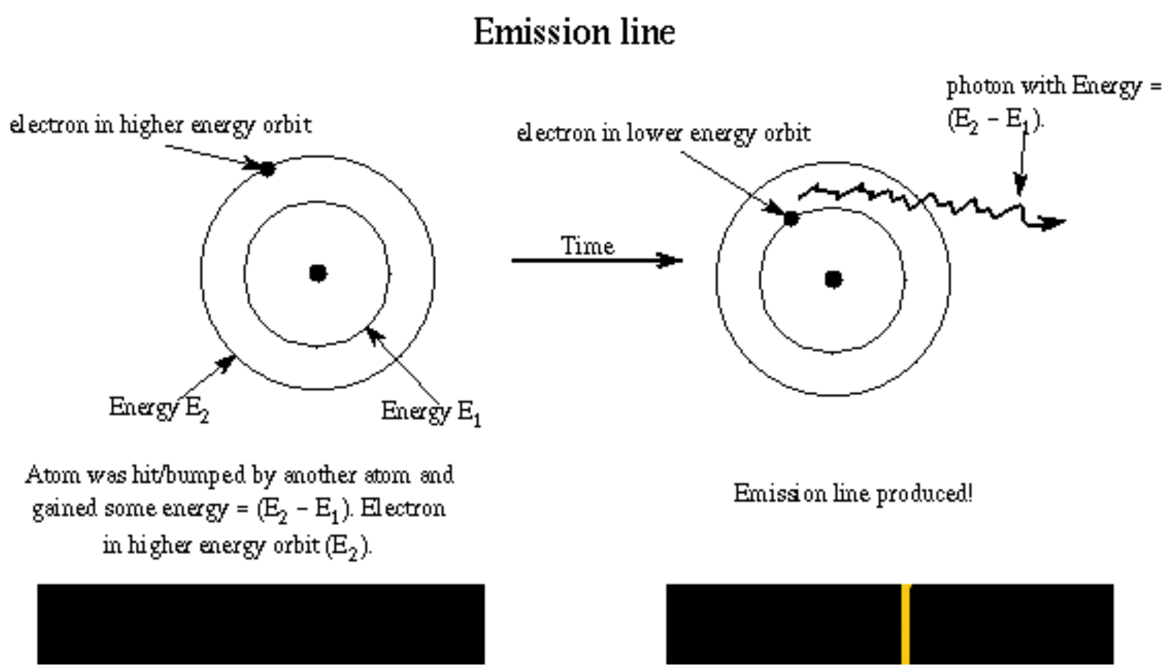


FIGURE 39.17 An atom can change stationary states by emitting or absorbing a photon or by undergoing a collision.



Emission



Emission line
electron in higher energy orbit
Energy E2
Energy E1
Atom was hit/bumped by another atom and gained some energy = (E2 – E1). Electron in higher energy orbit (E2).
Time
electron in lower energy orbit
photon with Energy = (E2 – E1).
Emission line produced!

Line Spectra

To explain discrete spectra, Bohr found that atoms obey three basic rules:

1. Electrons have only certain energies corresponding to particular distances from nucleus. As long as the electron is in one of those energy orbits, it will not lose or absorb any energy. The energy orbits are analogous to rungs on a ladder: electrons can be only on rungs of the ladder and not in between rungs.

2. The orbits closer to the nucleus have lower energy.

3. Atoms want to be in the lowest possible energy state called the **ground state** (all electrons as close to the nucleus as possible).

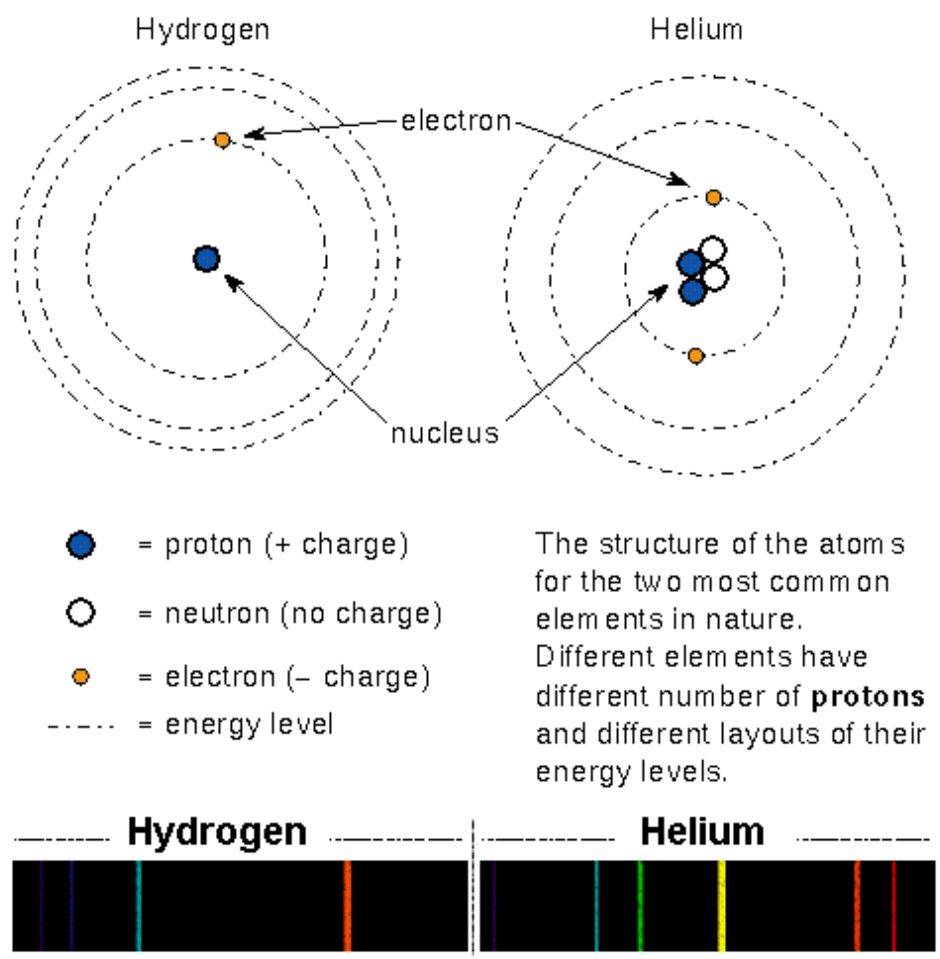
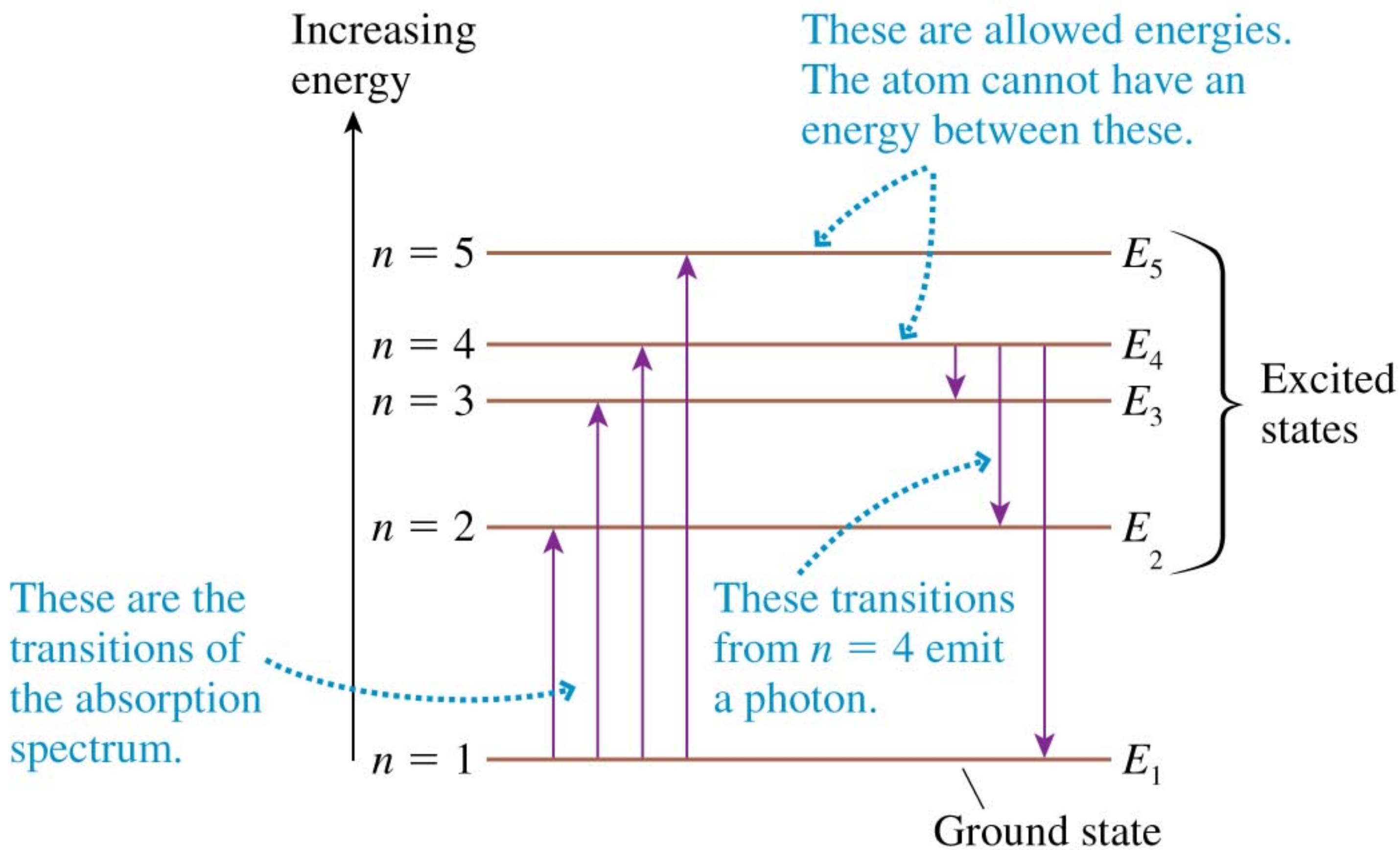
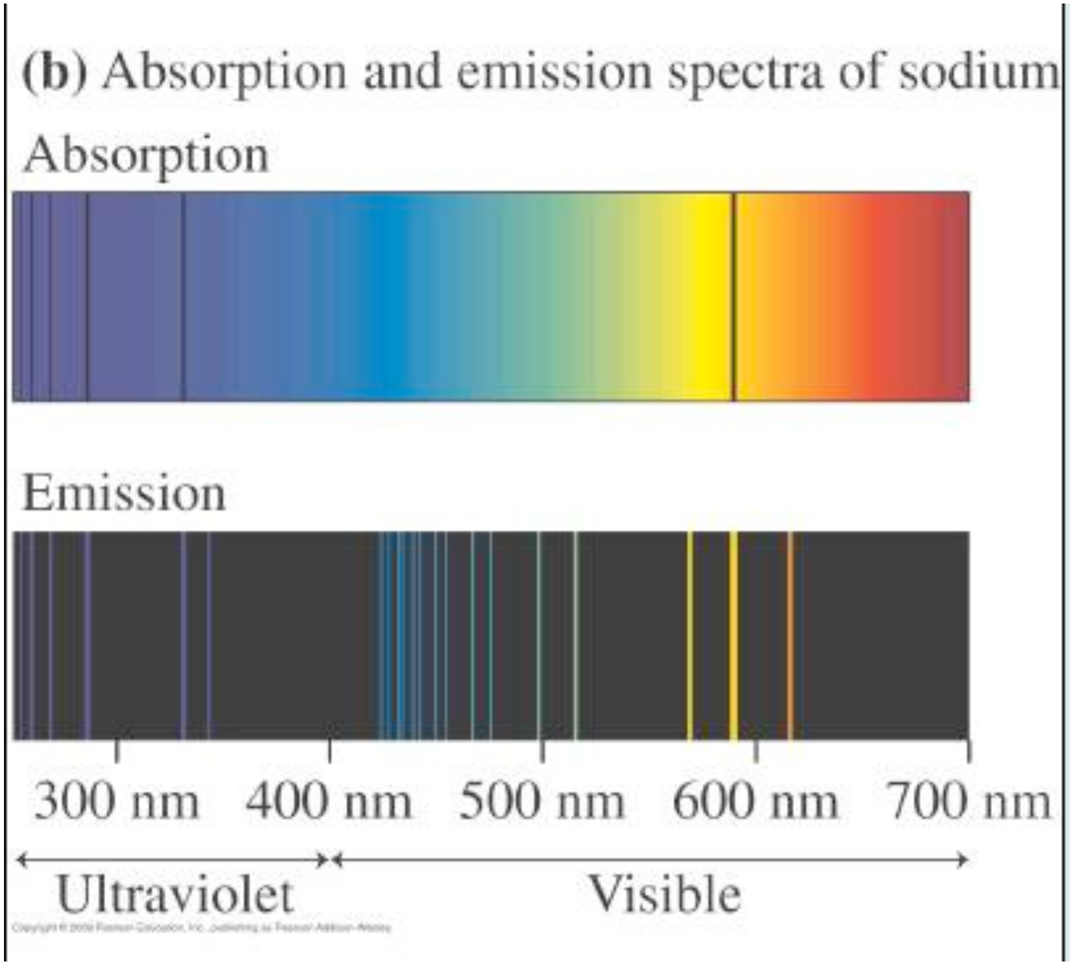


FIGURE 39.18 An energy-level diagram.



ABSORPTION AND EMISSION SPECTRA



Notice that there are fewer absorption than emission lines

Matter Waves and Energy Quantization

In 1924 de Broglie postulated that *if* a material particle of momentum $p = mv$ has a wave-like nature, then its wavelength must be given by

Photons

$$E = hf = pc$$

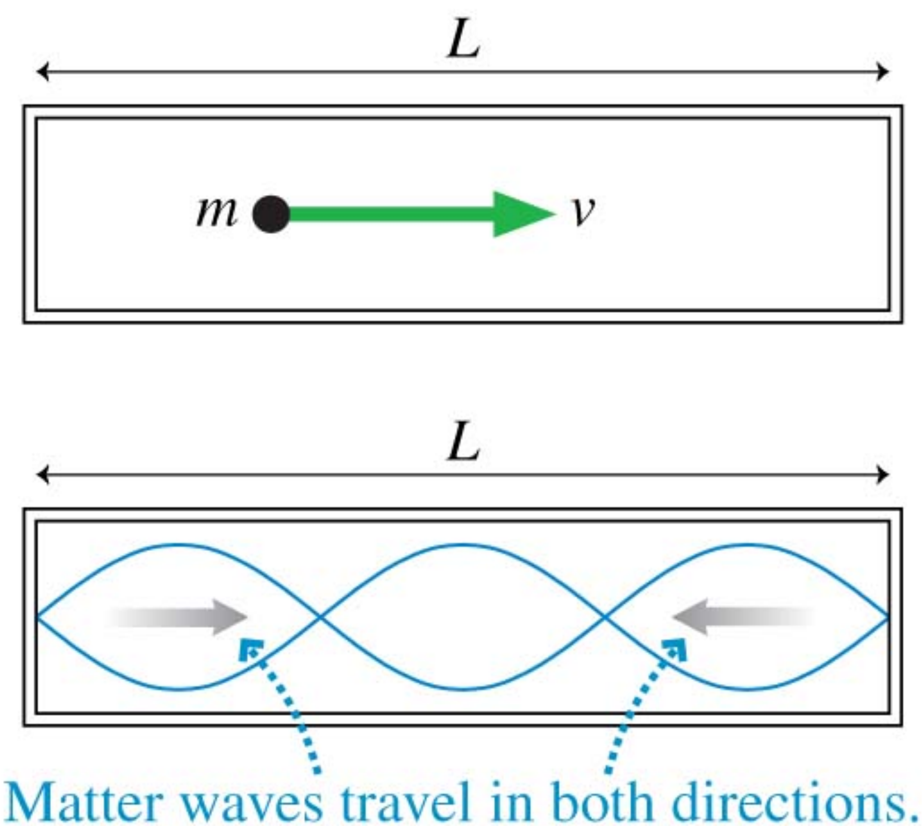
$$\lambda = c/f = h/p$$

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

where h is Planck's constant ($h = 6.63 \times 10^{-34}$ J s). This is called the **de Broglie wavelength.**

DB considered a matter wave to be a travelling one. Suppose however that the particle is confined in a small region and cannot escape. How do the wave properties show up?

FIGURE 39.15 A particle in a box creates a standing de Broglie wave as it reflects back and forth.



$$\lambda_n = 2L/n, n = 1,2,3,4...$$

$$\lambda = h/mv$$

$$v_n = n(h/2Lm), n = 1,2,3$$

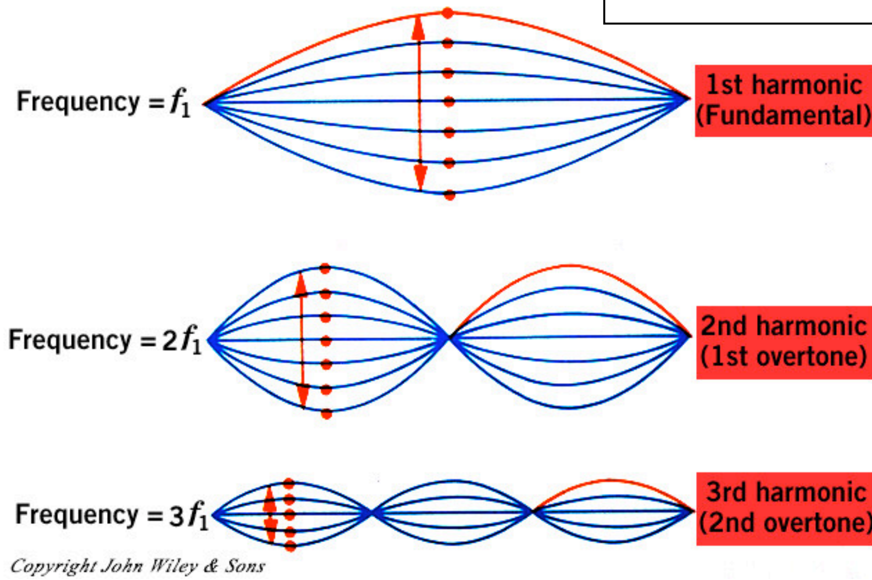
$$E_n = n^2(h^2/8mL^2) \equiv n^2 E_1$$

Probability

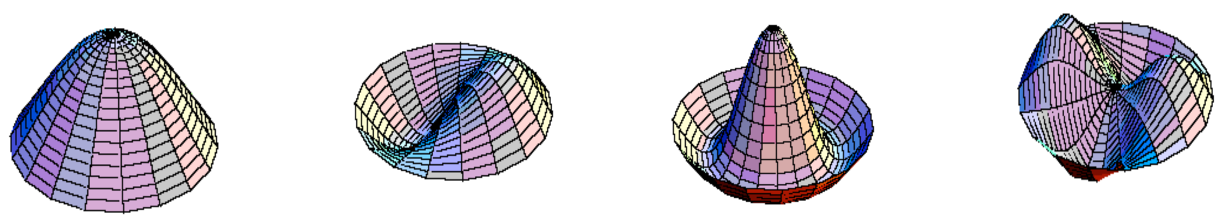
Waves with boundaries

Standing waves (harmonics)

$$y(x,t) = A \sin \omega_n t \sin \frac{n\pi x}{L}$$



Ends (or edges) must stay fixed. That's what we call a boundary condition.



This is an example of a Bessel function.

Quantization of Energy

- Consider a particle of mass *m* moving in one dimension as it bounces back and forth with speed *v* between the ends of a box of length *L*. We'll call this a *one-dimensional box;* its width isn't relevant.
- A wave, if it reflects back and forth between two fixed points, sets up a standing wave.
- A standing wave of length *L* *must* have a wavelength given by

$$\lambda_n = \frac{2L}{n} \qquad n = 1, 2, 3, 4, \ldots$$

Quantization of Energy

Using the de Broglie relationship $\lambda = h/mv$, a standing wave with wavelength λ_n forms when the particle has a speed

$$v_n = n\left(\frac{h}{2Lm}\right) \qquad n = 1, 2, 3, \ldots$$

Thus the particle's energy, which is purely kinetic energy, is

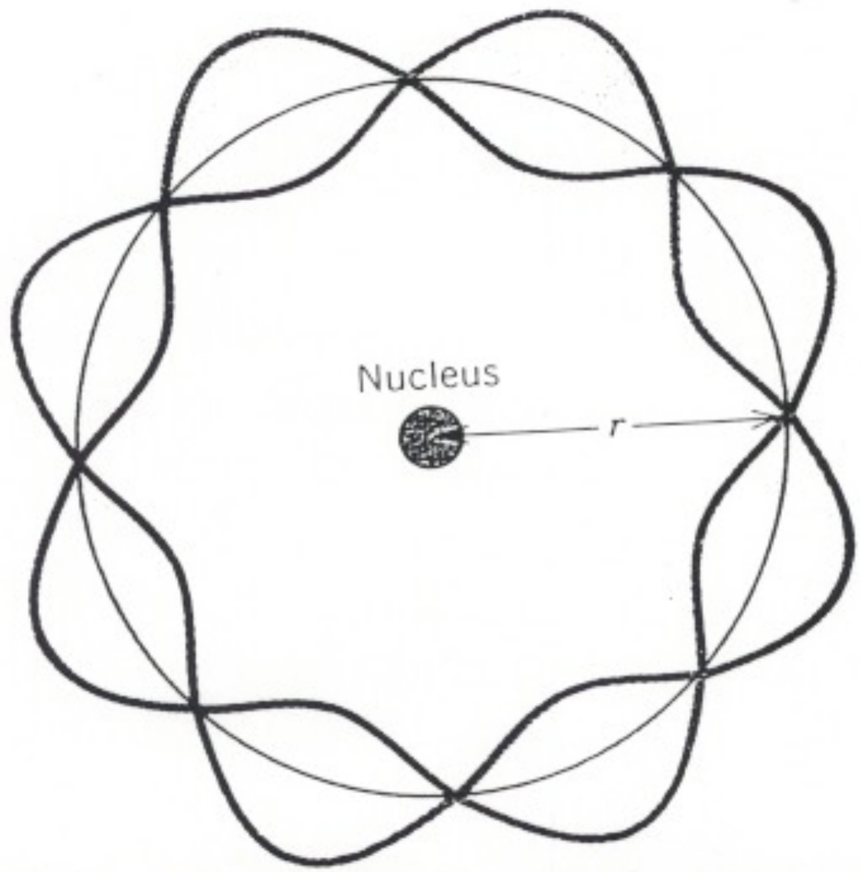
$$E_n = -\frac{E_1}{n^2}$$

$$E_n = \frac{1}{2}mv_n^2 = n^2\frac{h^2}{8mL^2} \qquad n = 1, 2, 3, \ldots$$

De Broglie's hypothesis about the wave-like properties of matter leads us to the remarkable conclusion that **the energy of a confined particle is quantized.**

Why quantization of angular momentum?

Phase cancellation



$$L = m_e vr = n\hbar \quad n = 1,2,3\ldots$$
$$\hbar = h/2\pi$$

an integer number of wavelengths fits into the circular orbit

$$\boxed{n\lambda = 2\pi r}$$

where

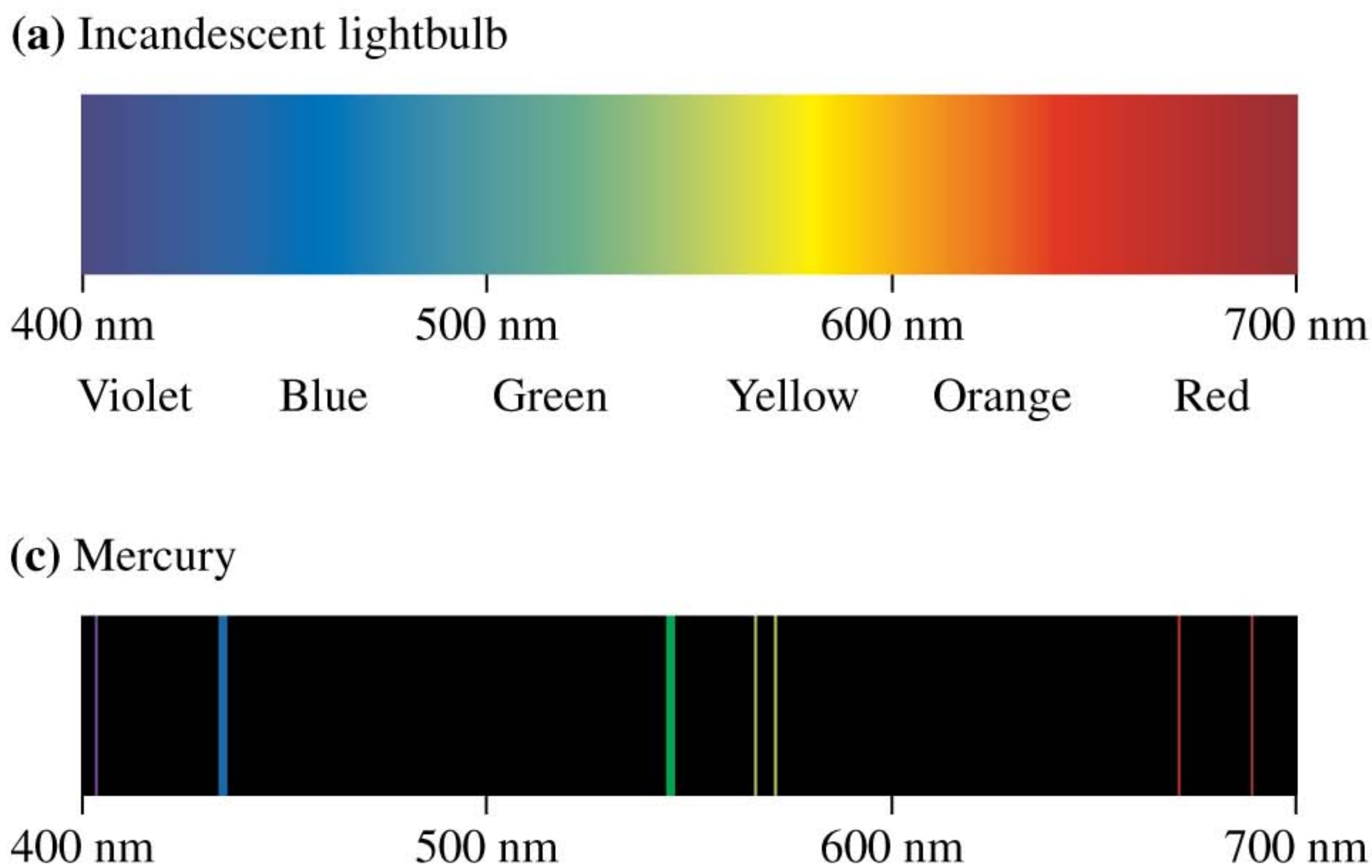
Photons
p=hf/c=
h/λ

$$\boxed{\lambda = \frac{h}{p}}$$

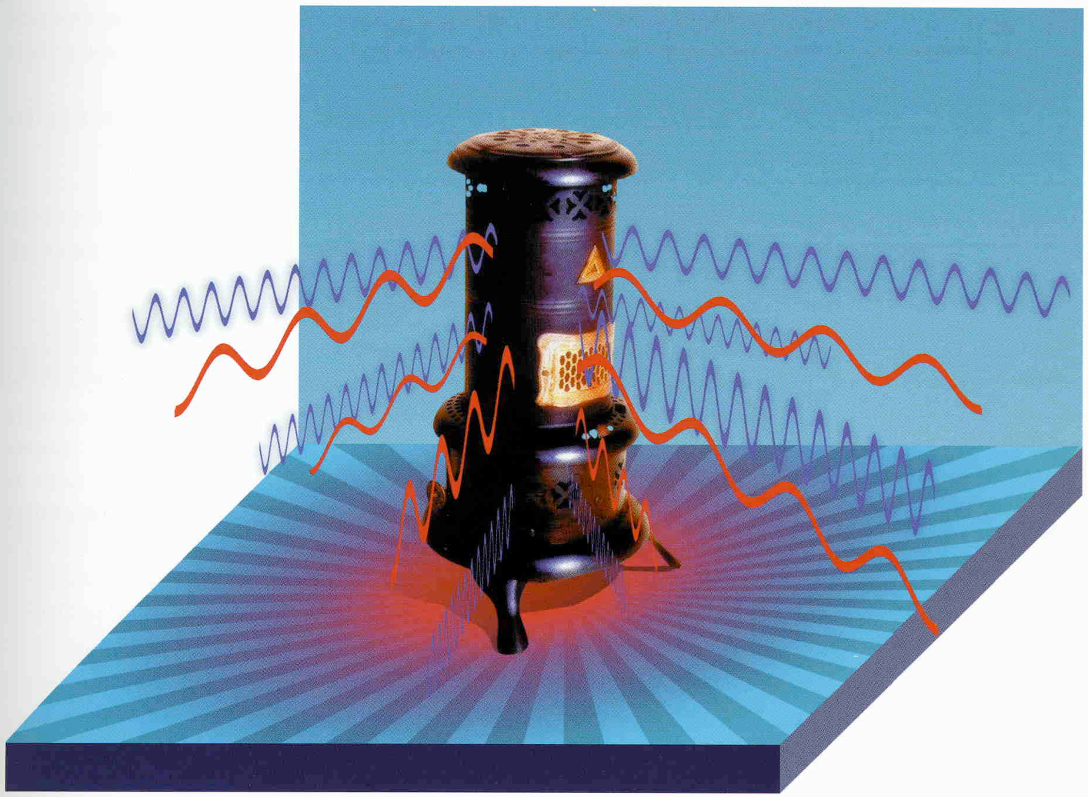
λ is the de Broglie wavelength

In order to understand quantum mechanics, you must understand waves!

FIGURE 25.2 Examples of spectra in the visible wavelength range 400–700 nm.



Black Body Radiation



A black body, approximated by this old-fashioned iron stove, radiates heat over all wavelengths. The dominant wavelength depends on its temperature.

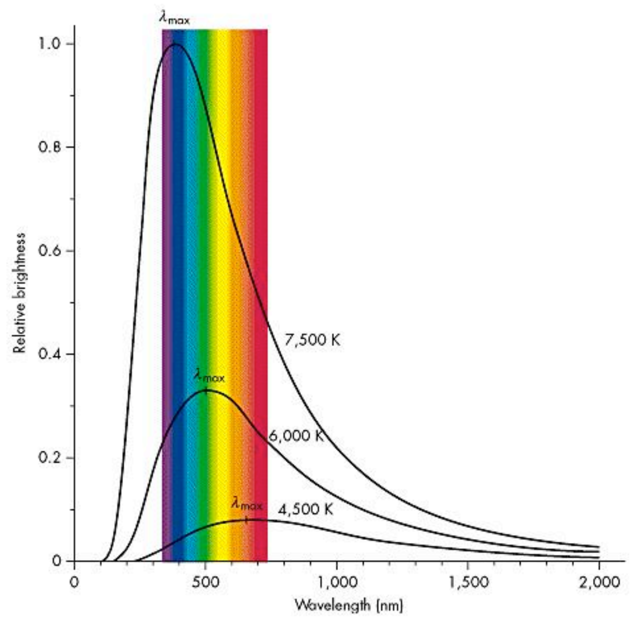
Blackbody Radiation

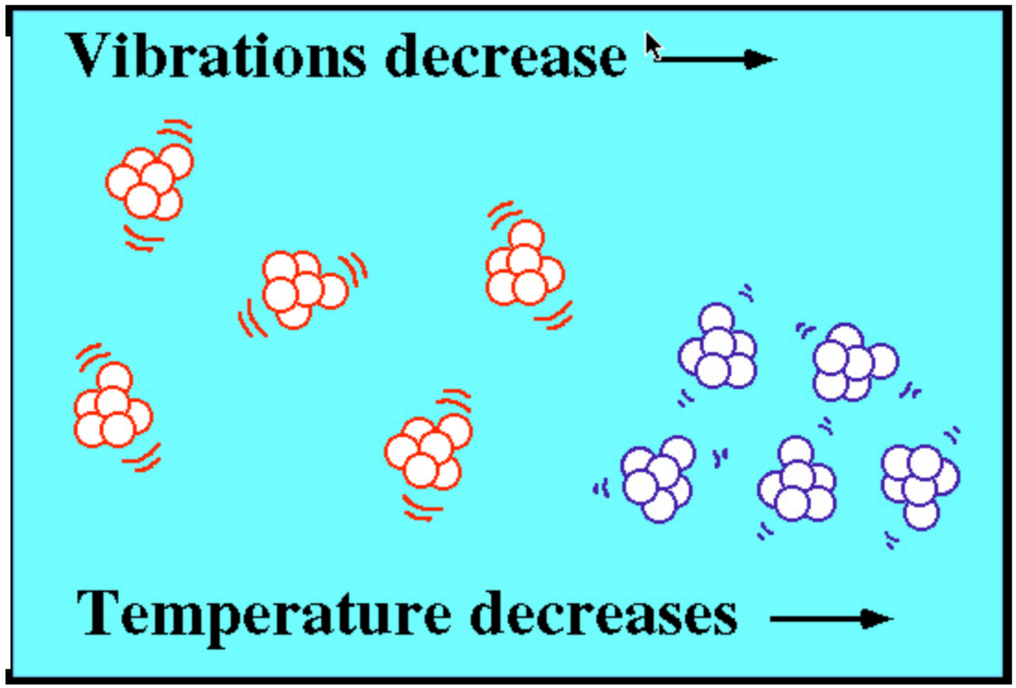
The heat energy *Q* radiated in a time interval Δt by an object with surface area *A* and absolute temperature *T* is given by

$$\frac{Q}{\Delta t} = e\sigma A T^4$$

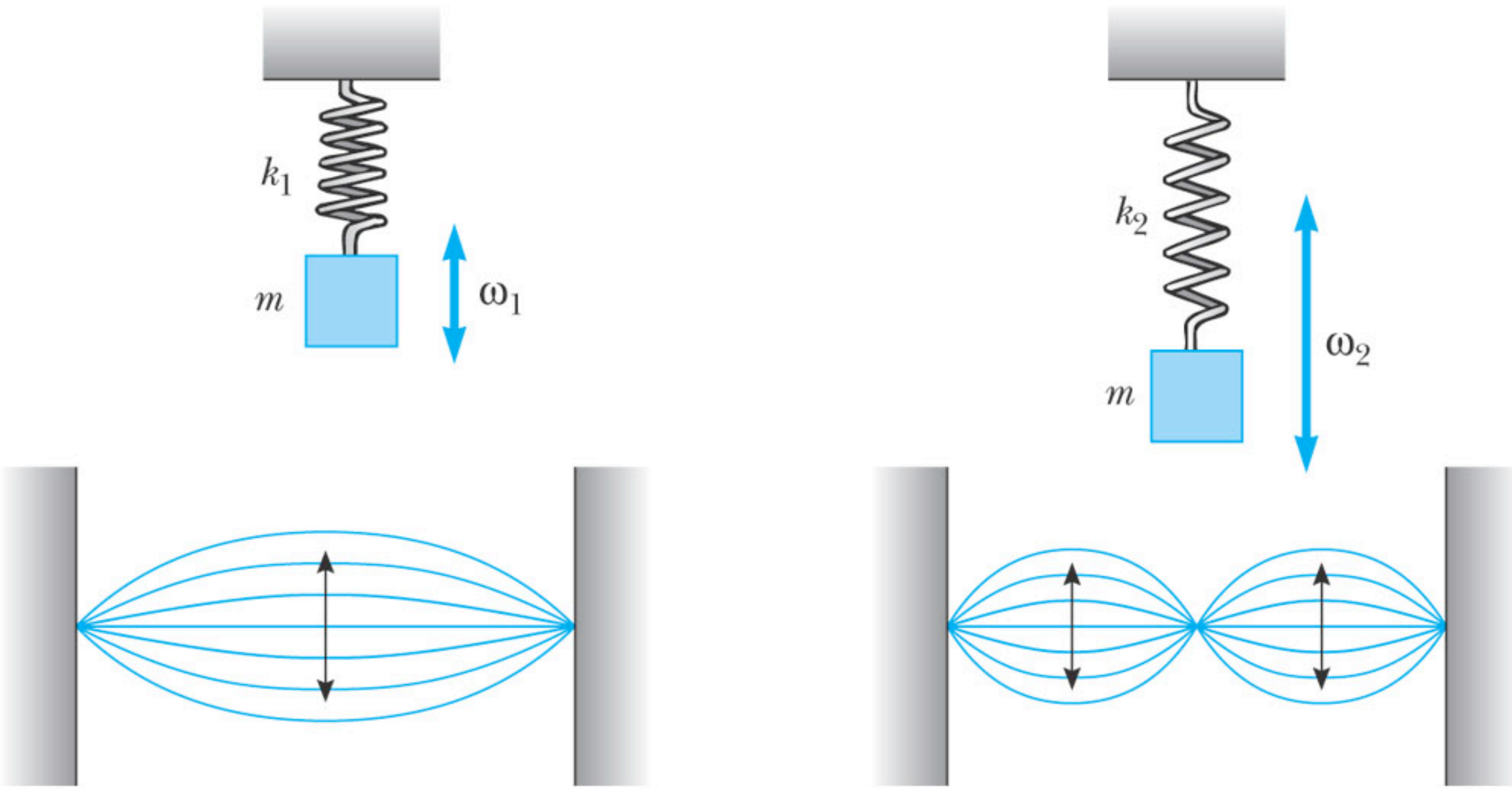
where $\sigma = 5.67 \times 10^{-8}\ \mathrm{W/m^2K^4}$ is the Stefan-Boltzmann constant. The parameter *e* is the *emissivity* of the surface, a measure of how effectively it radiates. The value of *e* ranges from 0 to 1. A perfectly absorbing—and thus perfectly emitting—object with $e = 1$ is called a *blackbody*, and the thermal radiation emitted by a blackbody is called **blackbody radiation.**

Blackbody radiation revisited





Wien' law $\lambda \sim 1/T$ or $f_m \sim T$



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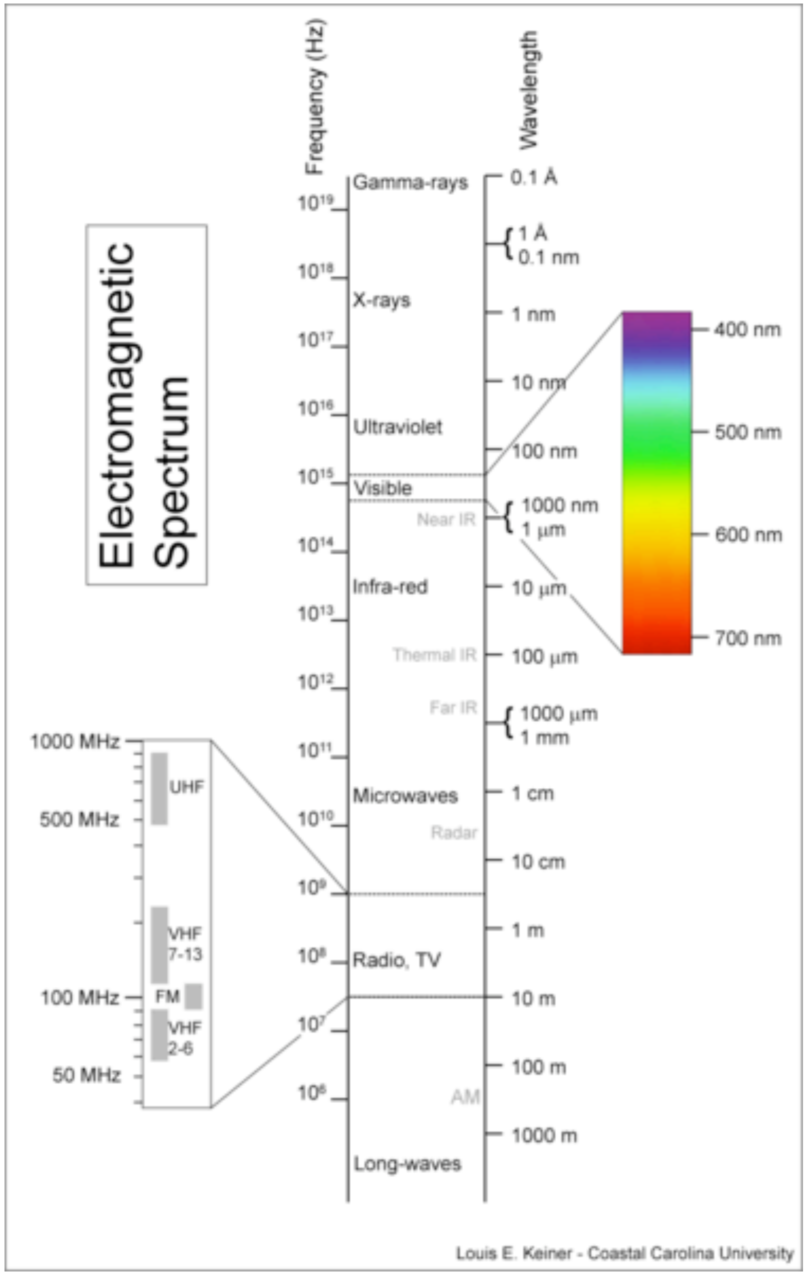
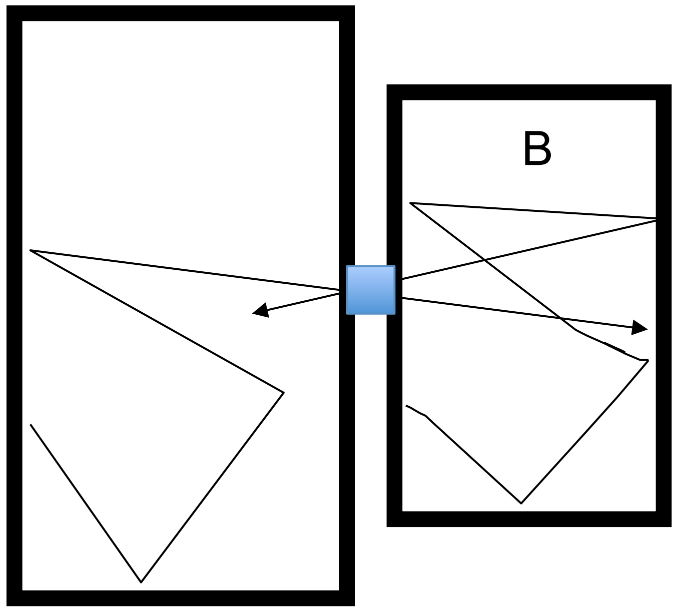
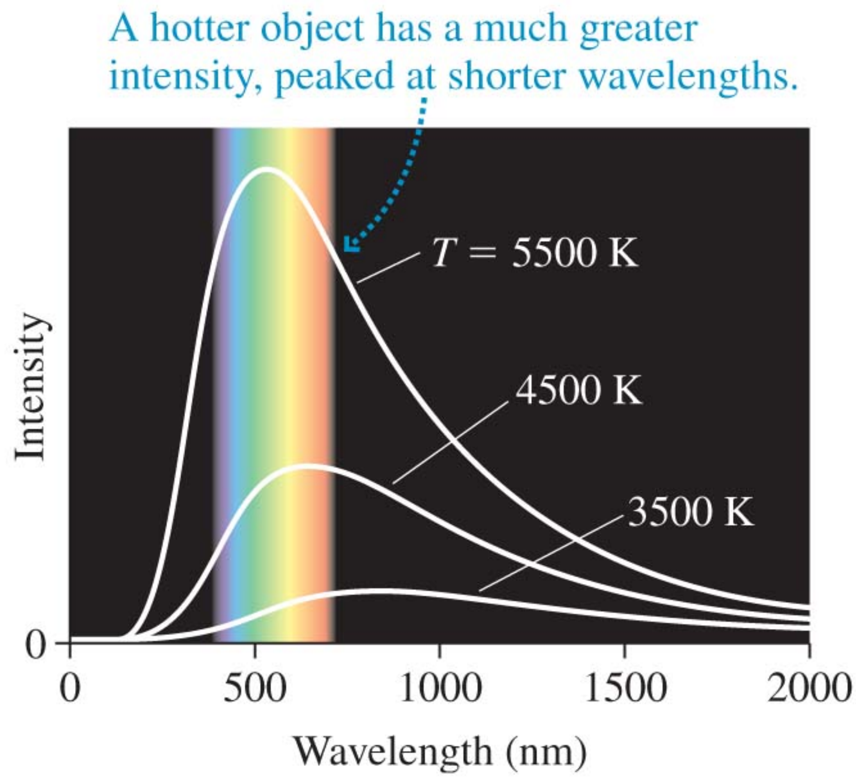


Red oscillators
Blue oscillators
Green oscillators
…


B

Lava glows when hot

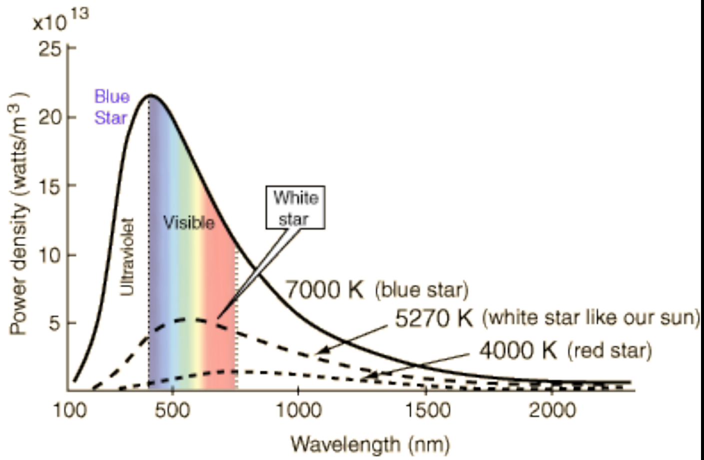
Blackbody Radiation

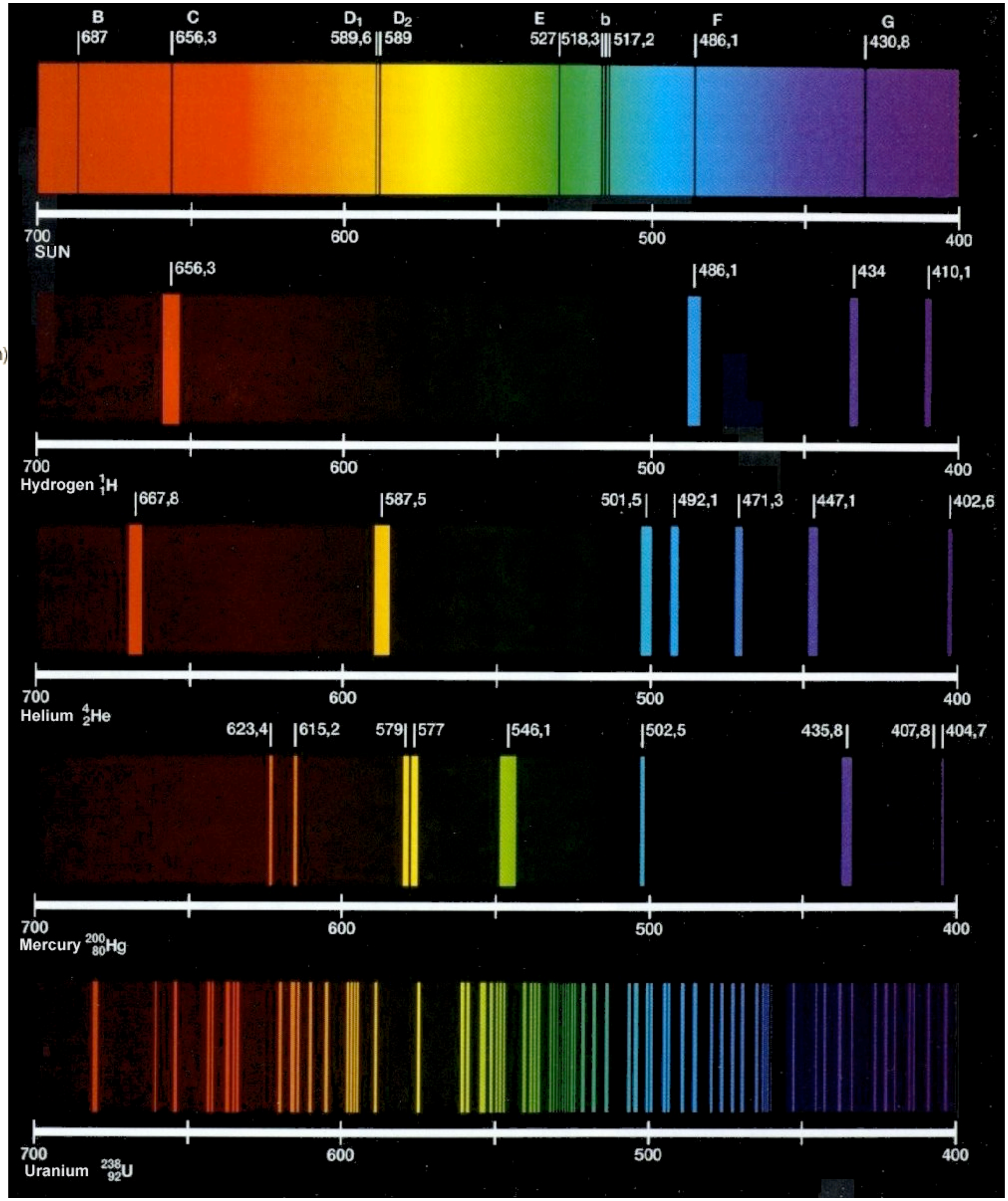


The wavelength of the peak in the intensity graph is given by Wien's law (T must be in kelvin):

$$\lambda_{\text{peak}}(\text{in nm}) = \frac{2.90 \times 10^6 \text{ nm K}}{T}$$

Wien's Displacement Law


x10^13
25
20
15
10
5
Power density (watts/m^3)
Blue Star
Ultraviolet
Visible
White star
7000 K (blue star)
5270 K (white star like our sun)
4000 K (red star)
100
500
1000
1500
2000
Wavelength (nm)


B
687
C
656,3
D1
589,6
D2
589
E
527
518,3
b
517,2
F
486,1
G
430,8
700
600
500
400
SUN
656,3
486,1
434
410,1
Hydrogen 1 1 H
667,8
587,5
501,5
492,1
471,3
447,1
402,6
Helium 4 2 He
623,4
615,2
579
577
546,1
502,5
435,8
407,8
404,7
Mercury 200 80 Hg
Uranium 238 92 U