$w = 2L \frac{\lambda}{a}$
Radians

The diameter \( w \) of the diffraction pattern increases with distance \( L \), showing that light spreads out behind the circular aperture, but it decreases if the size \( D \) of the circular aperture increases.

\[
\theta_1 = \frac{1.22\lambda}{D} \quad \text{Radians}
\]

\[
w = 2y_1 = 2L \tan \theta_1 \approx \frac{2.44\lambda L}{D}
\]
Two Slits With Finite Width $a$

With more than one slit having finite width $a$, we must consider:

1. Diffraction due to the individual slit
2. Interference of waves from different slits
Two Slits With Finite width $a$

Zero Order Maximum

First Diff. Minimum

$\alpha \sin \theta = \lambda$

First Order Maximum

$d \sin \theta = \lambda$

Diffraction envelope

Determined by slit width $a$

Interference "fine" structure

Determined by separation $d$

between slits
How small is the focal area

\[ \theta \approx \frac{2.44 \lambda}{D} \]

\[ w = f \theta = \frac{2.44 f \lambda}{D} \]

\[ f / D \leq 1 \]

\[ w_{\text{min}} \approx 2.5 \lambda \]

\[ I_F \approx I(A / 6.25 \lambda^2) \]
**INSTRUMENT RESOLUTION**

**FIGURE 24.19** Two images that are marginally resolved.

The maximum of image 2 falls on the first dark fringe of image 1. The images are marginally resolved.

Object 1

Object 2

Distant point sources

The image of each object is not a perfect point, but a small circular diffraction pattern.

**FIGURE 24.20** Enlarged photographs of the images of two closely spaced objects.

$\alpha > \theta_{\text{min}}$

Resolved

$\alpha = \theta_{\text{min}}$

Marginally resolved

$\alpha < \theta_{\text{min}}$

Not resolved

Raleigh’s criterion

$$\theta_{\text{min}} = \frac{1.22\lambda}{D}$$

(angular resolution of a lens)
Photons

Photon Momentum

\[ E = pc = hf \]
\[ p = hf / c \]

Photons are sometimes visualized as wave packets. The electromagnetic wave shown has a wavelength and a frequency, yet it is also discrete and fairly localized.

**FIGURE 39.11** A wave packet has wave-like and particle-like properties.

\[ P = dE / dt = (dN / dt)hf = R = hf \]
\[ E = N_f hf \]
Particle properties of light

Photomultiplier

Single photon produces single electron (photoelectric effect). Electron accelerates and produces secondary emission that exponentiates with further acceleration 1, 2, 4, 8, 16, ...

Low light full click no half clicks. The unit of light is a particle PHOTON.
Red photons
Blue photons
Green photon
X-ray photons
Gamma ray photons
etc
...

\[ E = pc = hf \]
\[ p = hf / c \]
FIGURE 39.1 Lenard’s experimental device to study the photoelectric effect.

Ultraviolet light causes the metal cathode to emit electrons. This is the photoelectric effect.

The photoelectrons form a current between the cathode and the anode.

The potential difference can be changed or reversed.

The current can be measured while the potential difference, the light frequency, and the light intensity are varied.
The observation that electromagnetic waves could eject electrons from the surface of a metal was first made by Hertz.

A simple experiment can be designed to measure the energy and intensity of the electrons ejected.

- Light shines on a metal plate emitting electrons
- The voltage on a battery can be gradually turned up until the electric field just stops the electrons from reaching the collector plate, thereby giving a measure of the kinetic energy.

The photoelectric effect or what makes you sunburn!!

The voltage on a battery can be gradually turned up until the electric field just stops the electrons from reaching the collector plate, thereby giving a measure of the kinetic energy.
Classical physics fails us.
The Classical Picture

• The energy in the light wave is spread out uniformly and continuously over the wavefront.

The maximum kinetic energy of an ejected electron is therefore

\[ K_{\text{max}} = C I A t - E_o \]

which depends on the light intensity and the time over which it is exposed.

• The intensity of a light wave is proportional to the square of the amplitude of the electric field.

...and therefore does not depend on frequency.

• The energy in the light wave is spread out uniformly and continuously over the wavefront.
The number of photoelectrons ejected depended on the intensity (as expected) but their maximum kinetic energy did not!

The maximum kinetic energy depended only on the frequency, the slope of the linear relationship between the energy and the frequency gives “Planck’s constant”, $h$.

The electrons were ejected immediately after the light started shining—the electron instantaneously absorbed enough energy to escape—provided there was enough energy to overcome the binding energy or “work function”.

Even a high intensity source of low frequency light cannot liberate electrons. THRESHOLD
In the photoelectric effect, these discrete localized quanta of energy, $hν$, are transferred entirely to the electron

$$K_{\text{max}} = h\nu - e\phi$$

Light is a particle

Instead of continuous waves we have to think of the energy as being localized in quanta.

$E_{\text{photon}} = h\nu$

$\nu_{\text{max}} = 6.22 \times 10^5 \text{ m/s}$

$K_{\text{max}} = 1.77 \text{ eV}$

$550 \text{ nm} \quad 2.25 \text{ eV}$

$700 \text{ nm} \quad 1.77 \text{ eV}$

$400 \text{ nm} \quad 3.1 \text{ eV}$

$\nu_{\text{max}} = 2.96 \times 10^5 \text{ m/s}$

Potassium - 2.0 eV needed to eject electron
FIGURE 39.8 The creation of a photoelectron.

One quantum of light with energy $E = hf \geq E_0$.

Work function $E_0$

An electron has absorbed the entire energy of the light quantum and has escaped.

$$K_m = hf - E_0$$
The Photon Model of Light

The **photon model** of light consists of three basic postulates:

1. Light consists of discrete, massless units called photons. A photon travels in vacuum at the speed of light, $3.00 \times 10^8$ m/s.

2. Each photon has energy $E_{\text{photon}} = hf$

where $f$ is the frequency of the light and $h$ is a *universal constant* called **Planck’s constant**. The value of Planck’s constant is $h = 6.63 \times 10^{-34}$ J s.

3. The superposition of a sufficiently large number of photons has the characteristics of a classical light wave.
Fig. 2.6 This sequence of photographs of a girl's face shows that photography is a quantum process. The probabilistic nature of quantum effects is evident from the first photographs in which the numbers of photons are very small. As the number of photons increases the photograph becomes more and more distinct until the optimum exposure is reached. The number of photons involved in these photographs ranges from about 3000 in the lowest exposure to about 30,000,000 in the final exposure.
FIGURE 25.9 Photographs made with increasing levels of light intensity.

The photo at very low light levels shows individual points, as if particles are arriving at the detector.

The particle-like behavior is not noticeable at higher light levels.

or exposure time
FIGURE 25.5 Atoms arranged in a cubic lattice.

(a) Molecular bonds

(b) The spacing between atomic planes is $d$. 

.1-1 nm
\[ \lambda = D \sin(2\theta) = 0.165 \text{ nm} \]
Matter Waves

• In 1927 Davisson and Germer were studying how electrons scatter from the surface of metals.
• They found that electrons incident normal to the crystal face at a speed of $4.35 \times 10^6$ m/s scattered at $\phi = 50^\circ$.
• This scattering can be interpreted as a mirror-like reflection from the atomic planes that slice diagonally through the crystal.
• The angle of incidence on this set of planes is the angle $\theta_m$ in $2d \cos \theta_m = m\lambda$, the Bragg condition for diffraction.
• Davisson and Germer found that the “electron wavelength” was

$$\lambda = D \sin(2\theta) = 0.165 \text{ nm}$$
Well if light is a particle (photon) is electron a wave?

When we send many electrons through slit they show interference – wave properties
Just as it happened when we send low intensity (single photon) light
The de Broglie Wavelength

De Broglie postulated that a particle of mass $m$ and momentum $p = mv$ has a wavelength

$$\lambda = \frac{h}{p}$$

where $h$ is Planck’s constant. This wavelength for material particles is now called the **de Broglie wavelength**. It depends *inversely* on the particle’s momentum, so the largest wave effects will occur for particles having the smallest momentum.
CONCEPTUAL INTERFERENCE EXPERIMENTS WITH WAVES AND PARTICLES
Fig. 1.7 A diagram of a double-slit experiment with bullets. The experimental set-up is shown on the left of the figure and the results of three different experiments indicated on the right. We have shown bullets that pass through slit 1 as open circles and bullets through slit 2 as black circles. The column labelled $P_1$ shows the distribution of bullets arriving at the detector boxes when slit 2 is closed and only slit 1 is open. Column $P_2$ shows a similar distribution obtained with slit 1 closed and slit 2 open. As can be seen, the maximum number of bullets appears in the boxes directly in line with the slit that is left open. The result obtained with both slits open is shown in the column labelled $P_{12}$. It is now a matter of chance through which slit a bullet will come and this is shown by the scrambled mixture of black and white bullets collected in each box. The important point to notice is that the total obtained in each box when both slits are open is just the sum of the numbers obtained when only one or other of the slits is open. This is obvious in the case of bullets since we know that bullets must pass through one of the slits to reach the detector boxes.

<table>
<thead>
<tr>
<th>Slit 1 open</th>
<th>Slit 2 closed</th>
<th>Slit 2 open</th>
<th>Slit 1 closed</th>
<th>Slits 1 and 2 open</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>$P_2$</td>
<td>$P_{12}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Number of bullets in each box after a fixed time
Fig. 1.9 A diagram of a double-slit experiment with water waves. The detectors are a line of small floating buoys whose jiggling up and down provides a measure of the wave energy. The wave crests spreading out from each slit are shown in the figure and can be compared with fig. 1.8. The column labelled \( I_1 \) shows the smoothly varying wave intensity obtained when only gap 1 is open. Notice that this is very similar to the pattern \( P_1 \) obtained with bullets in fig. 1.7 with only slit 1 open. Again it is largest at the detector directly in line with gap 1 and the source. The second column shows that a similar pattern, \( I_2 \), is obtained when gap 1 is closed and gap 2 is open. The final column, \( I_{12} \), shows the wave intensity pattern obtained with both slits open. It is dramatically different from the pattern obtained for bullets with both slits open. It is not equal to the sum of the patterns \( I_1 \) and \( I_2 \) obtained with one of the gaps closed. This rapidly varying intensity curve is called an interference pattern.
Fig. 1.11 A diagram of a double-slit experiment with electrons. Electrons always arrive with a flash at the phosphor detector at one point, in the same way that bullets always end up in just one of the detector boxes rather than the energy being spread out, as in a wave. The column marked \( P_1 \) shows the pattern obtained with only slit 1 open. Electrons that have gone through slit 1 are represented as open circles, like the bullets of fig. 1.7. Column \( P_2 \) shows the same thing with only slit 2 open and the electrons that have gone through slit 2 indicated by black circles. These two patterns are exactly the same as those obtained with bullets. The difference lies in the column headed \( P_{12} \), which shows the pattern obtained for electrons when both slits are open. This is just the interference pattern obtained with water waves and requires some kind of wave motion arising from each slit as indicated on the figure. It is not the sum of \( P_1 \) and \( P_2 \) and so we cannot say which slit any electron goes through. We have indicated this lack of knowledge by drawing the electrons, which still arrive like bullets, as half white and half black circles. This fact, that quantum objects such as electrons possess attributes of both wave and particle motion but behave like neither, is the central mystery of quantum mechanics.
Fig. 2.3 Sketch of the experimental set-up required to observe through which slit the electron passes in a double-slit experiment. Light, in the form of photons, is directed at the slits. In the figure a photon, represented as a small bullet, has hit an electron behind slit 1. The electron is disturbed slightly in its motion and the scattered photon is observed at the photon detectors. The electron patterns obtained with only one of the slits open are almost the same as before, when we did not observe the electron behind the slits. The surprise occurs with both slits open: there is no interference pattern. The small nudges given to the electrons in their collisions with the photons are always sufficient to wash out the interference pattern completely! We can now say with certainty through which slit the electron went but now the electrons are behaving just like bullets. The observed pattern is just the sum of the patterns for slit 1 and slit 2 separately.
FIGURE 39.15 A particle in a box creates a standing de Broglie wave as it reflects back and forth.

\[ L = \frac{n\lambda}{2} \]

Matter waves travel in both directions.

\[ \lambda_n = \frac{2L}{n} \quad n = 1, 2, 3, 4, \ldots \]
Quantization of Energy

• Consider a particle of mass $m$ moving in one dimension as it bounces back and forth with speed $v$ between the ends of a box of length $L$. We’ll call this a one-dimensional box; its width isn’t relevant.

• A wave, if it reflects back and forth between two fixed points, sets up a standing wave.

• A standing wave of length $L$ must have a wavelength given by

$$\lambda_n = \frac{2L}{n} \quad n = 1, 2, 3, 4, \ldots$$
Quantization of Energy

Using the de Broglie relationship $\lambda = \frac{h}{mv}$, a standing wave with wavelength $\lambda_n$ forms when the particle has a speed

$$v_n = n \left( \frac{h}{2Lm} \right) \quad n = 1, 2, 3, \ldots$$

Thus the particle’s energy, which is purely kinetic energy, is

$$E_n = \frac{1}{2}mv_n^2 = n^2 \frac{h^2}{8mL^2} \quad n = 1, 2, 3, \ldots$$

De Broglie’s hypothesis about the wave-like properties of matter leads us to the remarkable conclusion that the energy of a confined particle is quantized.
What’s wrong with this picture?

Accelerating charges radiate. Could this electromagnetic radiation be the source of the spectral lines?

No. This radiation must come at the expense of the kinetic energy of the orbiting electron!

It will eventually spiral into the nucleus. The atom would be unstable!
Bohr’s Model of Atomic Quantization

1. An atom consists of negative electrons orbiting a very small positive nucleus.

2. Atoms can exist only in certain **stationary states**. Each stationary state corresponds to a particular set of electron orbits around the nucleus. These states can be numbered 2, 3, 4, . . . , where \( n \) is the quantum number.

3. Each stationary state has an energy \( E_n \). The stationary states of an atom are numbered in order of increasing energy: \( E_1 < E_2 < E_3 < \ldots \)

4. The lowest energy state of the atom \( E_1 \) is *stable* and can persist indefinitely. It is called the **ground state** of the atom. Other stationary states with energies \( E_2, E_3, E_4, \ldots \) are called **excited states** of the atom.
Bohr Atom

\[ \vec{F}_{cb} = k \frac{q_1 q_2}{r^2} \hat{r} \]

\[ \hbar = \frac{\hbar}{2\pi} \]

\[ -k \frac{e^2}{r^2} + m \frac{\nu_\theta^2}{r} = 0 \]

\[ m \nu_\theta r = n \hbar, n = 1, 2, 3, 4... \]

\[ r = \frac{ke^2}{mv_\theta^2} \]

\[ v_\theta = \frac{n \hbar}{mr} \]

\[ r = n^2 \left( \frac{\hbar^2}{kme^2} \right) \]

\[ r = n^2 a_B, \]

\[ a_B = \frac{(\hbar/2\pi)^2}{(ke^2 m)} = 0.0529 \text{ nm} \]

\[ E_n = -E_1/n^2 \]

\[ E_1 = 13.6 \text{ eV} \]
Why quantization of angular momentum?

Phase cancellation

\[ L = m_e vr = n \hbar \quad n = 1, 2, 3, \ldots \]
\[ \hbar = h / 2\pi \]

An integer number of wavelengths fits into the circular orbit.

\[ n\lambda = 2\pi r \]

where

\[ \lambda = \frac{h}{p} \]

\( \lambda \) is the de Broglie wavelength.

In order to understand quantum mechanics, you must understand waves!
Bohr’s Model of Atomic Quantization

5. An atom can “jump” from one stationary state to another by emitting or absorbing a photon of frequency

\[
f_{\text{photon}} = \frac{\Delta E_{\text{atom}}}{h}
\]

where \( h \) is Planck’s constant and \( \Delta E_{\text{atom}} = |E_f - E_i| \).

\( E_f \) and \( E_i \) are the energies of the initial and final states. Such a jump is called a **transition** or, sometimes, a **quantum jump**.
**FIGURE 39.17** An atom can change stationary states by emitting or absorbing a photon or by undergoing a collision.

(a) Emission and absorption of light

**Emission**

- Excited-state electron
- Allowed orbits

The electron jumps to a lower-energy stationary state and emits a photon.

**Absorption**

- Approaching photon

The electron absorbs the photon and jumps to a higher-energy stationary state.
Emission

Emission line

Atom was hit/bumped by another atom and gained some energy $= (E_2 - E_1)$. Electron in higher energy orbit ($E_2$).

Emission line produced!

Photon with Energy $= (E_2 - E_1)$. 

electron in higher energy orbit

electron in lower energy orbit

Energy $E_2$

Energy $E_1$
To explain discrete spectra, Bohr found that atoms obey three basic rules:

1. Electrons have only certain energies corresponding to particular distances from nucleus. As long as the electron is in one of those energy orbits, it will not lose or absorb any energy. The energy orbits are analogous to rungs on a ladder: electrons can be only on rungs of the ladder and not in between rungs.

2. The orbits closer to the nucleus have lower energy.

3. Atoms want to be in the lowest possible energy state called the ground state (all electrons as close to the nucleus as possible).
Bohr’s Model of Atomic Quantization

6. An atom can move from a lower energy state to a higher energy state by absorbing energy $\Delta E_{\text{atom}} = E_f - E_i$ in an inelastic collision with an electron or another atom. This process, called collisional excitation, is shown.

**FIGURE 39.17** An atom can change stationary states by emitting or absorbing a photon or by undergoing a collision.

(b) Collisional excitation

- Approaching particle
- The particle transfers energy to the atom in the collision and excites the atom.
- Particle loses energy.
FIGURE 39.18 An energy-level diagram.

These are allowed energies. The atom cannot have an energy between these.

These transitions from \( n = 4 \) emit a photon.

These are the transitions of the absorption spectrum.

Increasing energy

\( n = 5 \)
\( n = 4 \)
\( n = 3 \)
\( n = 2 \)
\( n = 1 \)

Excited states

\( E_5 \)
\( E_4 \)
\( E_3 \)
\( E_2 \)
\( E_1 \)

Ground state
The Bohr Hydrogen Atom

The radius of the electron’s orbit in Bohr’s hydrogen atom is

\[ r_n = n^2 a_B \quad n = 1, 2, 3, \ldots \]

where \( a_B \) is the Bohr radius, defined as

\[ a_B = \text{Bohr radius} = \frac{4\pi\varepsilon_0 \hbar}{me^2} = 5.29 \times 10^{-11} \text{ m} = 0.0529 \text{ nm} \]

The possible electron speeds and energies are

\[ v_n = \frac{n\hbar}{mr_n} = \frac{1}{n} \frac{\hbar}{ma_B} = \frac{v_1}{n} \quad n = 1, 2, 3, \ldots \]

\[ E_n = -\frac{E_1}{n^2} = -\frac{13.60 \text{ eV}}{n^2} \quad n = 1, 2, 3, \ldots \]
The work function of metal A is 3.0 eV. Metals B and C have work functions of 4.0 eV and 5.0 eV, respectively. Ultraviolet light shines on all three metals, creating photoelectrons. Rank in order, from largest to smallest, the stopping potential for A, B, and C.

A. $V_C > V_B > V_A$
B. $V_A > V_B > V_C$
C. $V_A = V_B = V_C$
The work function of metal A is 3.0 eV. Metals B and C have work functions of 4.0 eV and 5.0 eV, respectively. Ultraviolet light shines on all three metals, creating photoelectrons. Rank in order, from largest to smallest, the stopping potential for A, B, and C.

A. $V_C > V_B > V_A$

B. $V_A > V_B > V_C$

C. $V_A = V_B = V_C$

$\Delta V = -\Delta V_{\text{stop}}$
The intensity of a beam of light is increased but the light’s frequency is unchanged. Which of the following is true?

A. The photons are larger.
B. There are more photons per second.
C. The photons travel faster.
D. Each photon has more energy.
The intensity of a beam of light is increased but the light’s frequency is unchanged. Which of the following is true?

A. The photons are larger.
B. **There are more photons per second.**
C. The photons travel faster.
D. Each photon has more energy.
What is the quantum number of this particle confined in a box?

A.  $n = 8$
B.  $n = 6$
C.  $n = 5$
D.  $n = 4$
E.  $n = 3$
What is the quantum number of this particle confined in a box?

A. $n = 8$
B. $n = 6$
C. $n = 5$
D. $n = 4$
E. $n = 3$

Correct answer: D. $n = 4$
A photon with a wavelength of 414 nm has energy $E_{\text{photon}} = 3.0 \text{ eV}$. Do you expect to see a spectral line with $\lambda = 414 \text{ nm}$ in the emission spectrum of the atom represented by this energy-level diagram? If so, what transition or transitions will emit it?

\begin{align*}
n = 3 & \quad 6.0 \text{ eV} \quad \text{A. Yes. } 1 \rightarrow 0 \\
n = 2 & \quad 5.0 \text{ eV} \quad \text{B. Yes. } 2 \rightarrow 0 \\
n = 1 & \quad 2.0 \text{ eV} \quad \text{C. Yes. } 2 \rightarrow 1 \\
n = 0 & \quad 0.0 \text{ eV} \quad \text{D. Yes. } 3 \rightarrow 1 \\
\end{align*}
A photon with a wavelength of 414 nm has energy $E_{\text{photon}} = 3.0 \text{ eV}$. Do you expect to see a spectral line with $\lambda = 414 \text{ nm}$ in the emission spectrum of the atom represented by this energy-level diagram? If so, what transition or transitions will emit it?

- $n = 3$ .................................................. 6.0 eV
- $n = 2$ .................................................. 5.0 eV
- $n = 1$ .................................................. 2.0 eV
- $n = 0$ .................................................. 0.0 eV

A. Yes. 1→0  
B. Yes. 2→0  
C. Yes. 2→1  
D. Yes. 3→1  
E. No
What is the quantum number of this hydrogen atom?

A. $n = 5$
B. $n = 4$
C. $n = 3$
D. $n = 2$
E. $n = 1$
What is the quantum number of this hydrogen atom?

A. $n = 5$
B. $n = 4$
C. $n = 3$
D. $n = 2$
E. $n = 1$