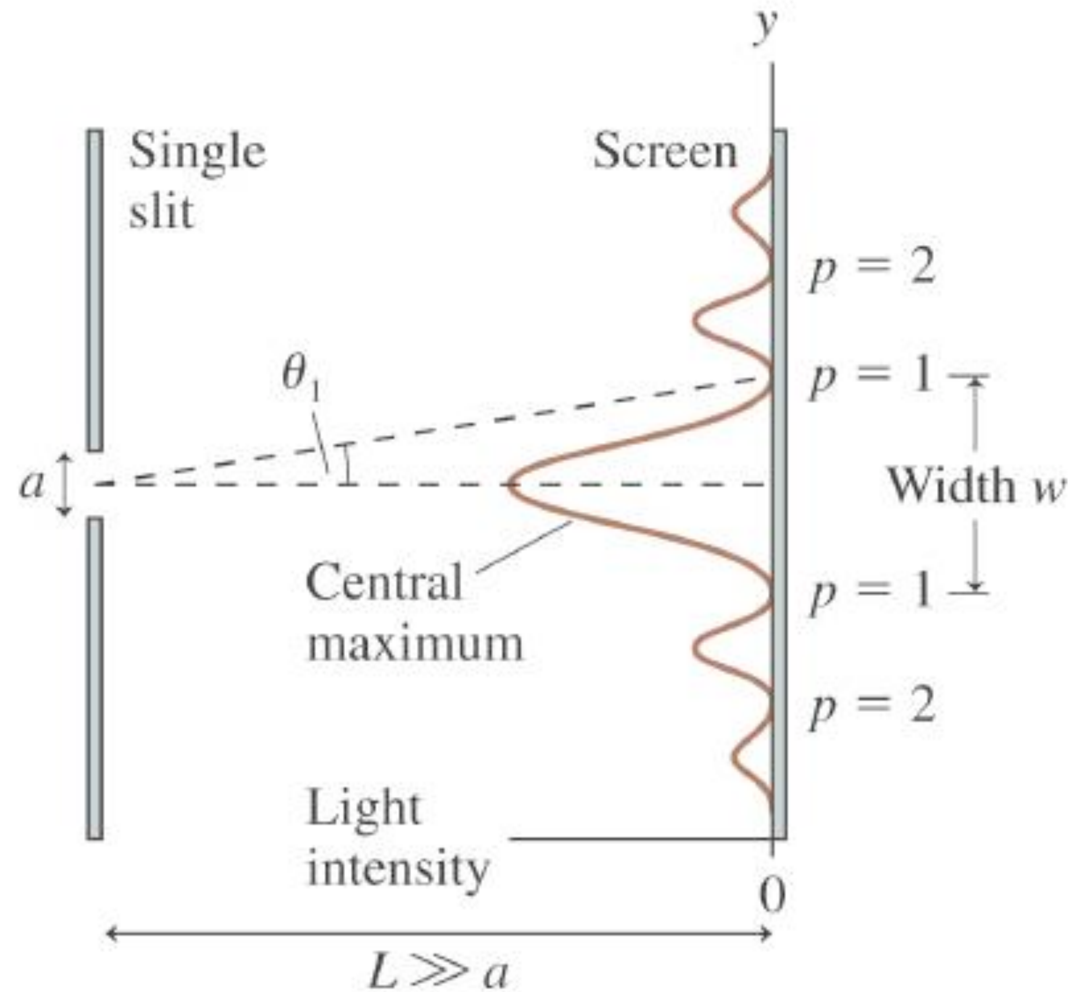
PHYS 270 – SUPPL. #18

DENNIS PAPADOPOULOS

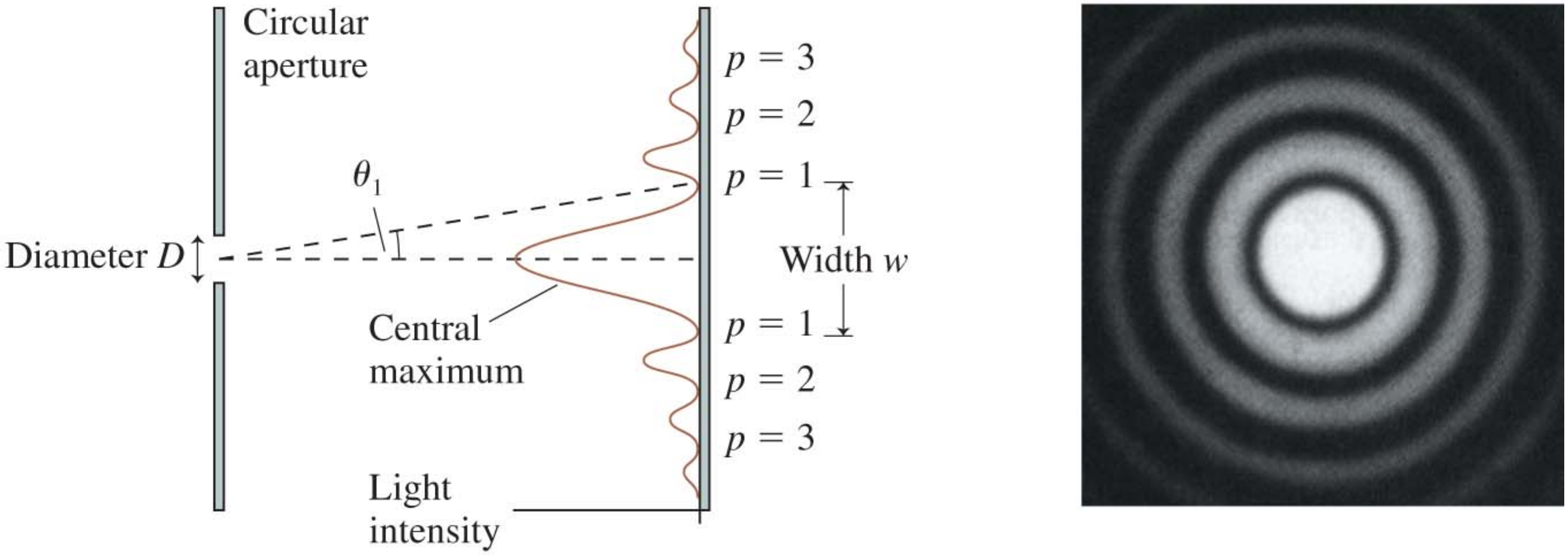
APRIL 7, 2011

RALEIGH'S CRITERION AND CHAPTER 39



$$w = 2L\frac{\lambda}{a}$$

FIGURE 22.15 The diffraction of light by a circular opening.



$$\theta_1 = \frac{1.22\lambda}{D}$$

Radians

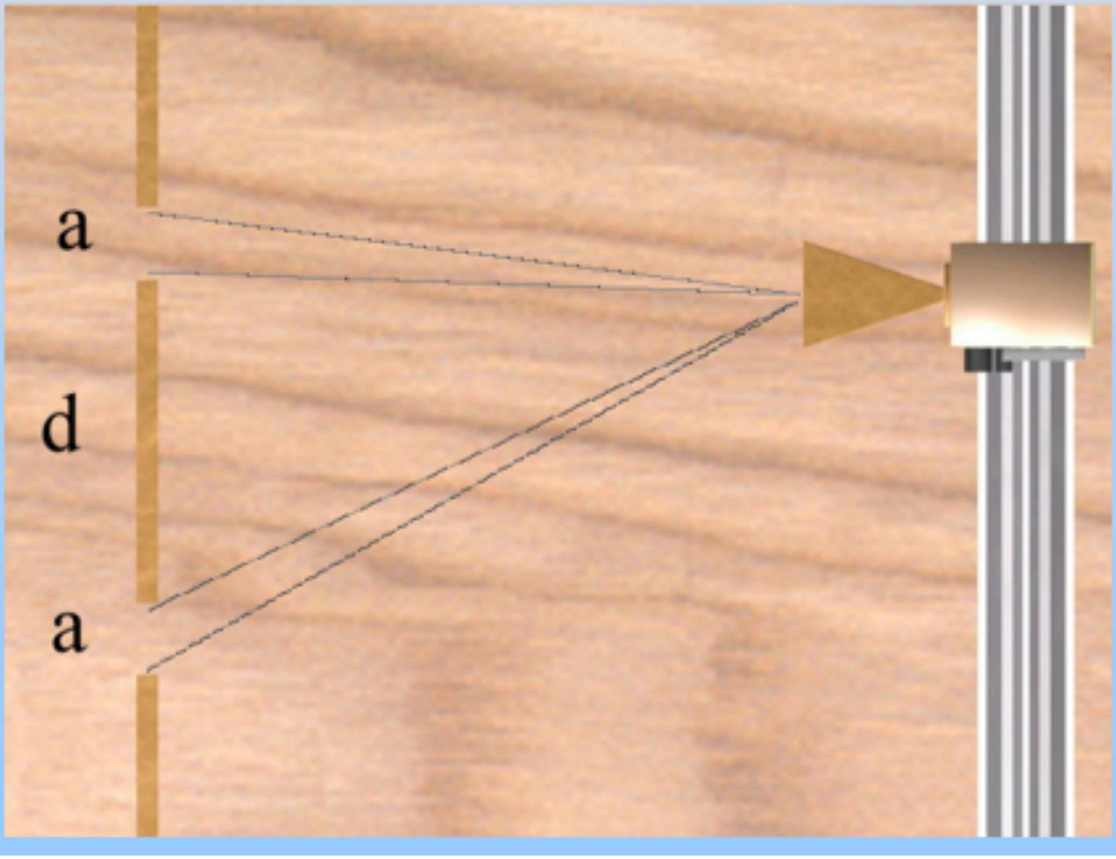
$$w = 2y_1 = 2L\tan\theta_1 \approx \frac{2.44\lambda L}{D}$$

The diameter w of the diffraction pattern increases with distance L, showing that light spreads out behind the circular aperture, but it decreases if the size D of the circular aperture increases.

Two Slits With Finite Width a

With more than one slit having finite width a, we must consider

1. Diffraction due to the individual slit
2. Interference of waves from different slits



Two Slits With Finite width *a*

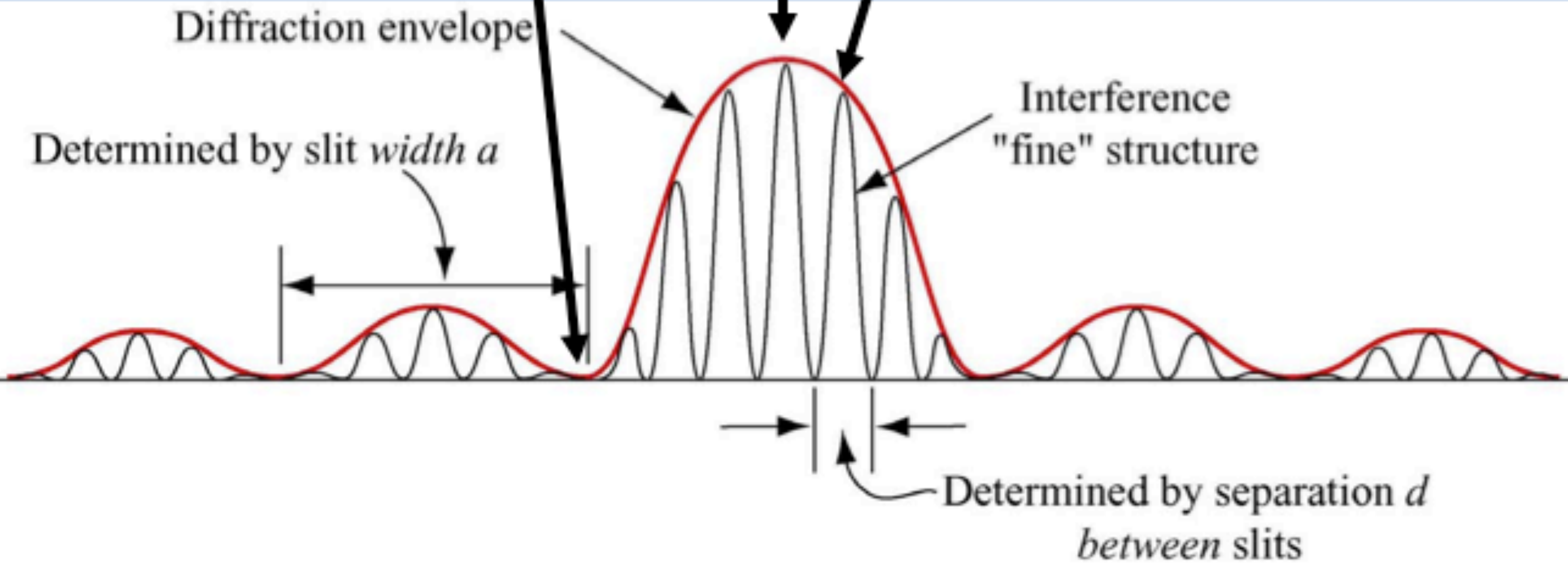
Zero Order Maximum

First Diff. Minimum

$$a \sin\theta = \lambda$$

First Order Maximum

$$d \sin\theta = \lambda$$



HOW SMALL IS THE FOCAL AREA

$$IA = I_F A_F$$

$$I_F = I(A/A_F)$$

$$\theta \approx \frac{2.44\lambda}{D}$$

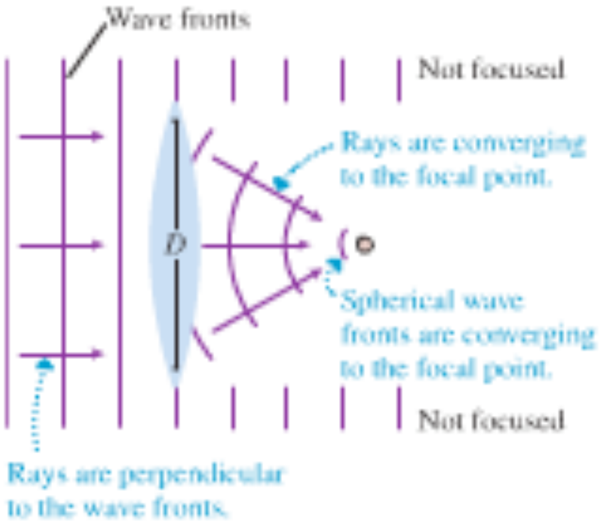
$$w = f\theta = \frac{2.44 f\lambda}{D}$$

$$f/D \leq 1$$

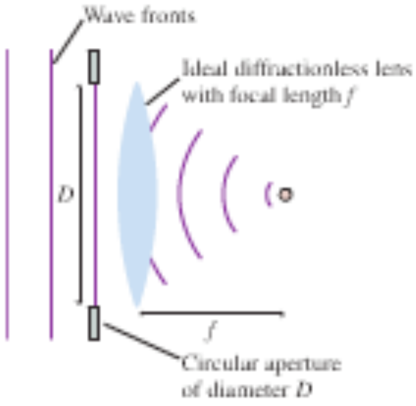
$$w_{\min} \approx 2.5\lambda$$

$$I_F \approx I(A/6.25\lambda^2)$$

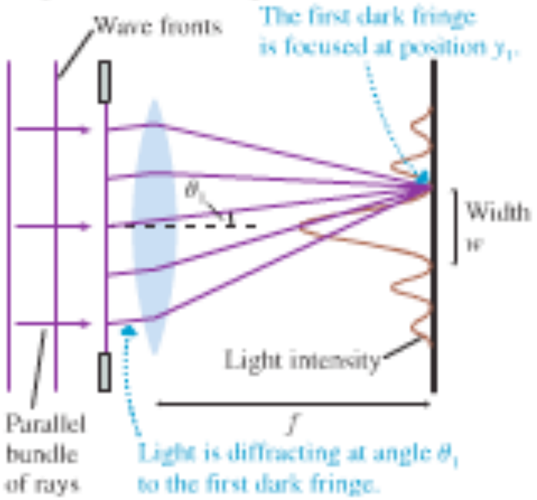
(a) A lens acts as a circular aperture.



(b) The aperture and focusing effects can be separated.

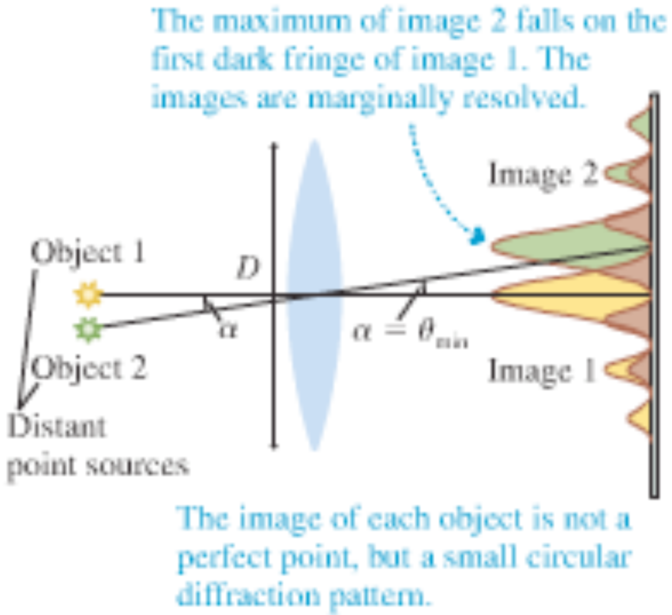


(c) The lens focuses the diffraction pattern in the focal plane.



INSTRUMENT RESOLUTION

FIGURE 24.19 Two images that are marginally resolved.



The maximum of image 2 falls on the first dark fringe of image 1. The images are marginally resolved.

The image of each object is not a perfect point, but a small circular diffraction pattern.

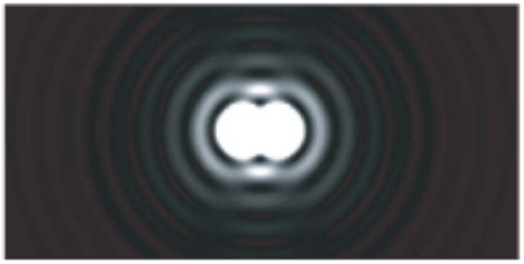
Raleigh's criterion

$$\theta_{\min} = \frac{1.22\lambda}{D} \qquad \text{(angular resolution of a lens)}$$

FIGURE 24.20 Enlarged photographs of the images of two closely spaced objects.

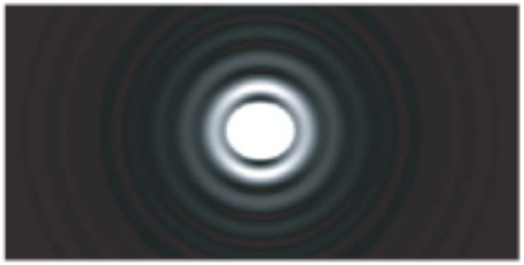
$\alpha > \theta_{\min}$

Resolved



$\alpha = \theta_{\min}$

Marginally resolved



$\alpha < \theta_{\min}$

Not resolved

Photons

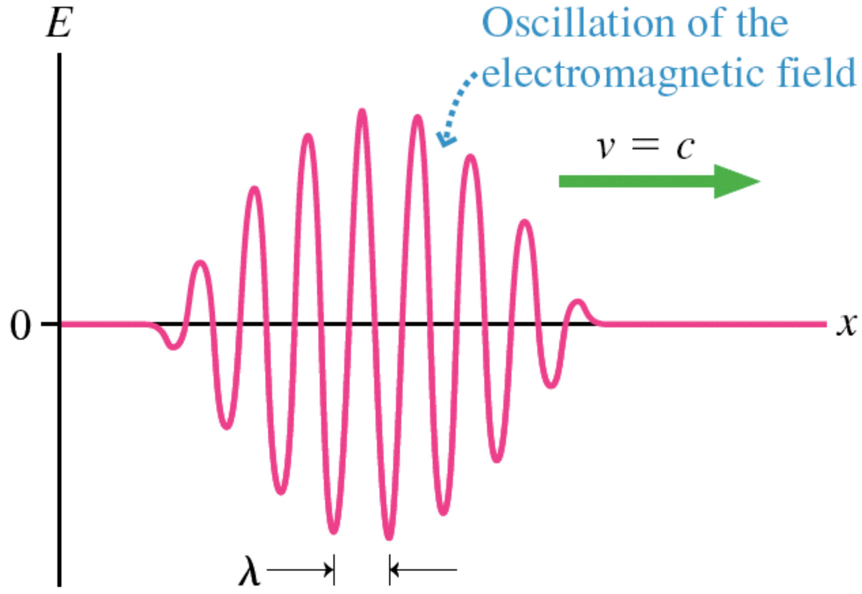
Photon Momentum

$$E = pc = hf$$

$$p = hf / c$$

Photons are sometimes visualized as **wave packets.** The electromagnetic wave shown has a wavelength and a frequency, yet it is also discrete and fairly localized.

FIGURE 39.11 A wave packet has wave-like and particle-like properties.



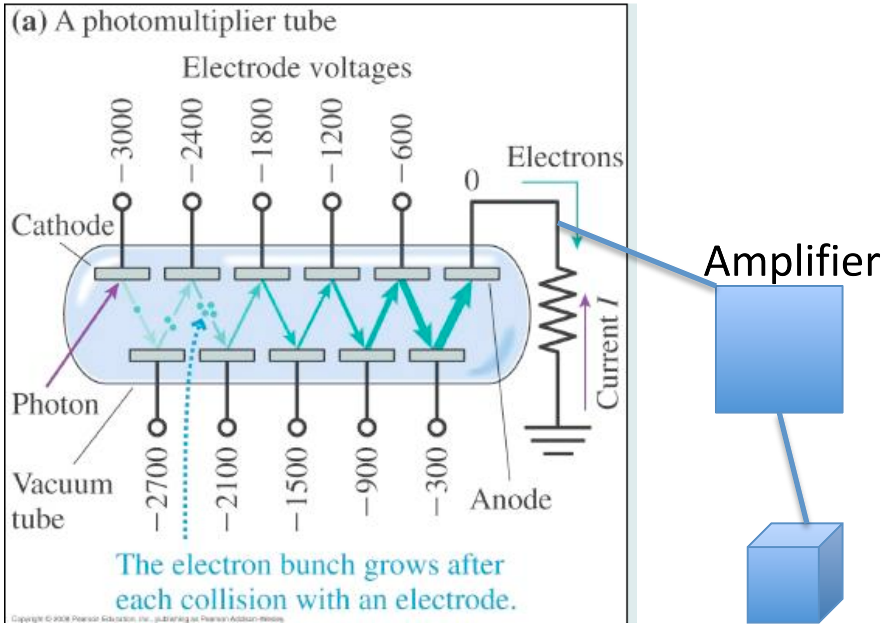
$$P = dE / dt = (dN / dt)hf = R = hf$$

$$E = N_T hf$$

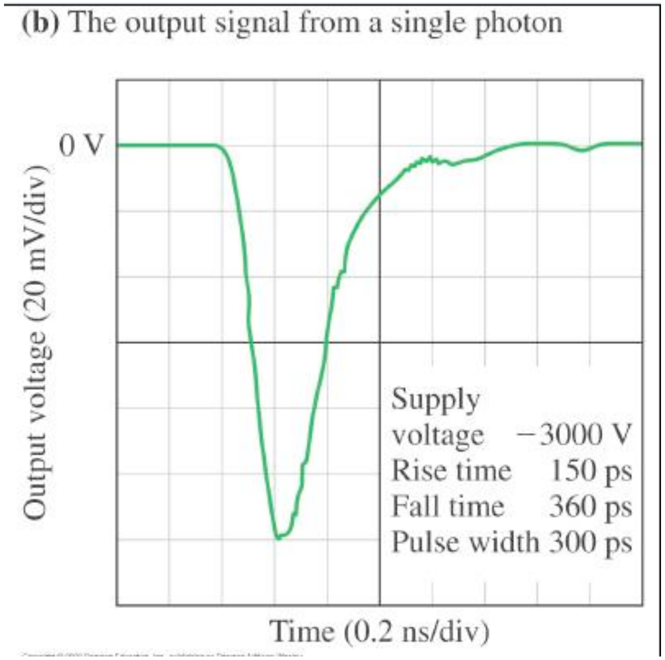
Particle properties of light

Photomultiplier

Single photon produces single electron (photoelectric effect). Electron accelerates and produces secondary emission that exponentiates with further acceleration 1,2,4,8,16,..

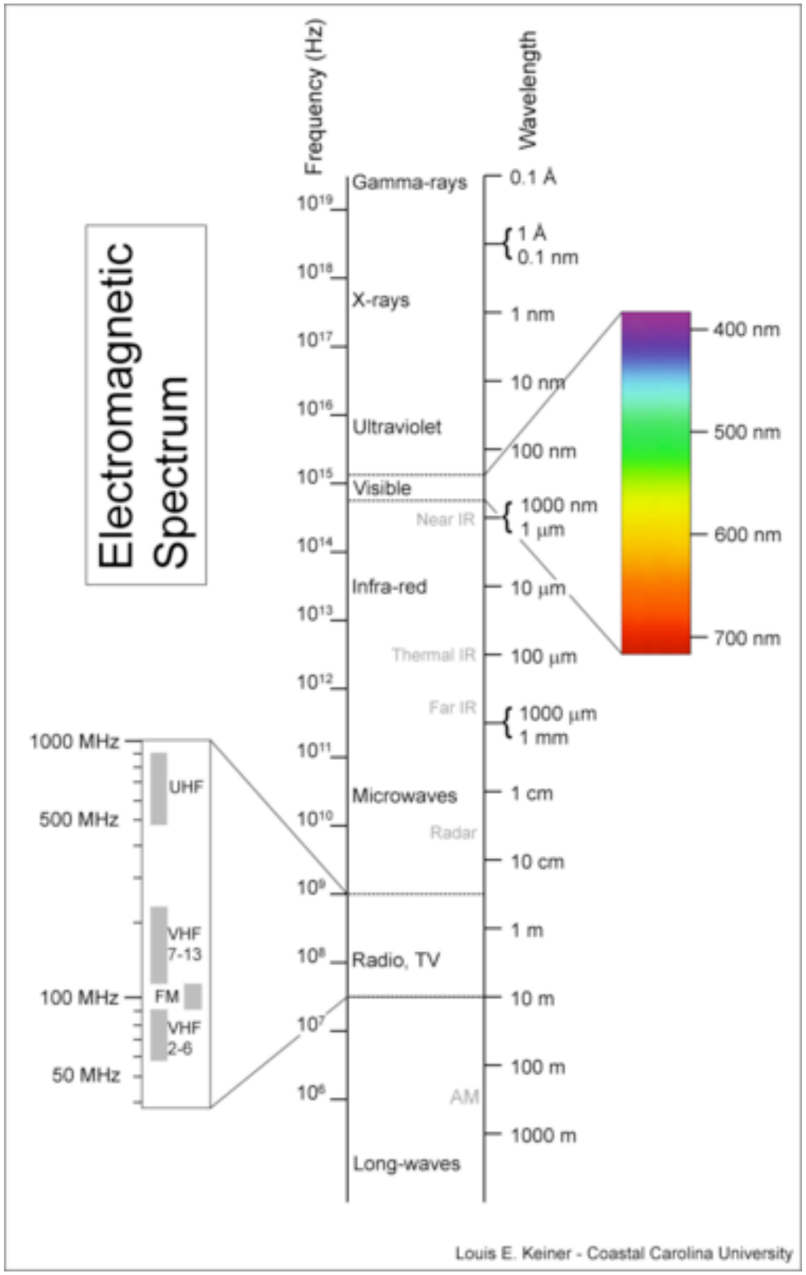


Speaker



Low light full click no half clicks.

The unit of light is a particle PHOTON.



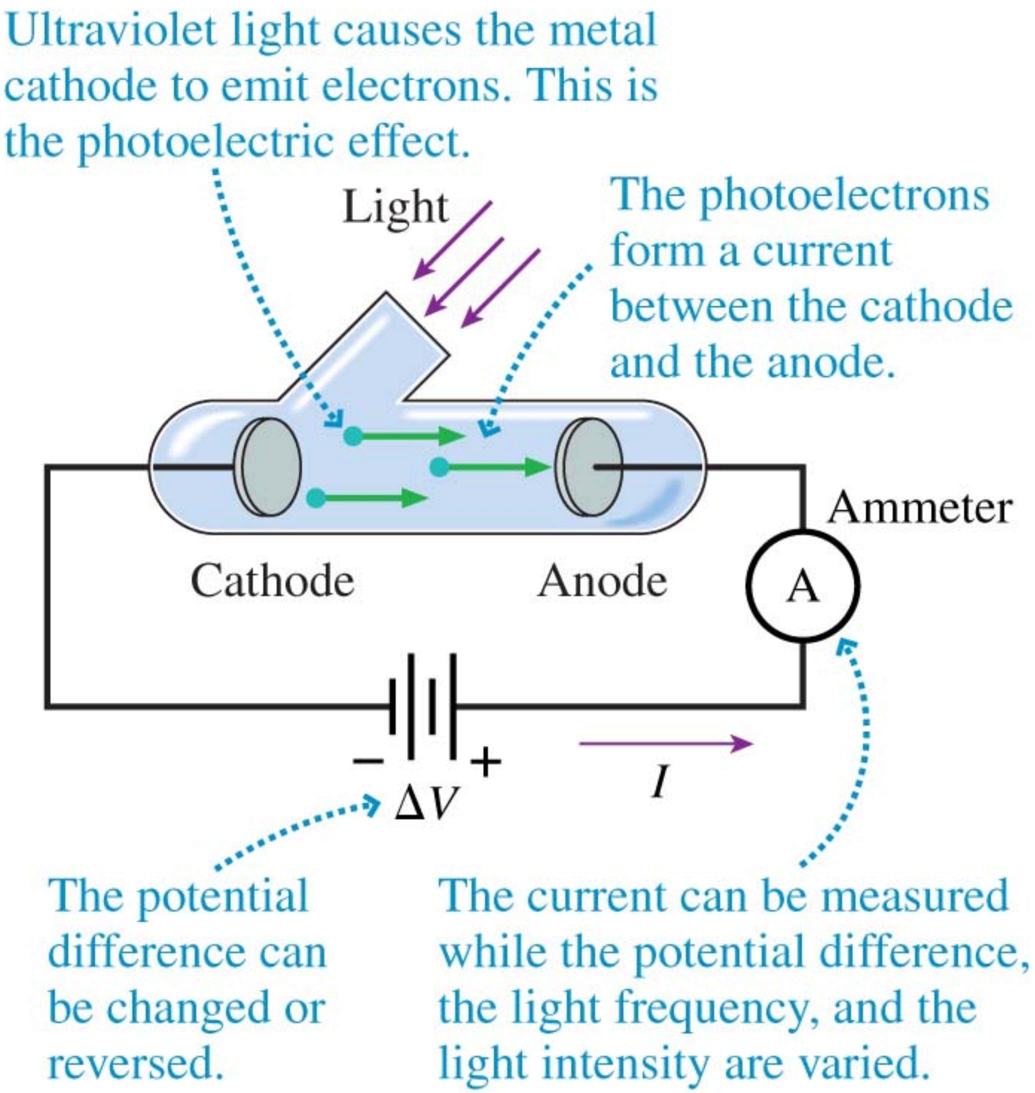
Louis E. Keiner - Coastal Carolina University

Red photons
Blue photons
Green photon
X-ray photons
Gamma ray photons
etc
…

$$E = pc = hf$$

$$p = hf/c$$

FIGURE 39.1 Lenard's experimental device to study the photoelectric effect.



Ultraviolet light causes the metal cathode to emit electrons. This is the photoelectric effect.

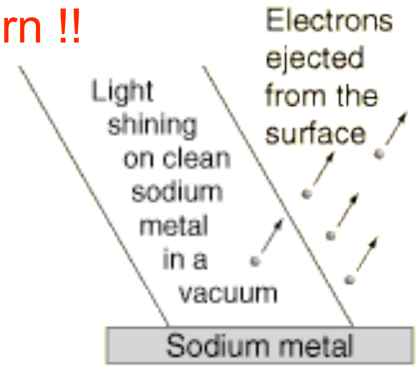
The photoelectrons form a current between the cathode and the anode.

The potential difference can be changed or reversed.

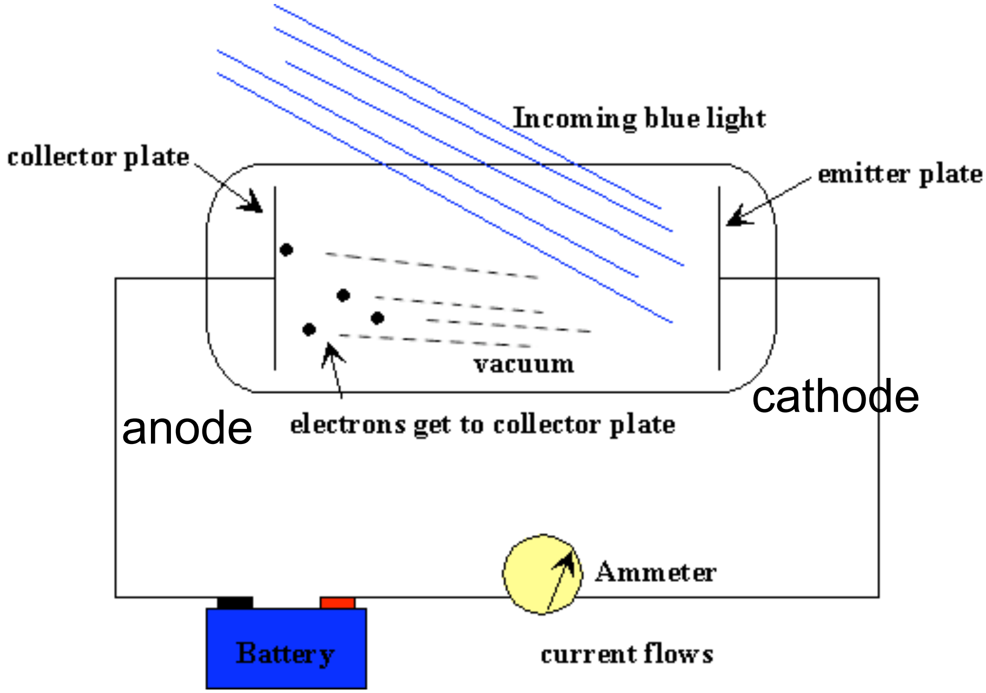
The current can be measured while the potential difference, the light frequency, and the light intensity are varied.

The photoelectric effect or what makes you sunburn !!

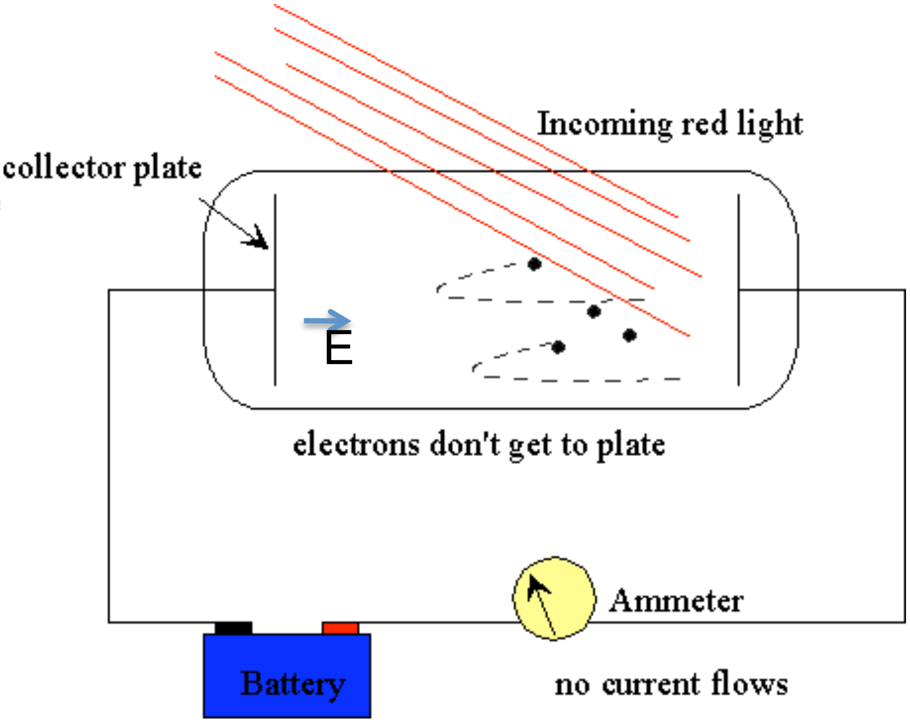
The observation that electromagnetic waves could eject electrons from the surface of a metal was first made by Hertz.



A simple experiment can be designed to measure the energy and intensity of the electrons ejected.



- Light shines on a metal plate emitting electrons



- The voltage on a battery can be gradually turned up until the electric field just stop the electrons from reaching the collector plate, thereby giving a measure of the kinetic energy.

Classical physics fails us.

The Classical Picture

- *The energy in the light wave is spread out uniformly and continuously over the wavefront.*

The maximum kinetic energy of an ejected electron is therefore

$$K_{\max} = \underset{\text{absorption coefficient}}{C}\,\overset{\text{light intensity}}{I}\,\underset{\text{cross sectional area of atom}}{A}\,\overset{\text{time}}{t} - E_o \longleftarrow \text{work function}$$

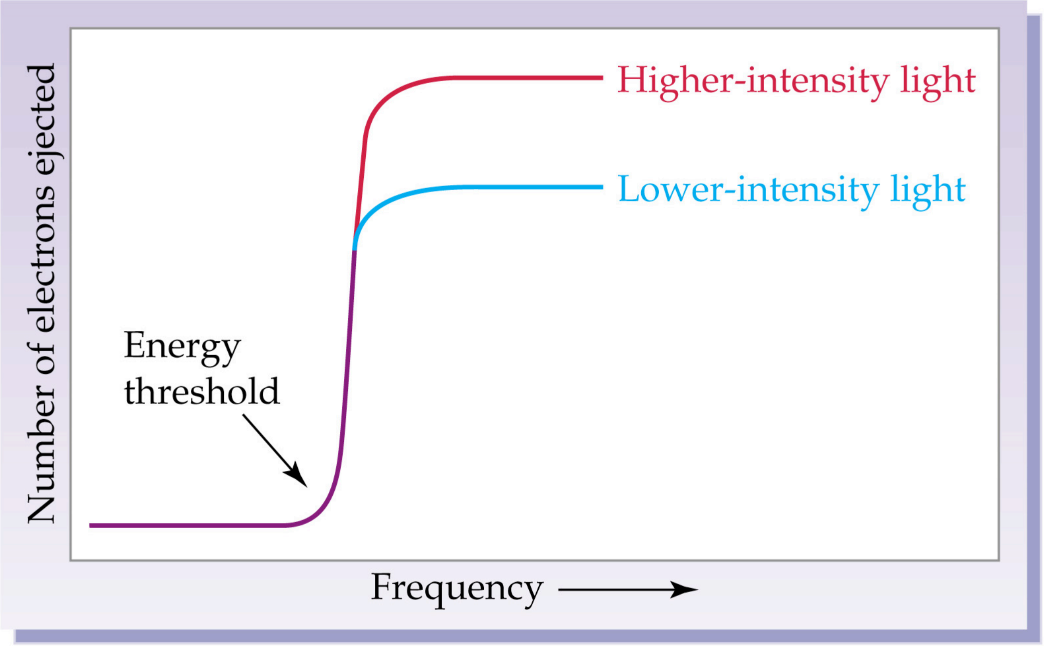
which depends on the light intensity and the time over which it is exposed.

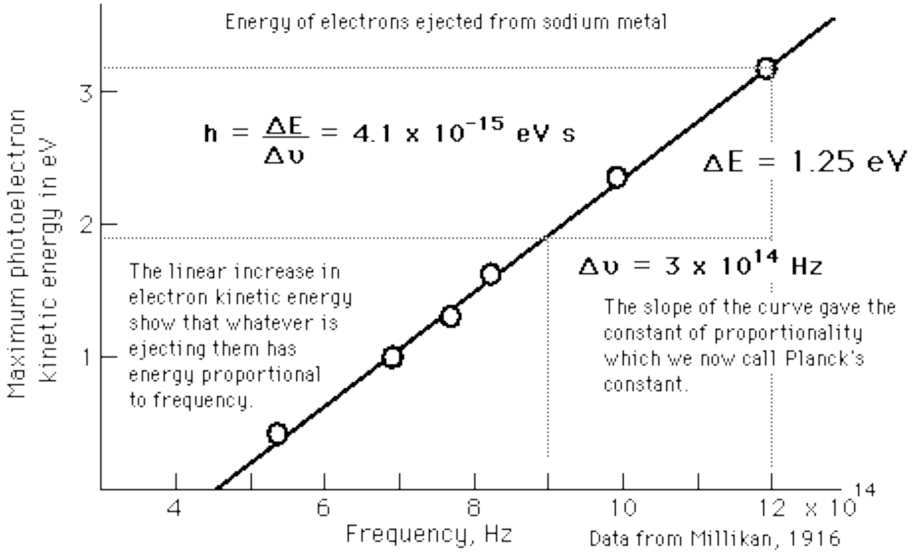
- *The intensity of a light wave is proportional to the square of the amplitude of the electric field.*

…and therefore does not depend on frequency.

- *The energy in the light wave is spread out uniformly and continuously over the wavefront.*

What the data was trying to tell them...





The linear increase in electron kinetic energy show that whatever is ejecting them has energy proportional to frequency.

The slope of the curve gave the constant of proportionality which we now call Planck's constant.

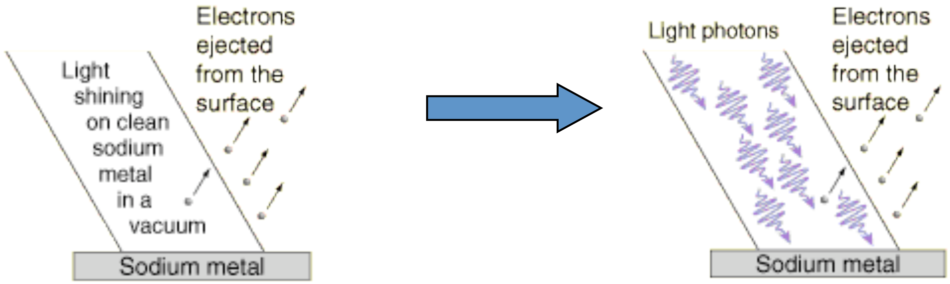
Data from Millikan, 1916

- *The number of photoelectrons ejected depended on the intensity (as expected) but their maximum kinetic energy did not!*
- *The maximum kinetic energy depended only on the frequency, the slope of the linear relationship between the energy and the frequency gives "Planck's constant", h.*
- *The electrons were ejected immediately after the light started shining—the electron instantaneously absorbed enough energy to escape- provided there was enough energy to overcome the binding energy or "work function".*
- *Even a high intensity source of low frequency light cannot liberate electrons. THRESHOLD*

Photoelectric Effect

Light is a particle

We have to change our way of thinking about this picture:



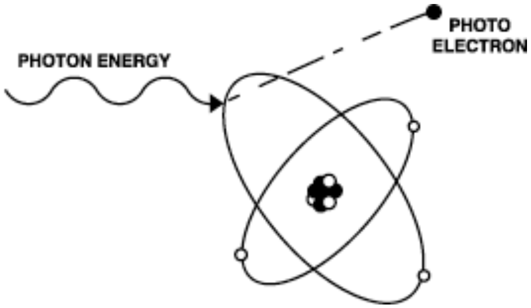
Instead of continuous waves we have to think of the energy as being localized in quanta.

Photon energy

$E = h\nu$

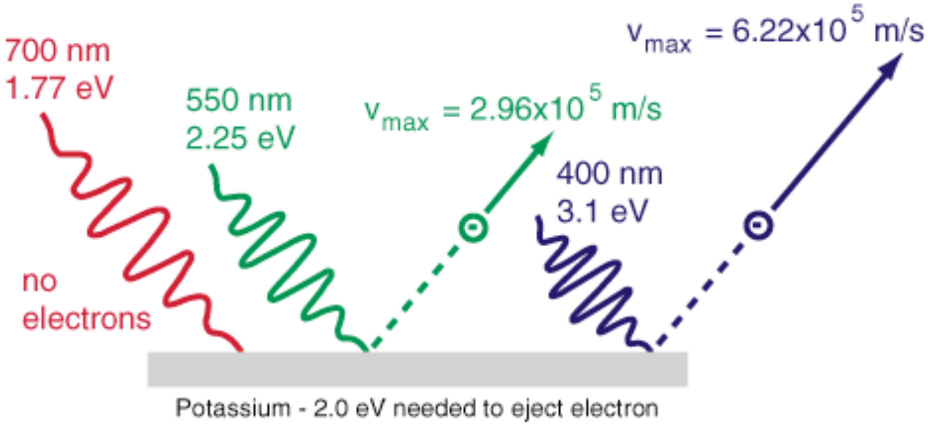
explains the experiment and shows that light behaves like particles.

In the photoelectric effect, these discrete localized quanta of energy, hν, are transferred entirely to the electron



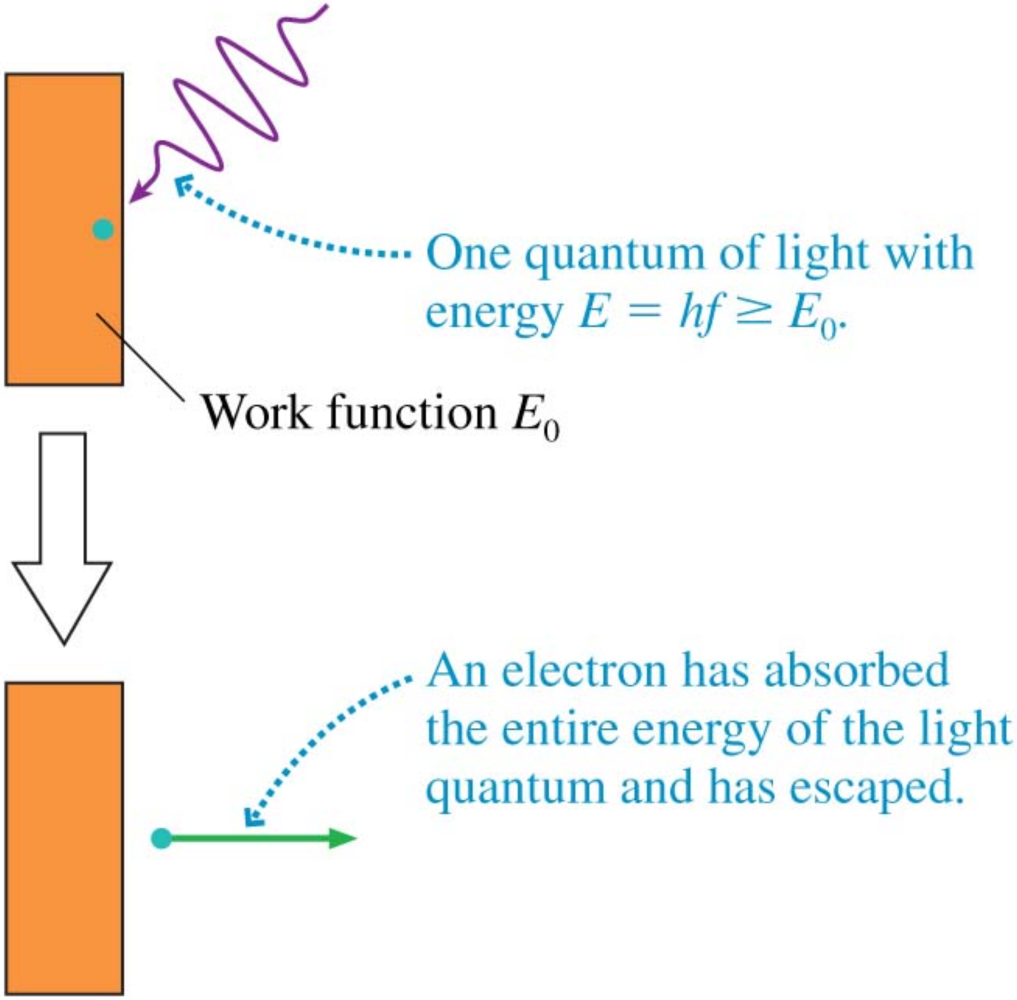
$$K_{\max} = h\nu - e\phi$$

$$E_{\text{photon}} = h\nu$$



Photoelectric effect

FIGURE 39.8 The creation of a photoelectron.



$$K_m = hf - E_o$$

The Photon Model of Light

The **photon model** of light consists of three basic postulates:

1. Light consists of discrete, massless units called photons. A photon travels in vacuum at the speed of light, 3.00×10^8 m/s.
2. Each photon has energy

$$E_{\text{photon}} = hf$$

where f is the frequency of the light and h is a *universal constant* called **Planck's constant.** The value of Planck's constant is $h = 6.63 \times 10^{-34}$ J s.

3. The superposition of a sufficiently large number of photons has the characteristics of a classical light wave.

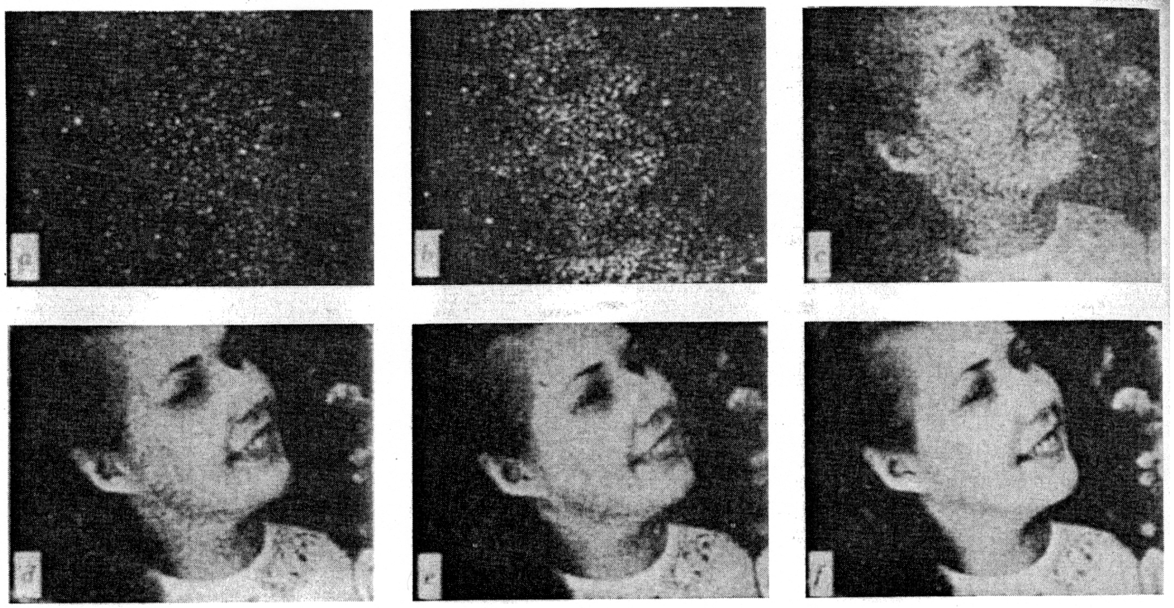
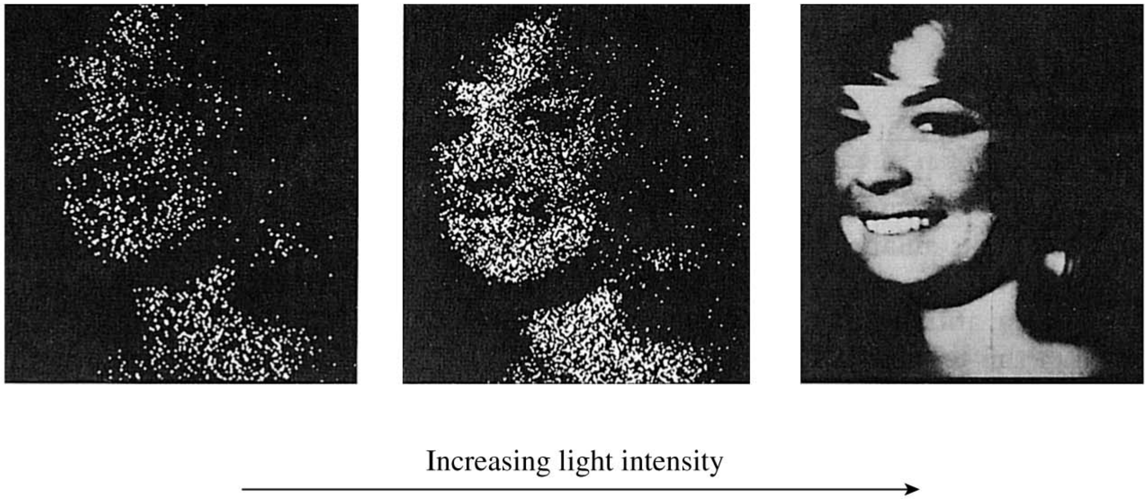


Fig. 2.6 This sequence of photographs of a girl's face shows that photography is a quantum process. The probabilistic nature of quantum effects is evident from the first photographs in which the numbers of photons are very small. As the number of photons increases the photograph becomes more and more distinct until the optimum exposure is reached. The number of photons involved in these photographs ranges from about 3000 in the lowest exposure to about 30 000 000 in the final exposure.

FIGURE 25.9 Photographs made with increasing levels of light intensity.


The photo at very low light levels shows individual points, as if particles are arriving at the detector.

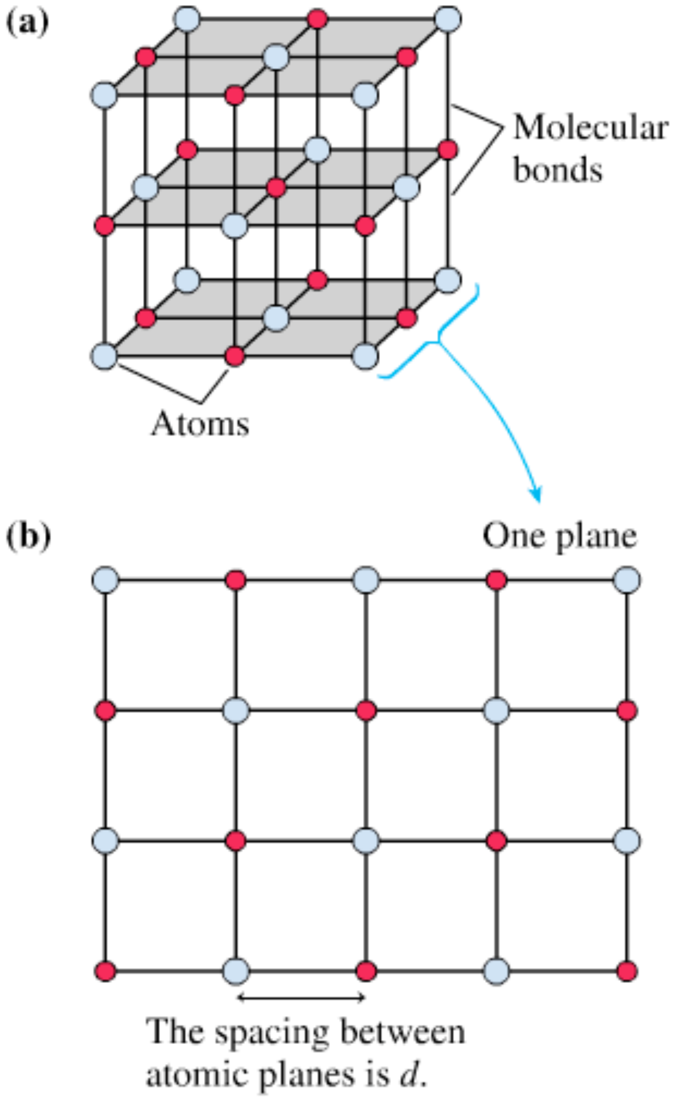
The particle-like behavior is not noticeable at higher light levels.

Increasing light intensity

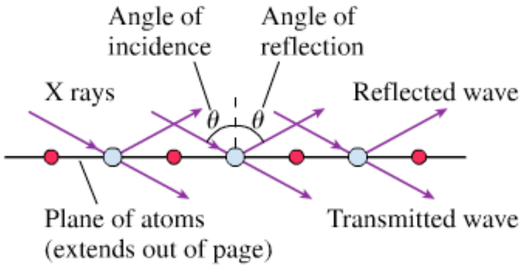
or exposure time

.1-1 nm

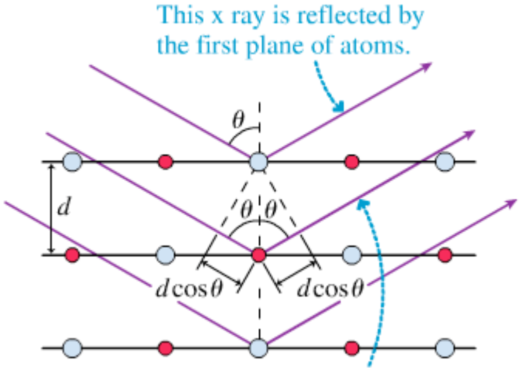
FIGURE 25.5 Atoms arranged in a cubic lattice.



(a) X rays are transmitted and reflected at one plane of atoms.

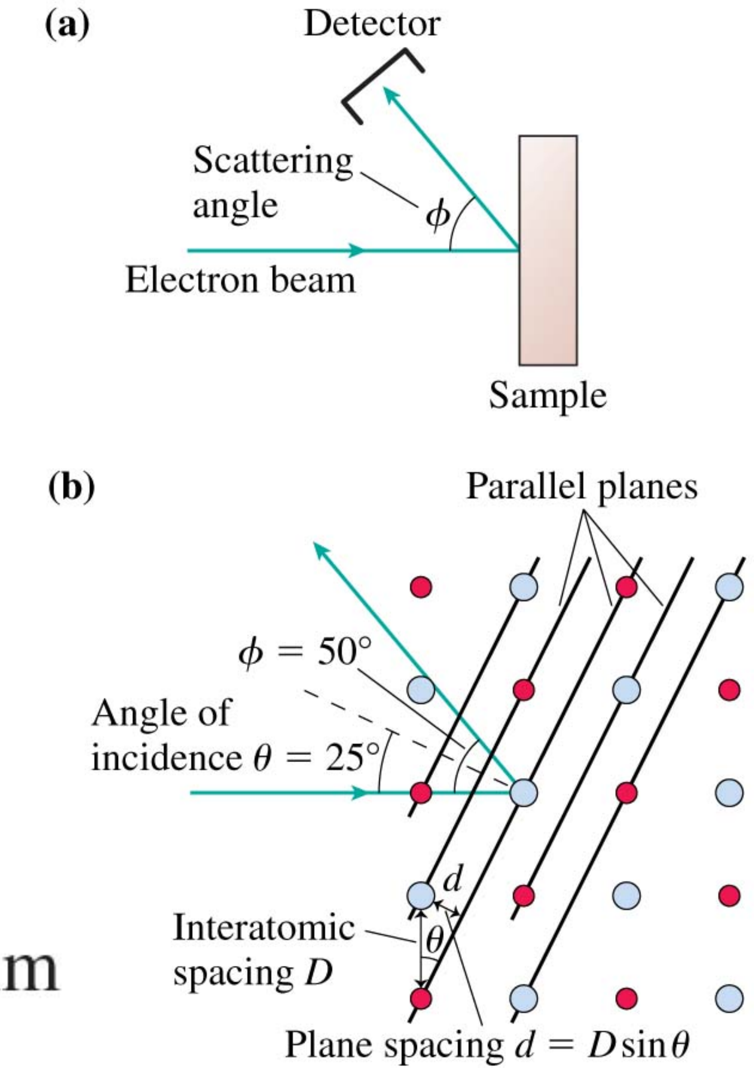


(b) The reflections from parallel planes interfere.



$$\lambda = D\sin(2\theta) = 0.165 \text{ nm}$$

FIGURE 25.11 The Davisson-Germer experiment to study electrons scattered from metal surfaces.



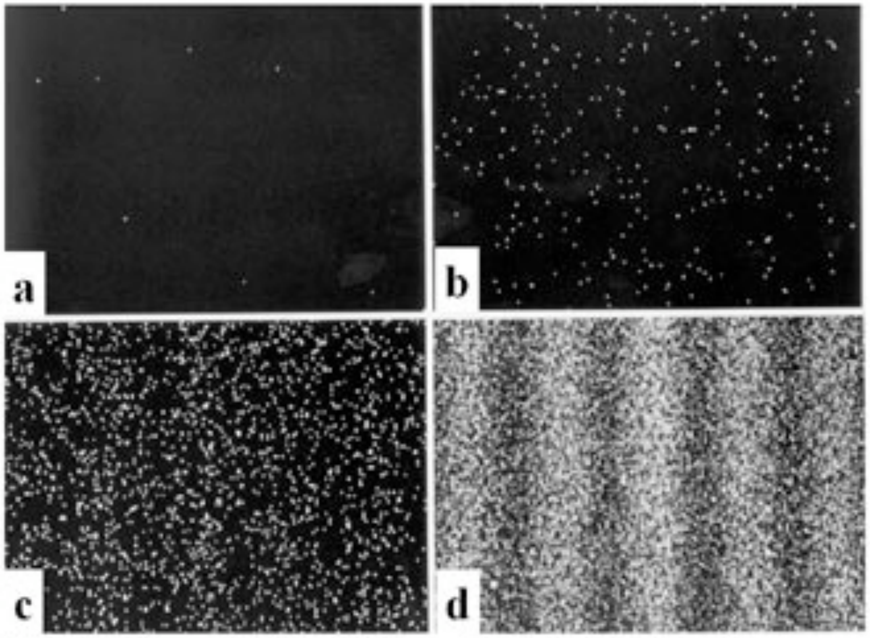
Matter Waves

- In 1927 Davisson and Germer were studying how electrons scatter from the surface of metals.
- They found that electrons incident normal to the crystal face at a speed of 4.35×10^6 m/s scattered at $\phi = 50°$.
- This scattering can be interpreted as a mirror-like reflection from the atomic planes that slice diagonally through the crystal.
- The angle of incidence on this set of planes is the angle θ_m in $2d \cos \theta_m = m\lambda$, the Bragg condition for diffraction.
- Davisson and Germer found that the "electron wavelength" was

$$\lambda = D \sin(2\theta) = 0.165 \text{ nm}$$

Well if light is a particle (photon) is electron a wave?


Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.



Single-electron Build-up of Interference Pattern

Electron Interferometry

When we send many electrons through slit they show interference – wave properties
Just as it happened when we send low intensity (single photon) light

The de Broglie Wavelength

De Broglie postulated that a particle of mass *m* and momentum $p = mv$ has a wavelength

$$\lambda = \frac{h}{p}$$

where *h* is Planck’s constant. This wavelength for material particles is now called the **de Broglie wavelength.** It depends *inversely* on the particle’s momentum, so the largest wave effects will occur for particles having the smallest momentum.

CONCEPTUAL INTERFERENCE EXPERIMENTS WITH WAVES AND PARTICLES

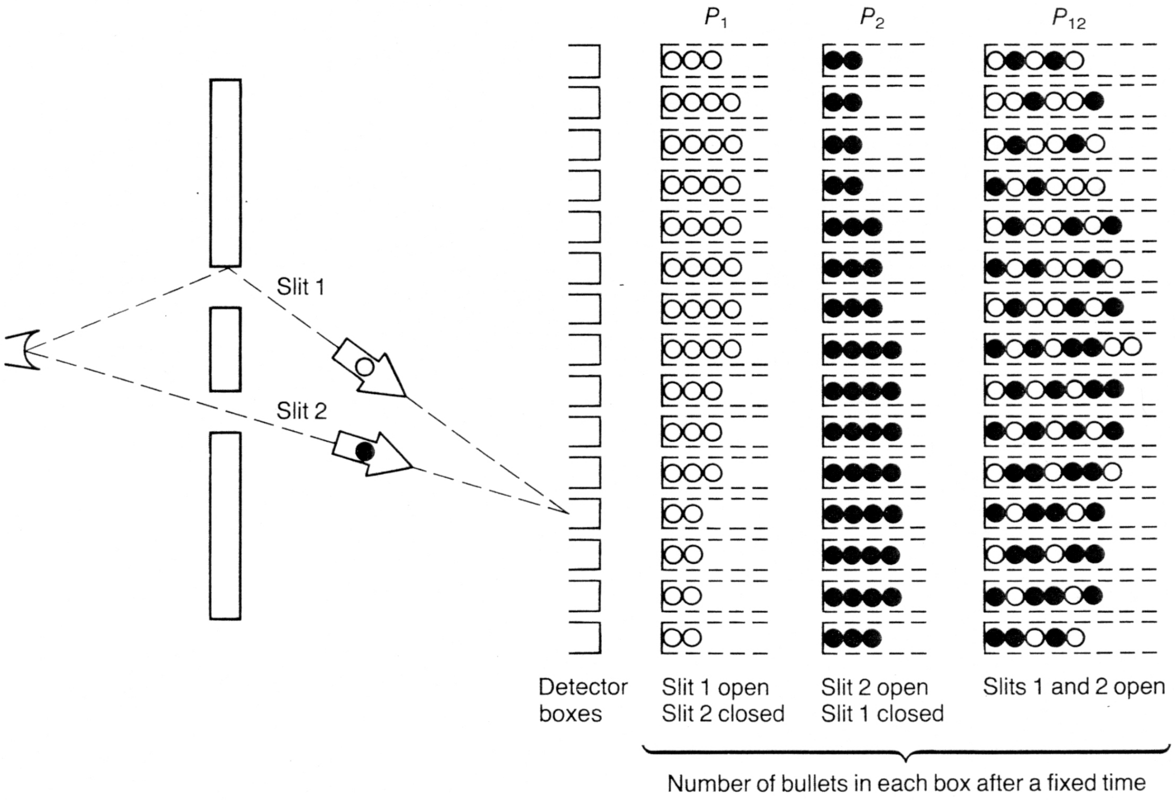


Fig. 1.7 A diagram of a double-slit experiment with bullets. The experimental set-up is shown on the left of the figure and the results of three different experiments indicated on the right. We have shown bullets that pass through slit 1 as open circles and bullets through slit 2 as black circles. The column labelled P_1 shows the distribution of bullets arriving at the detector boxes when slit 2 is closed and only slit 1 is open. Column P_2 shows a similar distribution obtained with slit 1 closed and slit 2 open. As can be seen, the maximum number of bullets appears in the boxes directly in line with the slit that is left open. The result obtained with both slits open is shown in the column labelled P_{12}. It is now a matter of chance through which slit a bullet will come and this is shown by the scrambled mixture of black and white bullets collected in each box. The important point to notice is that the total obtained in each box when both slits are open is just the sum of the numbers obtained when only one or other of the slits is open. This is obvious in the case of bullets since we know that bullets must pass through one of the slits to reach the detector boxes.

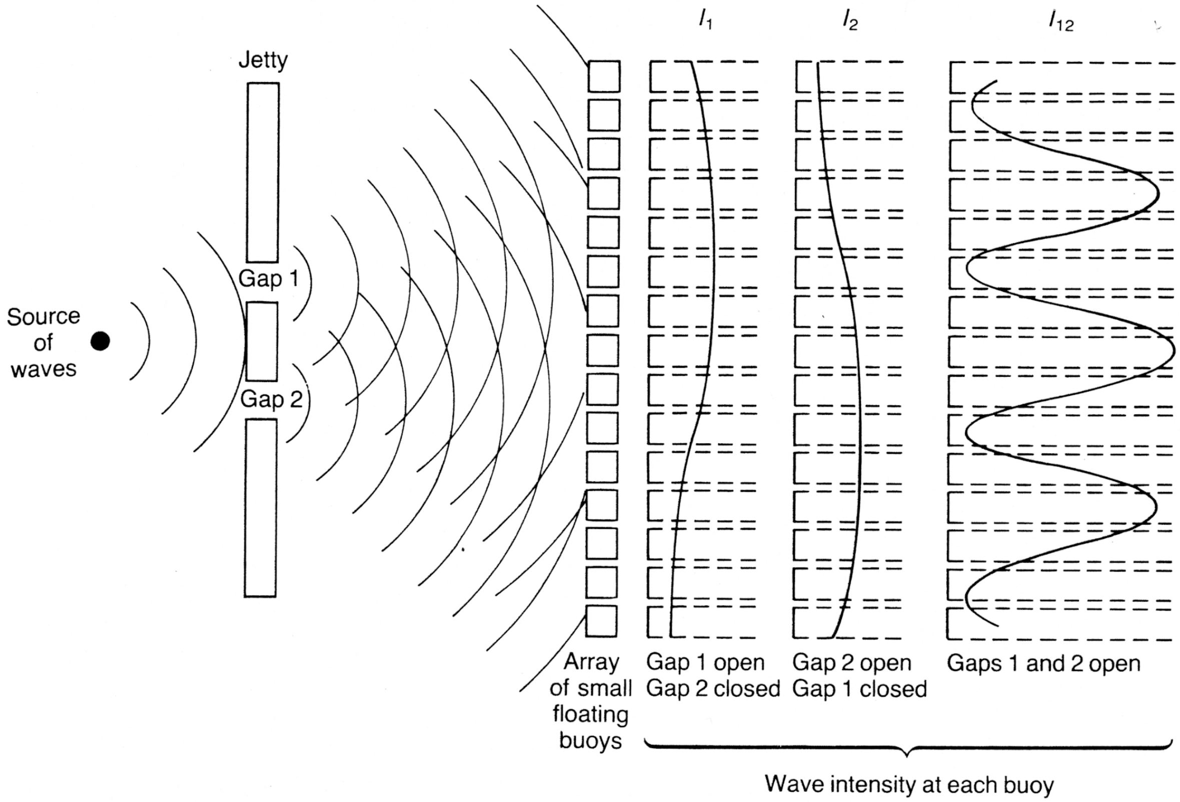


Fig. 1.9 A diagram of a double-slit experiment with water waves. The detectors are a line of small floating buoys whose jiggling up and down provides a measure of the wave energy. The wave crests spreading out from each slit are shown in the figure and can be compared with fig. 1.8. The column labelled I_1 *shows the smoothly varying wave intensity obtained when only gap 1 is open. Notice that this is very similar to the pattern* P_1 *obtained with bullets in fig. 1.7 with only slit 1 open. Again it is largest at the detector directly in line with gap 1 and the source. The second column shows that a similar pattern,* I_2*, is obtained when gap 1 is closed and gap 2 is open. The final column,* I_{12}*, shows the wave intensity pattern obtained with both slits open. It is dramatically different from the pattern obtained for bullets with both slits open. It is not equal to the sum of the patterns* I_1 *and* I_2 *obtained with one of the gaps closed. This rapidly varying intensity curve is called an interference pattern.*

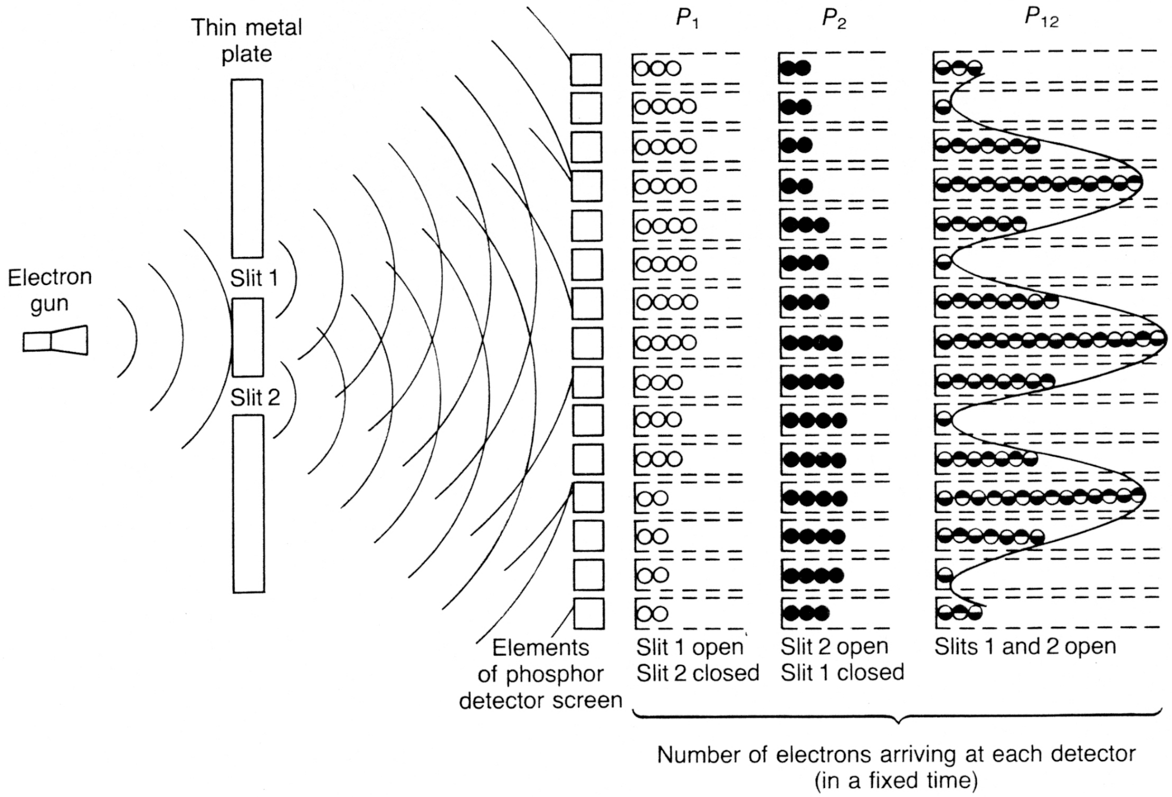


Fig. 1.11 A diagram of a double-slit experiment with electrons. Electrons always arrive with a flash at the phosphor detector at one point, in the same way that bullets always end up in just one of the detector boxes rather than the energy being spread out, as in a wave. The column marked P_1 *shows the pattern obtained with only slit 1 open. Electrons that have gone through slit 1 are represented as open circles, like the bullets of fig. 1.7. Column* P_2 *shows the same thing with only slit 2 open and the electrons that have gone through slit 2 indicated by black circles. These two patterns are exactly the same as those obtained with bullets. The difference lies in the column headed* P_{12}*, which shows the pattern obtained for electrons when both slits are open. This is just the interference pattern obtained with water waves and requires some kind of wave motion arising from each slit as indicated on the figure. It is not the sum of* P_1 *and* P_2 *and so we cannot say which slit any electron goes through. We have indicated this lack of knowledge by drawing the electrons, which still arrive like bullets, as half white and half black circles. This fact, that quantum objects such as electrons possess attributes of both wave and particle motion but behave like neither, is the central mystery of quantum mechanics.*

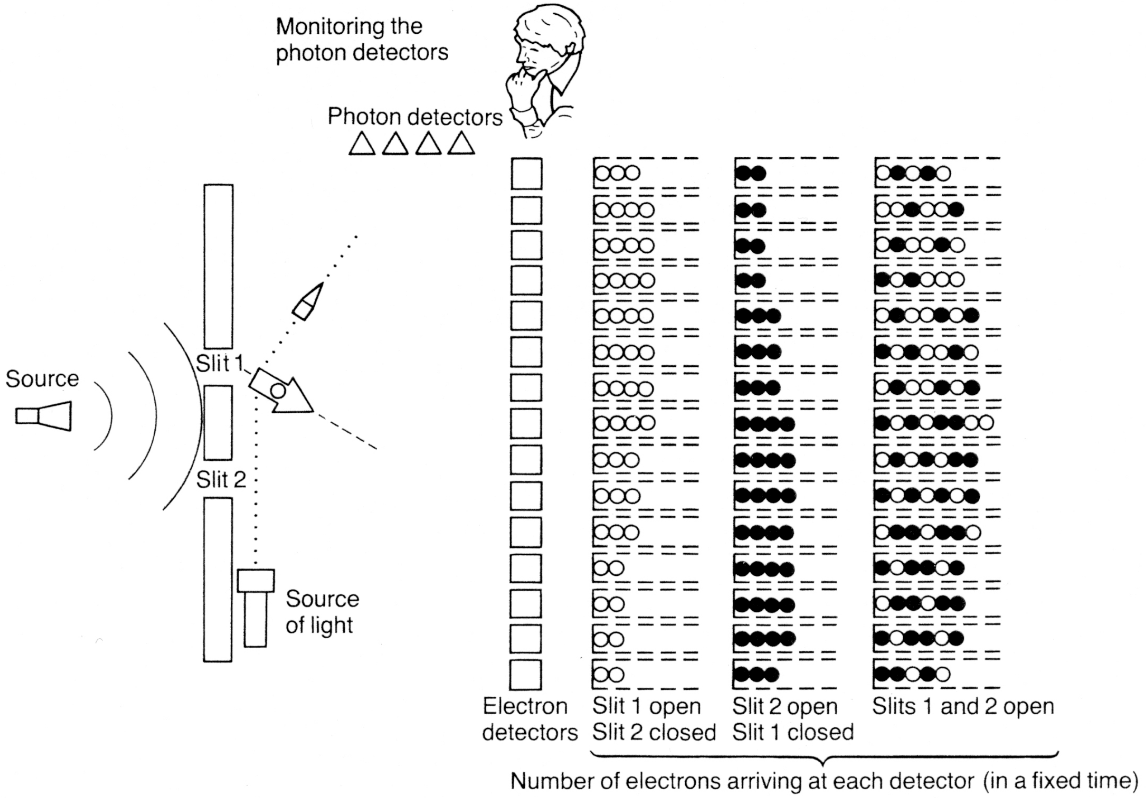
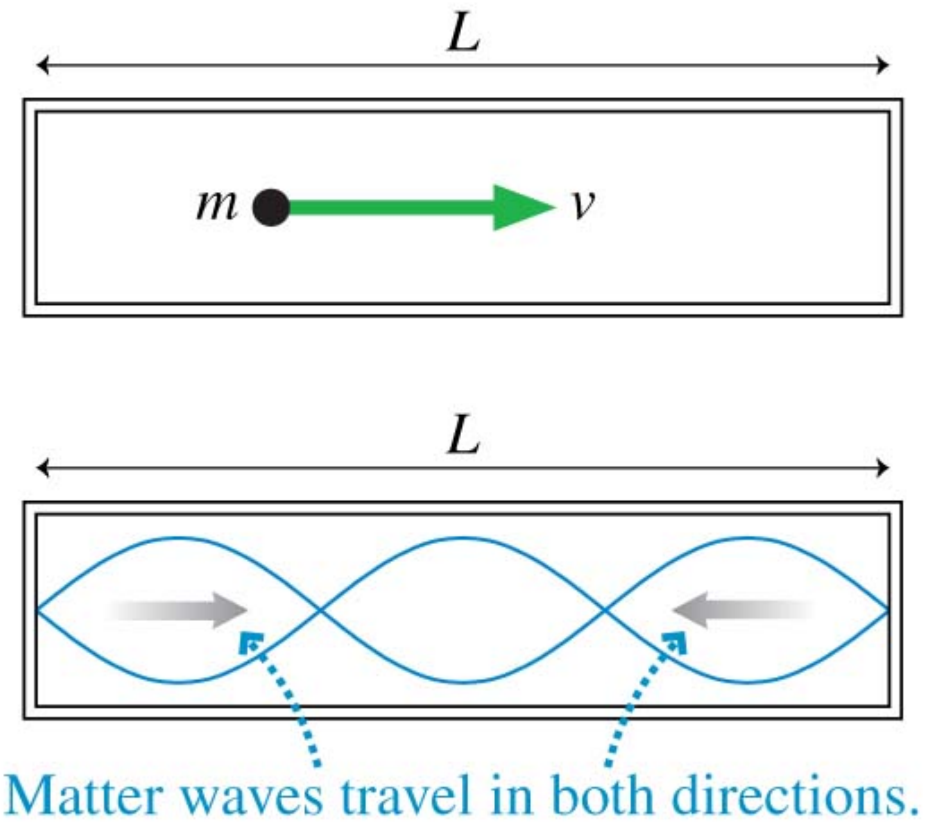


Fig. 2.3 Sketch of the experimental set-up required to observe through which slit the electron passes in a double-slit experiment. Light, in the form of photons, is directed at the slits. In the figure a photon, represented as a small bullet, has hit an electron behind slit 1. The electron is disturbed slightly in its motion and the scattered photon is observed at the photon detectors. The electron patterns obtained with only one of the slits open are almost the same as before, when we did not observe the electron behind the slits. The surprise occurs with both slits open: there is no interference pattern. The small nudges given to the electrons in their collisions with the photons are always sufficient to wash out the interference pattern completely! We can now say with certainty through which slit the electron went but now the electrons are behaving just like bullets. The observed pattern is just the sum of the patterns for slit 1 and slit 2 separately.

FIGURE 39.15 A particle in a box creates a standing de Broglie wave as it reflects back and forth.



Matter waves travel in both directions.

$$L = n\lambda/2$$

$$\lambda_n = \frac{2L}{n} \qquad n = 1, 2, 3, 4, \ldots$$

Quantization of Energy

- Consider a particle of mass *m* moving in one dimension as it bounces back and forth with speed *v* between the ends of a box of length *L.* We'll call this a *one-dimensional box;* its width isn't relevant.
- A wave, if it reflects back and forth between two fixed points, sets up a standing wave.
- A standing wave of length *L must* have a wavelength given by

$$\lambda_n = \frac{2L}{n} \qquad n = 1, 2, 3, 4, \ldots$$

Quantization of Energy

Using the de Broglie relationship $\lambda = h/mv$, a standing wave with wavelength λ_n forms when the particle has a speed

$$v_n = n\left(\frac{h}{2Lm}\right) \qquad n = 1, 2, 3, \ldots$$

Thus the particle's energy, which is purely kinetic energy, is

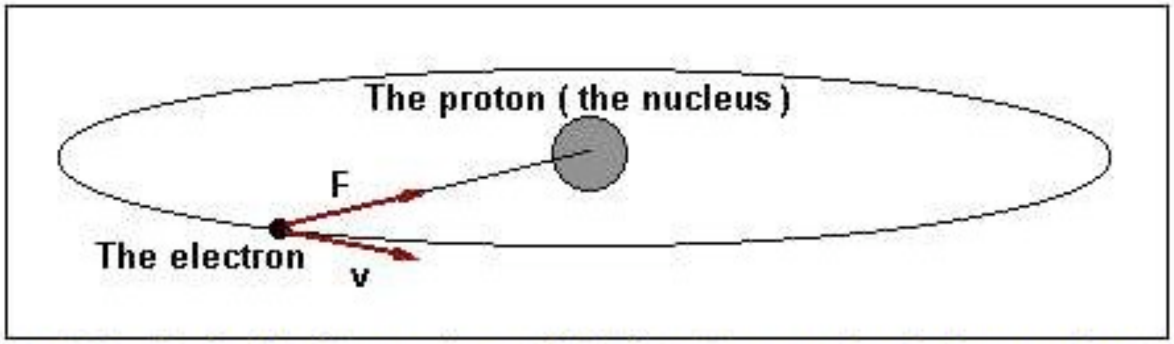
$$E_n = \frac{1}{2}mv_n^2 = n^2\frac{h^2}{8mL^2} \qquad n = 1, 2, 3, \ldots$$

De Broglie's hypothesis about the wave-like properties of matter leads us to the remarkable conclusion that **the energy of a confined particle is quantized.**

The Planetary Model

The attractive Coulomb force between the positive nucleus and the orbiting electron could provide the attractive force which keeps the electron in it's orbit, much as the planets orbit the sun with gravity providing the centripetal force.

$$F_{centripetal} = \frac{m_e v_0^2}{r} = m_e \omega_0^2 r$$



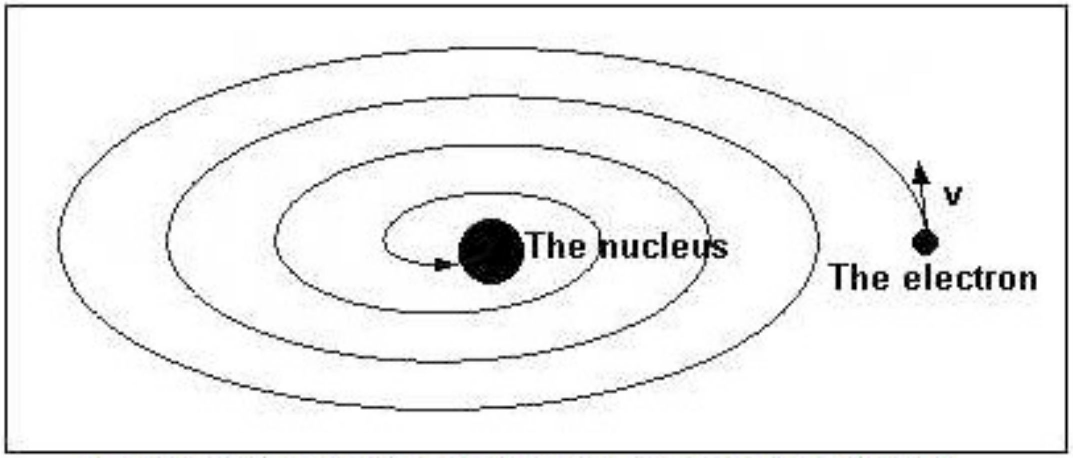
The Rutherford's atomic model. The electron circulating on the orbit around the nucleus with the velocity v is attracted by it with the force F

What's wrong with this picture?

Accelerating charges radiate. Could this electromagnetic radiation be the source of the spectral lines?

No. This radiation must come at the expense of the kinetic energy of the orbiting electron!

It will eventually spiral into the nucleus. The atom would be unstable!



In the planetary model of atom, the electron should emit energy and spirally fall on the nucleus.

Bohr's Model of Atomic Quantization

1. An atom consists of negative electrons orbiting a very small positive nucleus.
2. Atoms can exist only in certain **stationary states.** Each stationary state corresponds to a particular set of electron orbits around the nucleus. These states can be numbered 2, 3, 4, . . . , where *n* is the *quantum number.*
3. Each stationary state has an energy E_n. The stationary states of an atom are numbered in order of increasing energy: $E_1 < E_2 < E_3 < \ldots$
4. The lowest energy state of the atom E_1 is *stable* and can persist indefinitely. It is called the **ground state** of the atom. Other stationary states with energies E_2, E_3, E_4,. . . are called **excited states** of the atom.

Bohr Atom

$$\vec{F}_{cb} = k\frac{q_1 q_2}{r^2}\hat{r}$$

$$-k\frac{e^2}{r^2} + m\frac{v_\theta^2}{r} = 0$$

$$r = \frac{ke^2}{mv_\theta^2}$$

$$\hbar \equiv h/2\pi$$

$$mv_\theta r = n\hbar, n = 1, 2, 3, 4\ldots$$

$$v_\theta = n\hbar / mr$$

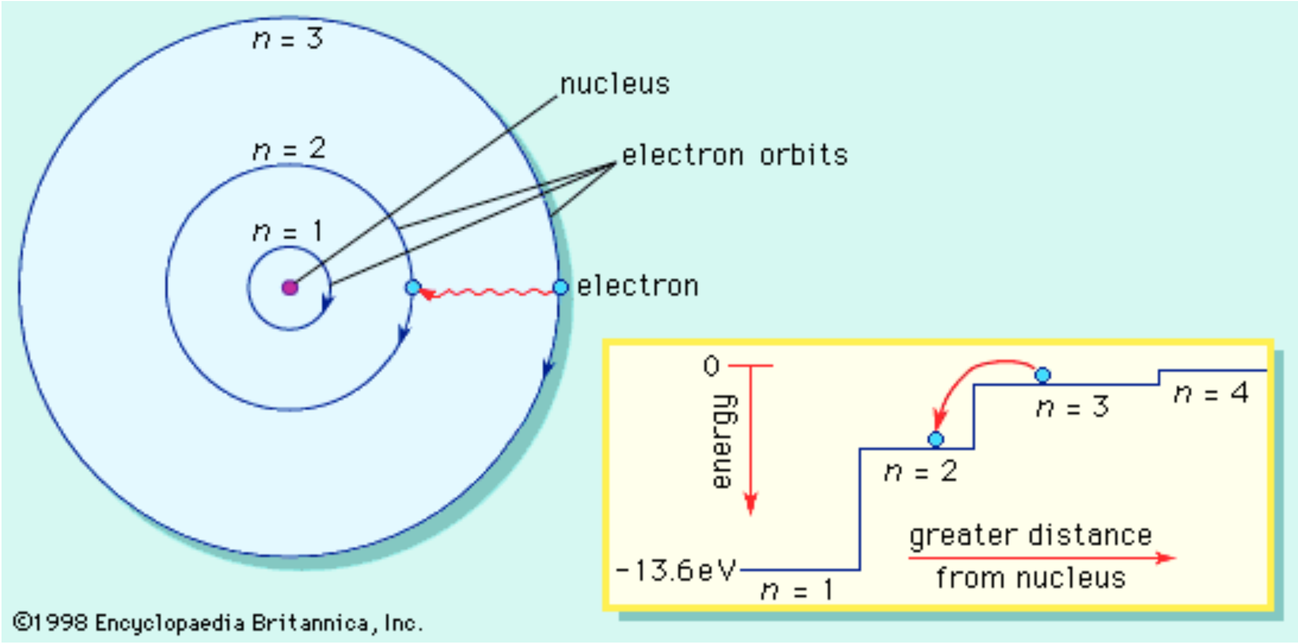
$$r = n^2(\hbar^2 / kme^2)$$

$r=n^2a_B$,

$a_B= (h/2\pi)^2/(ke^2m)=.0529$ nm

$E_n=-E_1/n^2$

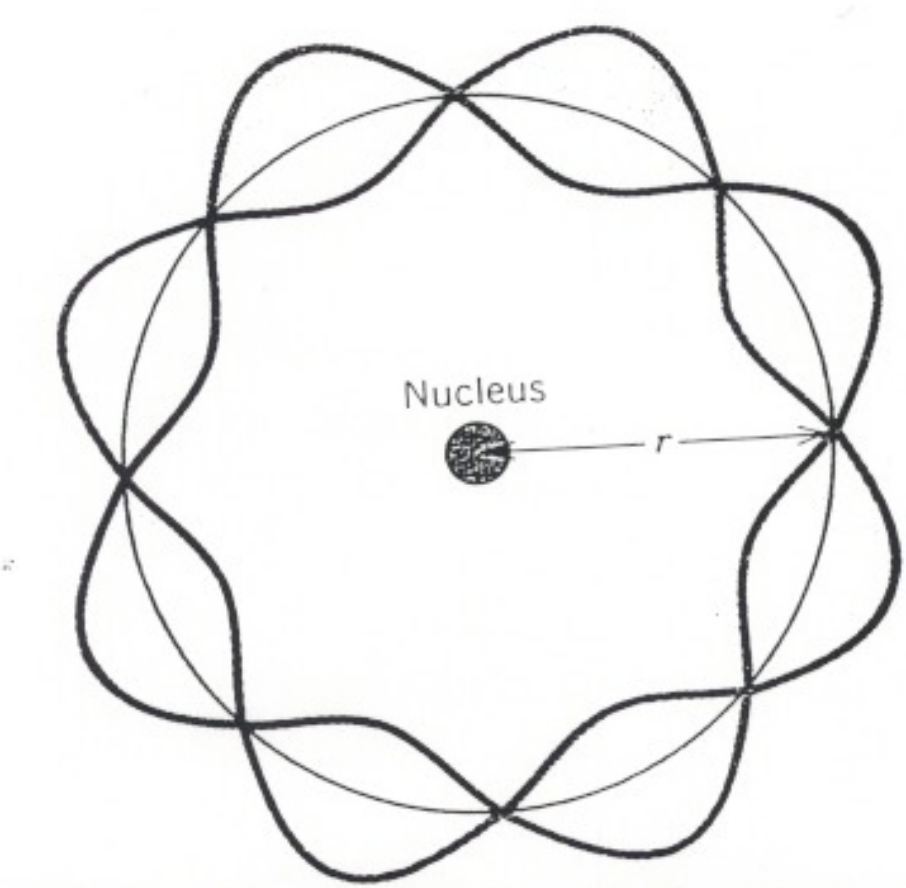
$E_1=13.6$ eV



©1998 Encyclopaedia Britannica, Inc.

Why quantization of angular momentum?

Phase cancellation



$$\boxed{L = m_e vr = n\hbar \quad n = 1,2,3\ldots \\ \hbar = h/2\pi}$$

an integer number of wavelengths fits into the circular orbit

$$\boxed{n\lambda = 2\pi r}$$

where

Photons p=hf/c= h/λ → $\boxed{\lambda = \frac{h}{p}}$

λ is the de Broglie wavelength

In order to understand quantum mechanics, you must understand waves!

Bohr’s Model of Atomic Quantization

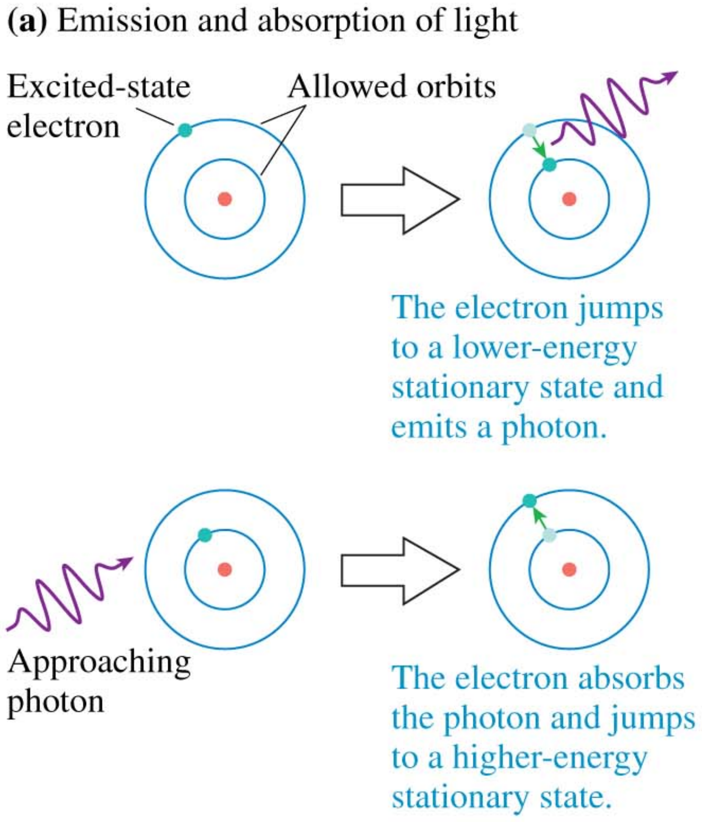
5. An atom can “jump” from one stationary state to another by emitting or absorbing a photon of frequency

$$f_{\text{photon}} = \frac{\Delta E_{\text{atom}}}{h}$$

where h is Planck’s constant and and $\Delta E_{\text{atom}} = |E_f - E_i|$.

E_f and E_i are the energies of the initial and final states. Such a jump is called a **transition** or, sometimes, a **quantum jump.**

FIGURE 39.17 An atom can change stationary states by emitting or absorbing a photon or by undergoing a collision.



(a) Emission and absorption of light

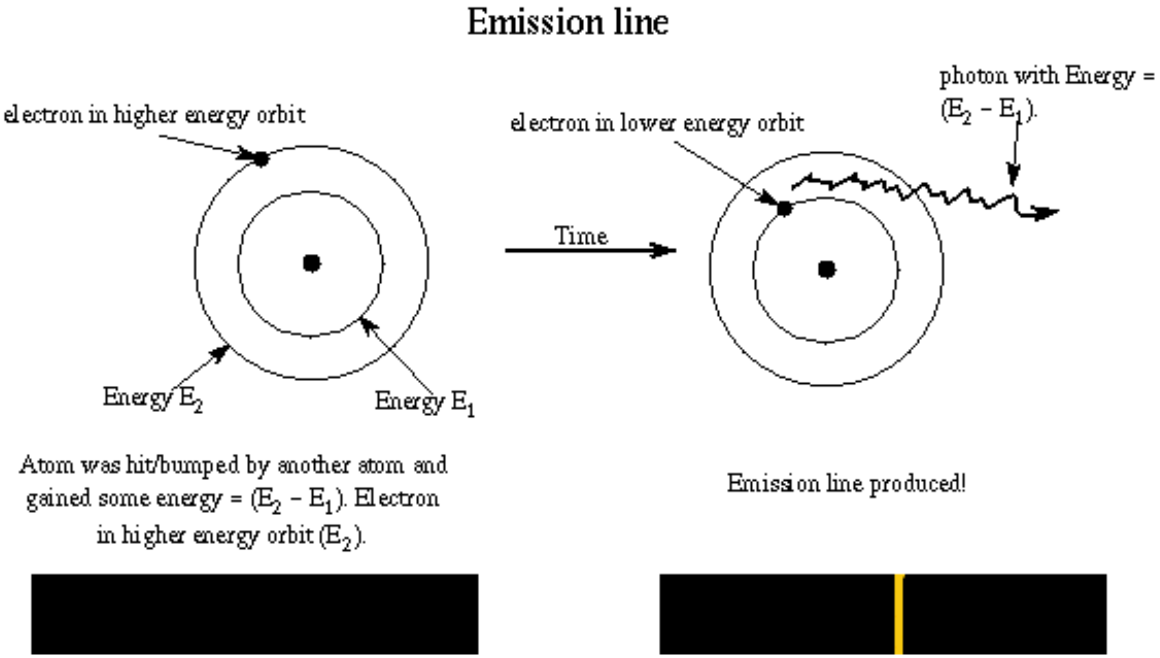
Emission

The electron jumps to a lower-energy stationary state and emits a photon.

Absorption

The electron absorbs the photon and jumps to a higher-energy stationary state.

Emission


Emission line
electron in higher energy orbit
Energy E_2
Energy E_1
Time
electron in lower energy orbit
photon with Energy = (E_2 − E_1).
Atom was hit/bumped by another atom and gained some energy = (E_2 − E_1). Electron in higher energy orbit (E_2).
Emission line produced!

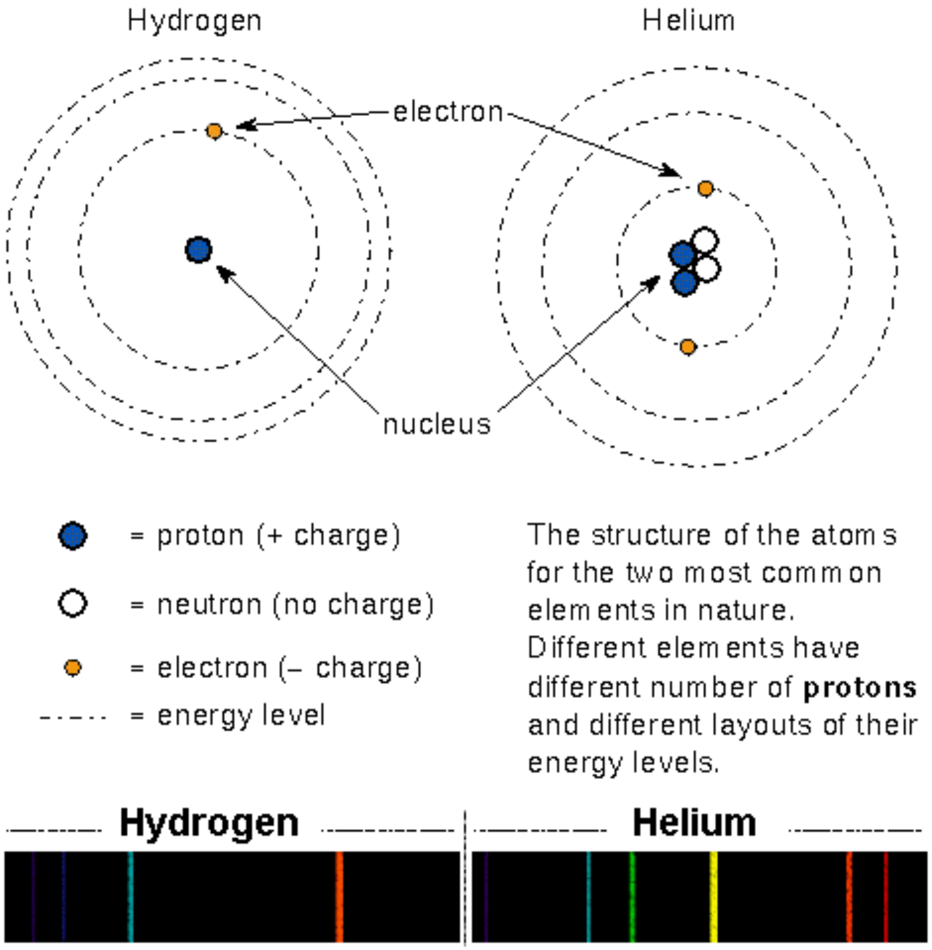
Line Spectra

To explain discrete spectra, Bohr found that atoms obey three basic rules:

1. Electrons have only certain energies corresponding to particular distances from nucleus. As long as the electron is in one of those energy orbits, it will not lose or absorb any energy. The energy orbits are analogous to rungs on a ladder: electrons can be only on rungs of the ladder and not in between rungs.

2. The orbits closer to the nucleus have lower energy.

3. Atoms want to be in the lowest possible energy state called the **ground state** (all electrons as close to the nucleus as possible).



The structure of the atoms for the two most common elements in nature. Different elements have different number of **protons** and different layouts of their energy levels.

Bohr's Model of Atomic Quantization

6. An atom can move from a lower energy state to a higher energy state by absorbing energy $\Delta E_{\text{atom}} = E_f - E_i$ in an inelastic collision with an electron or another atom.

This process, called **collisional excitation,** is shown.

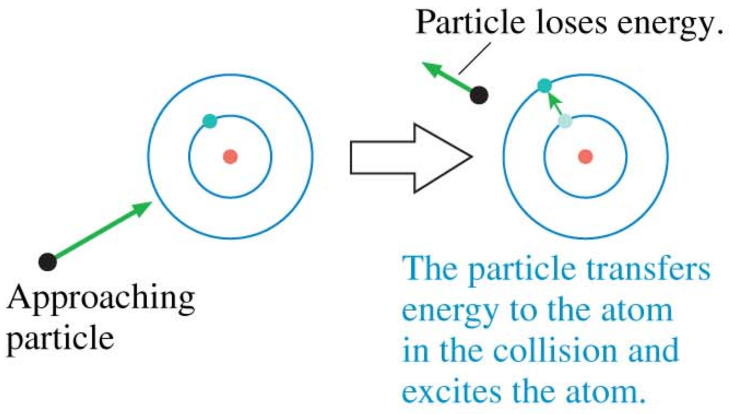
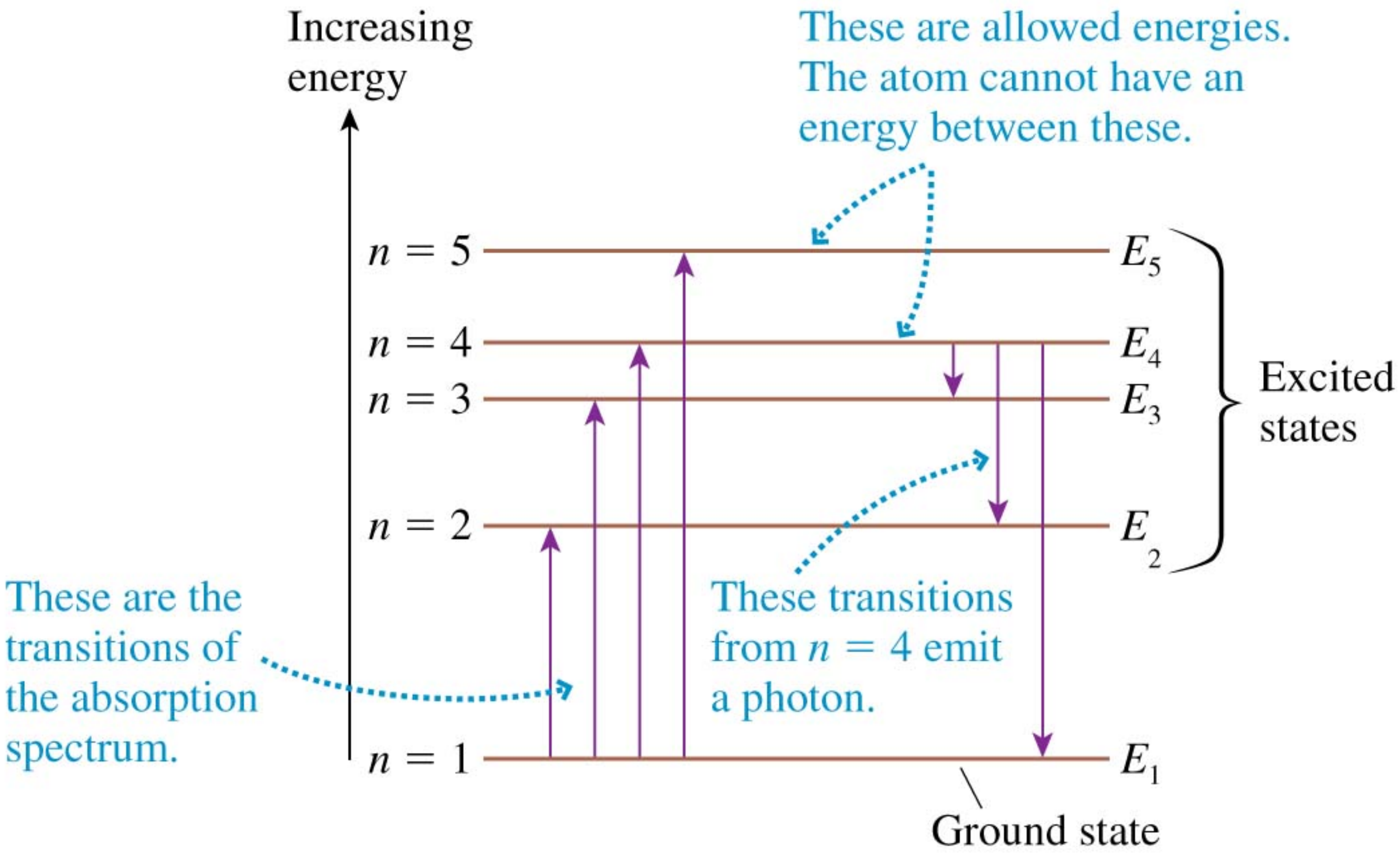


FIGURE 39.17 An atom can change stationary states by emitting or absorbing a photon or by undergoing a collision.

(b) Collisional excitation

The particle transfers energy to the atom in the collision and excites the atom.

FIGURE 39.18 An energy-level diagram.



The Bohr Hydrogen Atom

The radius of the electron's orbit in Bohr's hydrogen atom is

$$r_n = n^2 a_{\mathrm{B}} \qquad n = 1, 2, 3, \ldots$$

where a_B is the Bohr radius, defined as

$$a_{\mathrm{B}} = \text{Bohr radius} \equiv \frac{4\pi\epsilon_0\hbar}{me^2} = 5.29 \times 10^{-11}\ \mathrm{m} = 0.0529\ \mathrm{nm}$$

The possible electron speeds and energies are

$$v_n = \frac{n\hbar}{mr_n} = \frac{1}{n}\frac{\hbar}{ma_{\mathrm{B}}} = \frac{v_1}{n} \qquad n = 1, 2, 3, \ldots$$

$$E_n = -\frac{E_1}{n^2} = -\frac{13.60\ \mathrm{eV}}{n^2} \qquad n = 1, 2, 3, \ldots$$

The work function of metal A is 3.0 eV. Metals B and C have work functions of 4.0 eV and 5.0 eV, respectively. Ultraviolet light shines on all three metals, creating photoelectrons. Rank in order, from largest to smallest, the stopping potential for A, B, and C.

A. $V_C > V_B > V_A$
B. $V_A > V_B > V_C$
C. $V_A = V_B = V_C$

The work function of metal A is 3.0 eV. Metals B and C have work functions of 4.0 eV and 5.0 eV, respectively. Ultraviolet light shines on all three metals, creating photoelectrons. Rank in order, from largest to smallest, the stopping potential for A, B, and C.

A. $V_C > V_B > V_A$
[✓ **B.** $\boldsymbol{V_A > V_B > V_C}$]
C. $V_A = V_B = V_C$

$\Delta V = -\Delta V_{stop}$

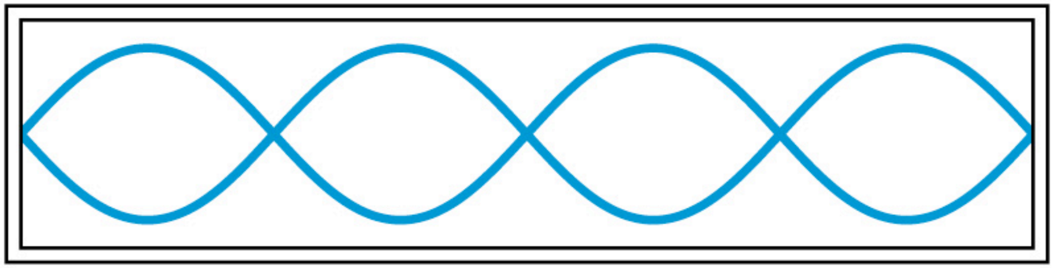
The intensity of a beam of light is increased but the light’s frequency is unchanged. Which of the following is true?

A. The photons are larger.
B. There are more photons per second.
C. The photons travel faster.
D. Each photon has more energy.

The intensity of a beam of light is increased but the light’s frequency is unchanged. Which of the following is true?

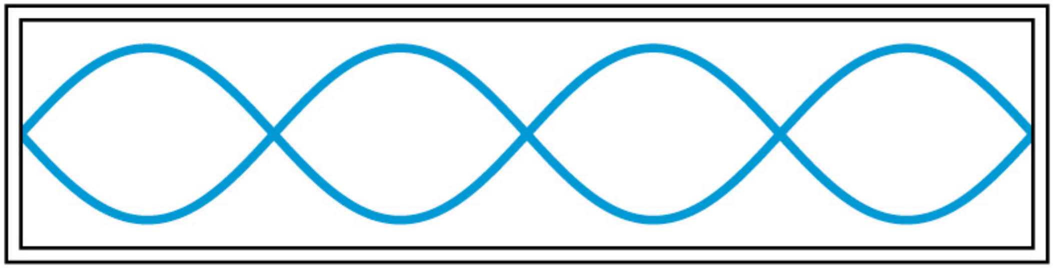
A. The photons are larger.
✓ **B. There are more photons per second.**
C. The photons travel faster.
D. Each photon has more energy.

What is the quantum number of this particle confined in a box?



- A. $n = 8$
- B. $n = 6$
- C. $n = 5$
- D. $n = 4$
- E. $n = 3$

What is the quantum number of this particle confined in a box?



A. $n = 8$
B. $n = 6$
C. $n = 5$
[✓ D. **$n = 4$**]
E. $n = 3$

A photon with a wavelength of 414 nm has energy E_{photon} = 3.0 eV. Do you expect to see a spectral line with = 414 nm in the emission spectrum of the atom represented by this energy-level diagram? If so, what transition or transitions will emit it?

A. Yes. 1→0

B. Yes. 2→0

C. Yes. 2→1

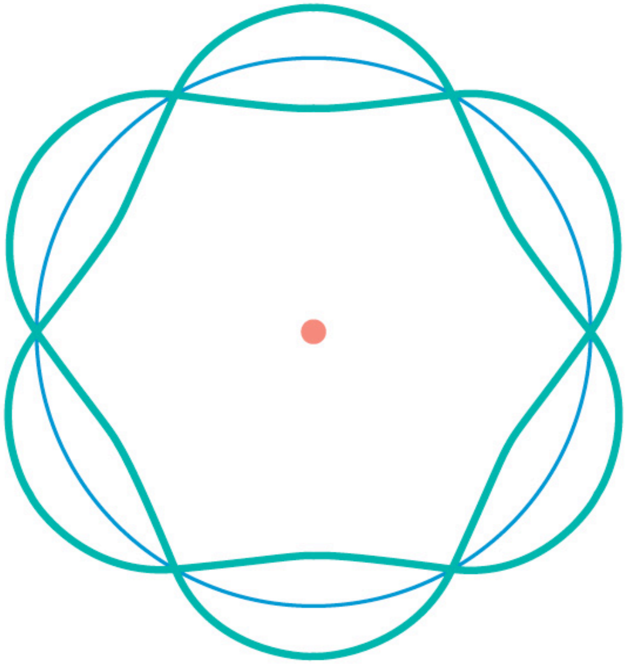
D. Yes. 3→1

E. No

A photon with a wavelength of 414 nm has energy E_{photon} = 3.0 eV. Do you expect to see a spectral line with λ = 414 nm in the emission spectrum of the atom represented by this energy-level diagram? If so, what transition or transitions will emit it?

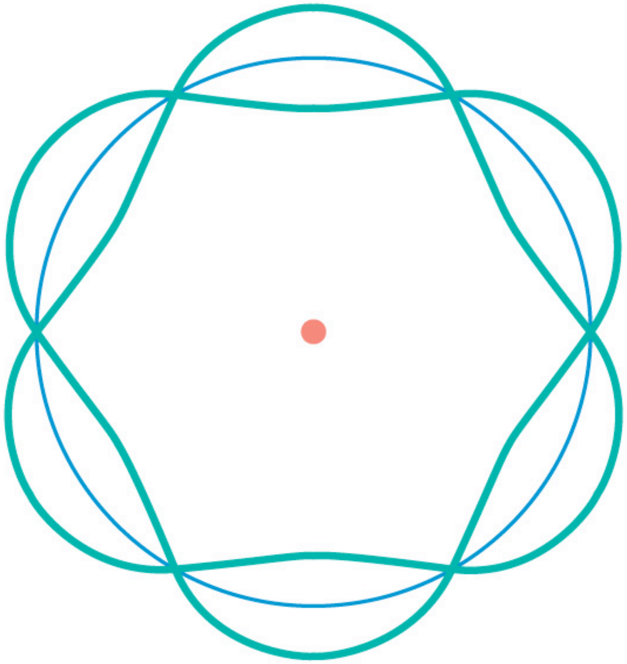
A. Yes. 1→0
B. Yes. 2→0
[✓ **C. Yes. 2→1**]
D. Yes. 3→1
E. No

What is the quantum number of this hydrogen atom?



A. $n = 5$
B. $n = 4$
C. $n = 3$
D. $n = 2$
E. $n = 1$

What is the quantum number of this hydrogen atom?



A. $n = 5$
B. $n = 4$
✓ **C. $n = 3$**
D. $n = 2$
E. $n = 1$