PHYS 270-SPRING 2011

LECTURE # 17

Modern Optics

Matter Waves

April 5, 2011

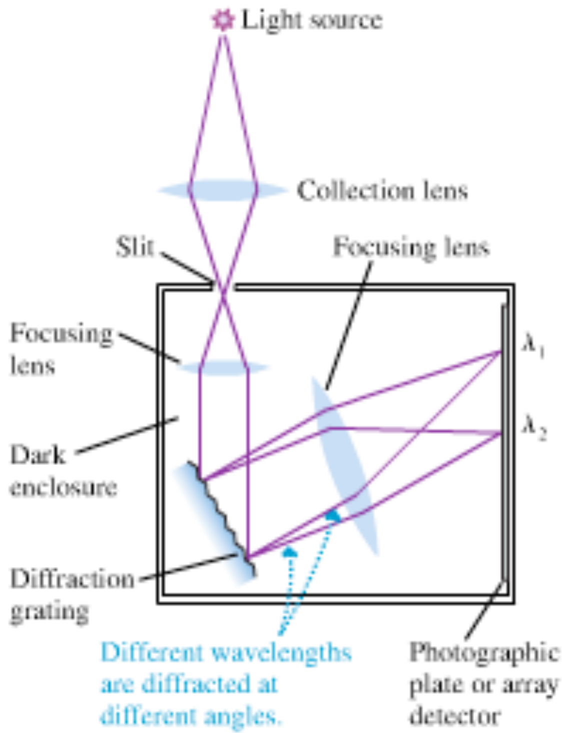
Spectroscopy: Unlocking the Structure of Atoms

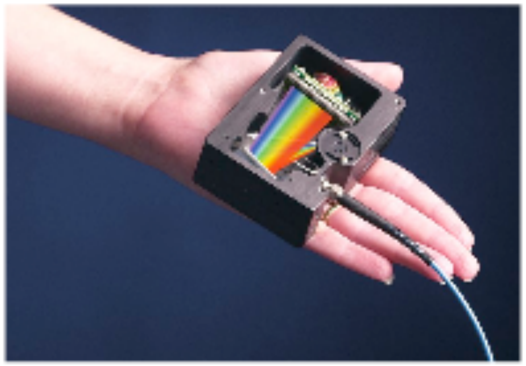
There are two types of spectra, continuous spectra and discrete spectra:

- Hot, self-luminous objects, such as the sun or an incandescent light bulb, emit a *continuous spectrum* in which a rainbow is formed by light being emitted at every possible wavelength.
- In contrast, the light emitted by a gas discharge tube (such as those used to make neon signs) contains only certain discrete, individual wavelengths. Such a spectrum is called a **discrete spectrum.**

HOW TO MEASURE SPECTRA

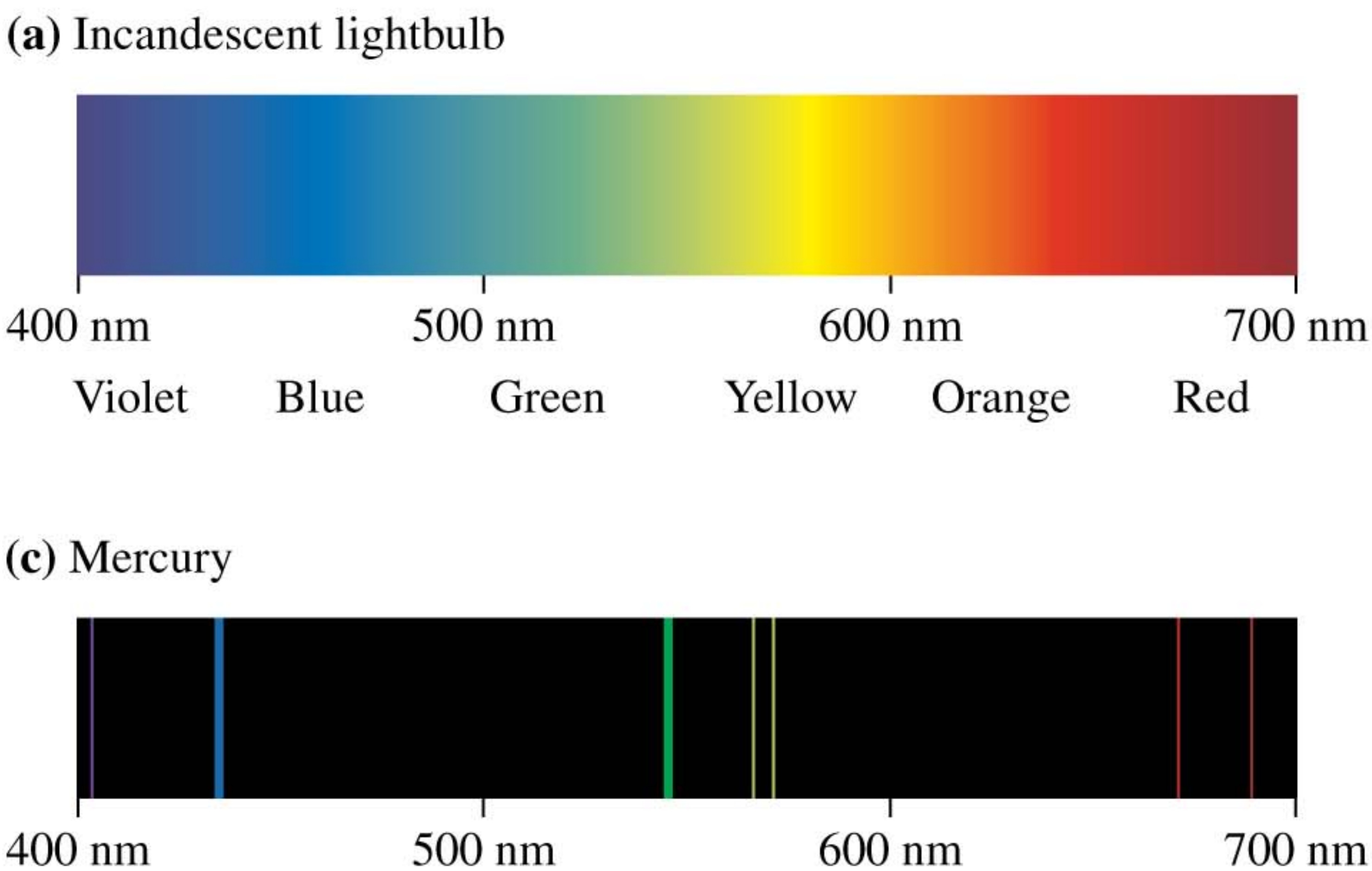
FIGURE 25.1 A diffraction spectrometer for the accurate measurement of wavelengths.





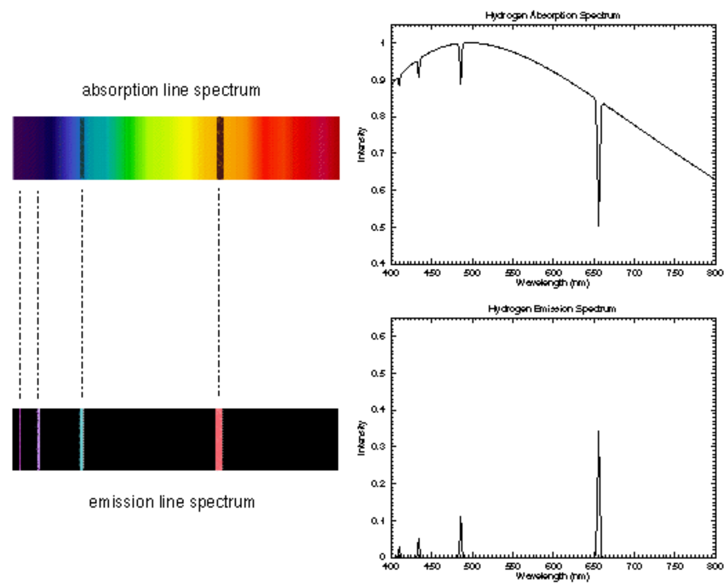
Some modern spectrometers are small enough to hold in your hand. (The rainbow has been superimposed to show how it works.)

FIGURE 25.2 Examples of spectra in the visible wavelength range 400–700 nm.



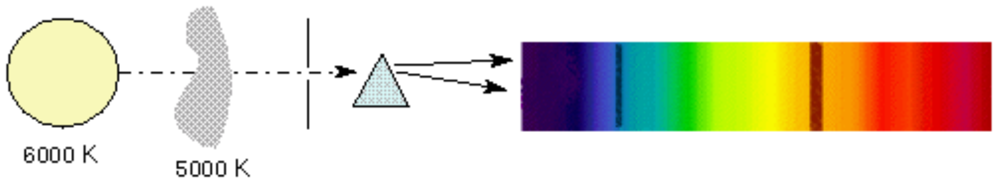
Absorption and Emission Lines

Close examination of the spectra from the Sun and other stars reveals that the rainbow of colors has many dark lines in it, called **absorption lines**. They are produced by the cooler thin gas in the upper layers of the stars absorbing certain colors of light produced by the hotter dense lower layers. You can also see them in the reflected light spectrum from planets. Some of the colors in the sunlight reflecting off the planets are absorbed by the molecules on the planet's surface or in its atmosphere. The spectra of hot, thin (low density) gas clouds are a series of bright lines called **emission lines**. In both of these types of spectra you see spectral features at certain, discrete wavelengths (or colors) and no where else.

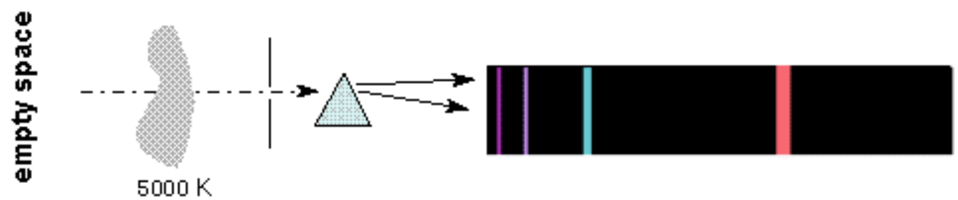


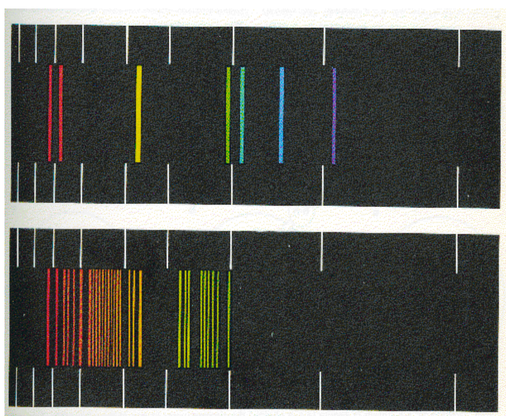
Two ways of showing the same spectra: on the **left** are pictures of the dispersed light and on the **right** are plots of the intensity vs. wavelength. Notice that the pattern of spectral lines in the absorption and emission line spectra are the **same** since the gas is the same.

See also http://www.learner.org/teacherslab/science/light/color/spectra/spectra_1.html



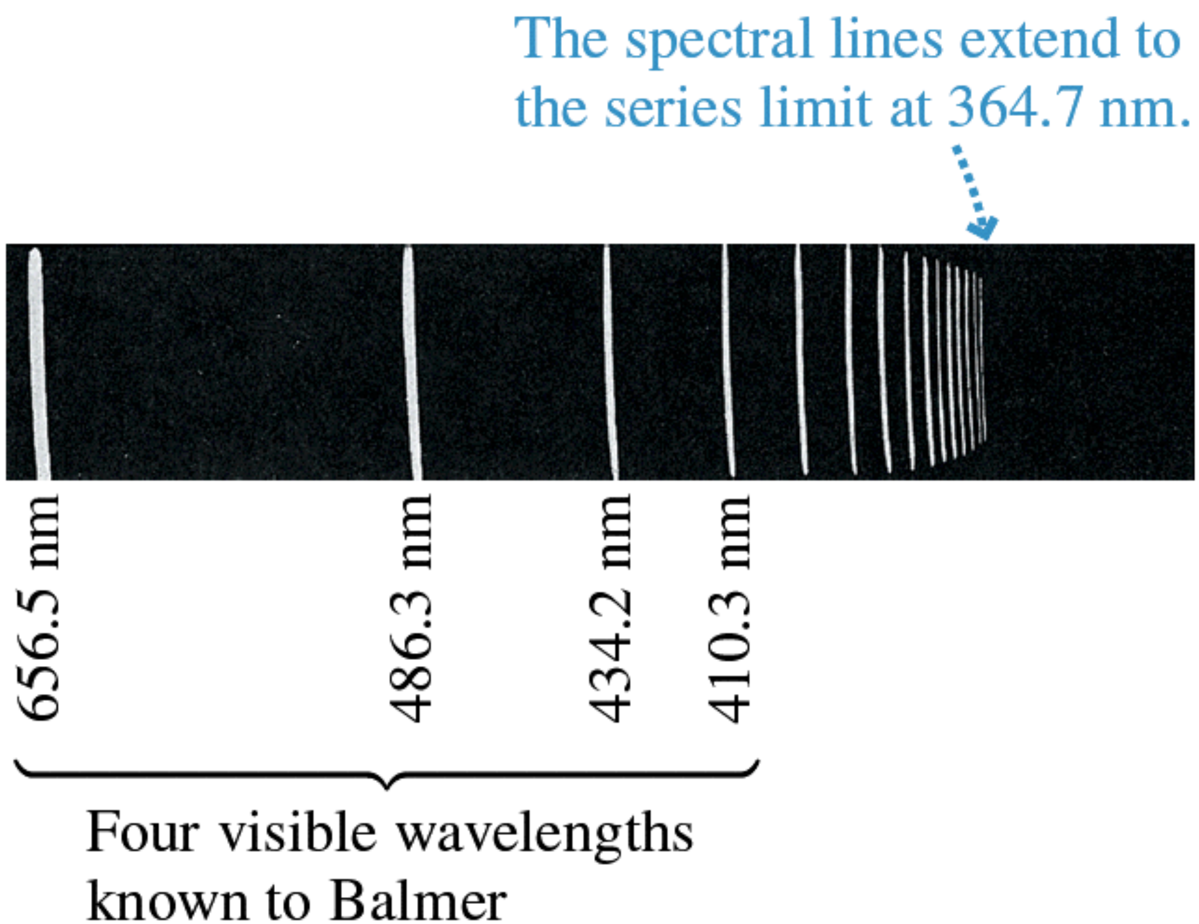
Type of spectrum seen depends on the temperature of the thin gas **relative to** the background. TOP: thin gas is *cooler* so **absorption lines** are seen. BOTTOM: thin gas is *hotter* so **emission lines** are seen.



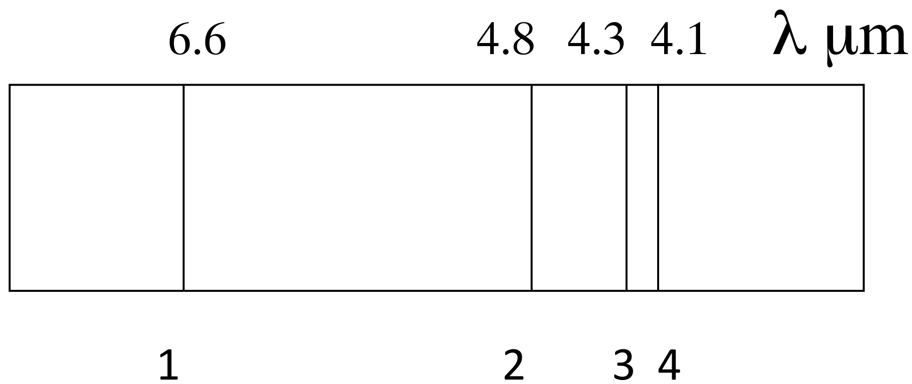


When the light from luminous gases is dispersed by a glass prism it is found to produce a line spectrum. Above: helium spectrum. Below: neon spectrum.

ATOMIC HYDROGEN SPECTRUM



Balmer - Numerology



Hydrogen Spectrum

Balmer math schoolteacher liked to amuse himself by taking four numbers and then finding an equation that described their relationship. While playing with the four numbers he found something very interesting about the number a=3.645 μm
They followed the progression

9*a*/5, 16*a*/12, 25*a*/21, 36*a*/32 $\longrightarrow$ $\frac{3^2}{3^2-4}a, \frac{4^2}{4^2-4}a, \frac{5^2}{5^2-4}a, \frac{6^2}{6^2-4}a$

$$\lambda = \frac{n^2}{n^2-4}a, (n = 3, 4, 5)$$

The Spectrum of Hydrogen

- Hydrogen is the simplest atom, with one electron orbiting a proton, and it also has the simplest atomic spectrum.
- The emission lines have wavelengths which correspond to two integers, *m* and *n*.
- Every line in the hydrogen spectrum has a waveler

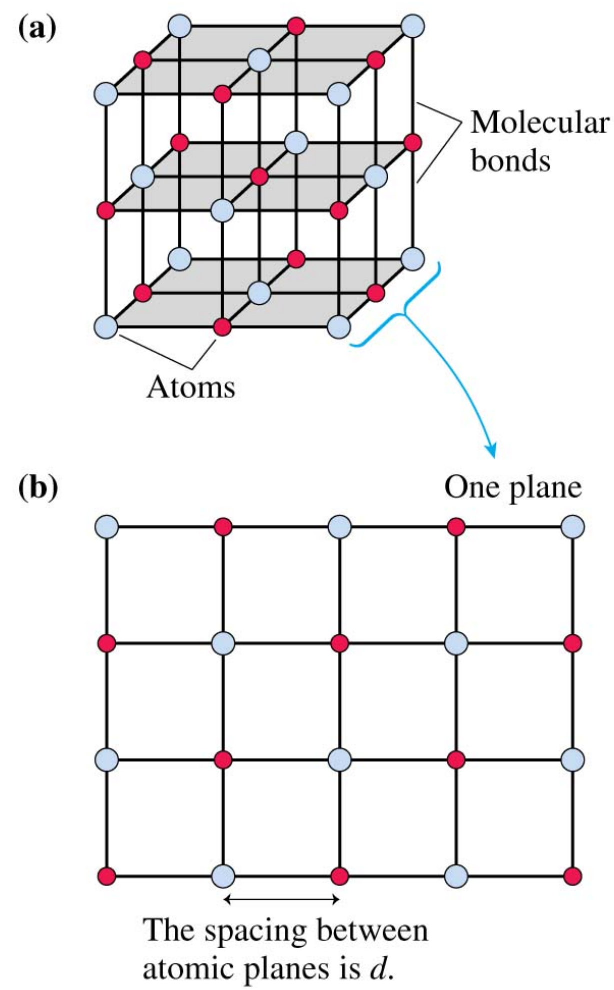
$$\lambda = \frac{91.18\text{ nm}}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)} \qquad \begin{cases} m = 1 & \text{Lyman series} \\ m = 2 & \text{Balmer series} \\ m = 3 & \text{Paschen series} \\ \quad\vdots & \end{cases}$$

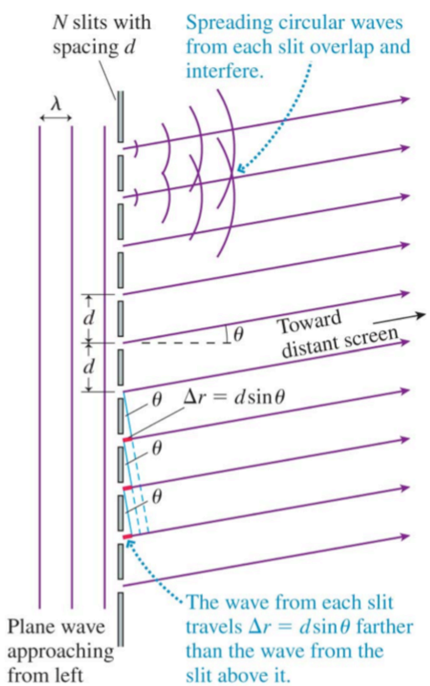
$$n = m + 1, m + 2, \ldots$$

X-Ray Diffraction

The figure shows a simple cubic lattice of atoms. The crystal structure of most materials is more complex than this, but a cubic lattice will help you understand the ideas of x-ray diffraction.

FIGURE 25.5 Atoms arranged in a cubic lattice.

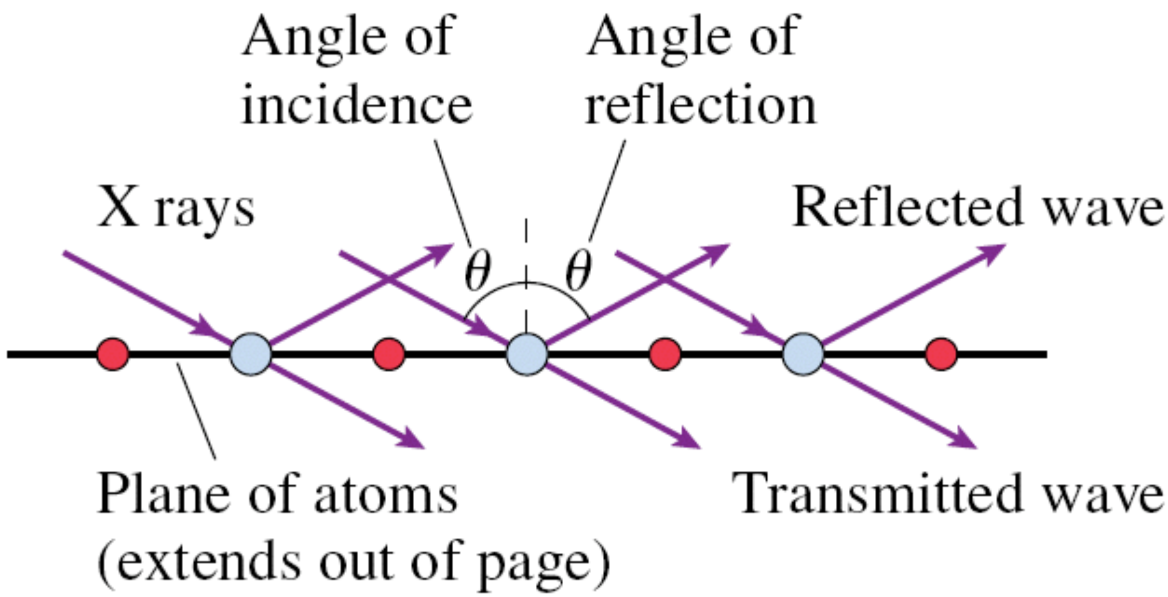




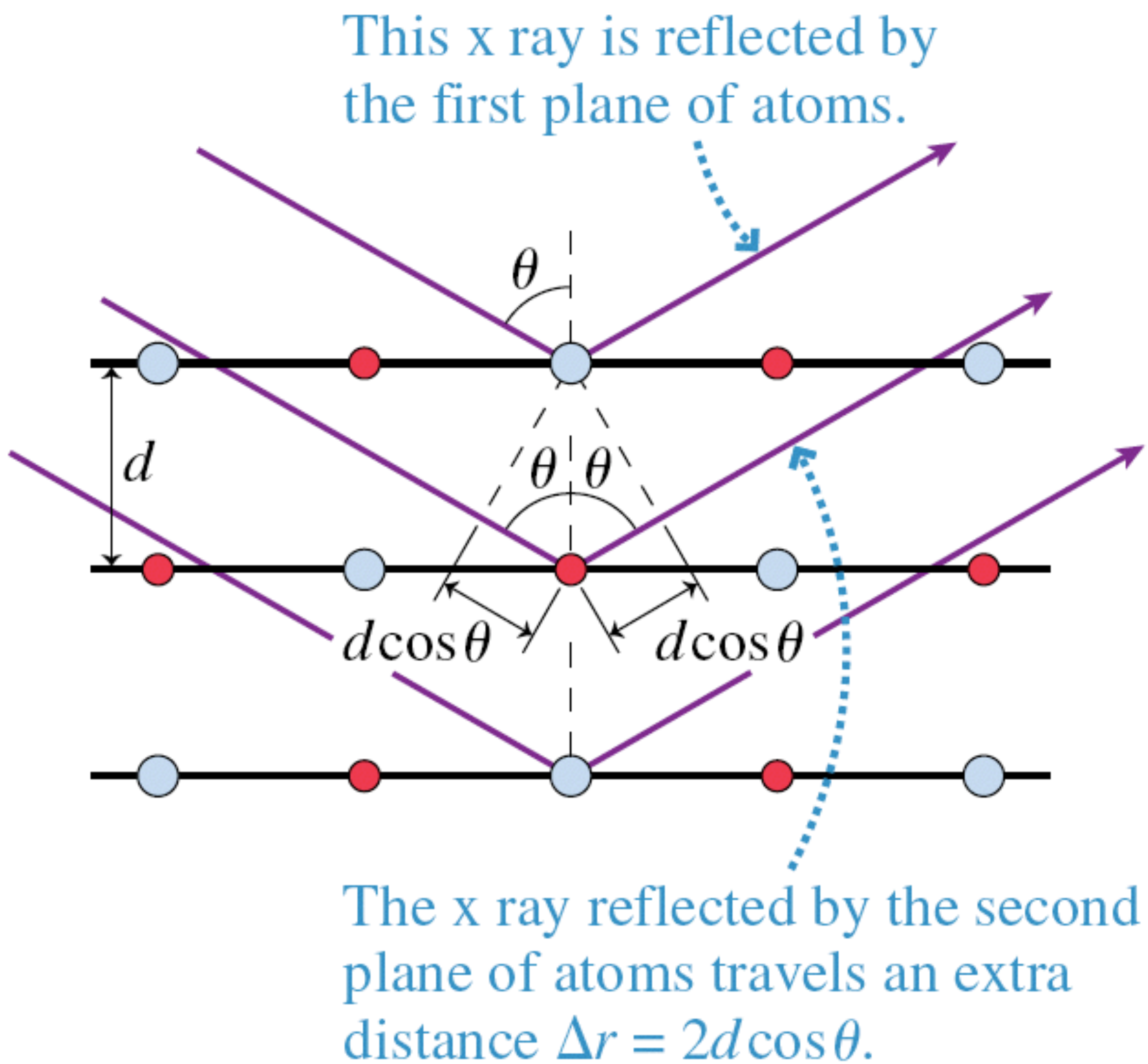
Crystal is a stack of diffraction gratings with very small d consistent with X-ray wavelength.

FIGURE 25.6 The x-ray reflections from parallel atomic planes interfere constructively to cause strong reflections for certain angles of incidence.

(a) X rays are transmitted and reflected at one plane of atoms.



(b) The reflections from parallel planes interfere.



As in gratings maxima when $2d\cos\theta = m\lambda$

X-Ray Diffraction

- The wave reflecting from any particular plane travels an extra distance $\Delta r = 2d\cos\theta$ before combining with the reflection from the plane immediately above it, where d is the spacing between the atomic planes.
- If $\Delta r = m\lambda$, these two waves will be in phase when they recombine.
- Consequently, x rays will strongly reflect from the crystal when the angle of incidence θ_m satisfies

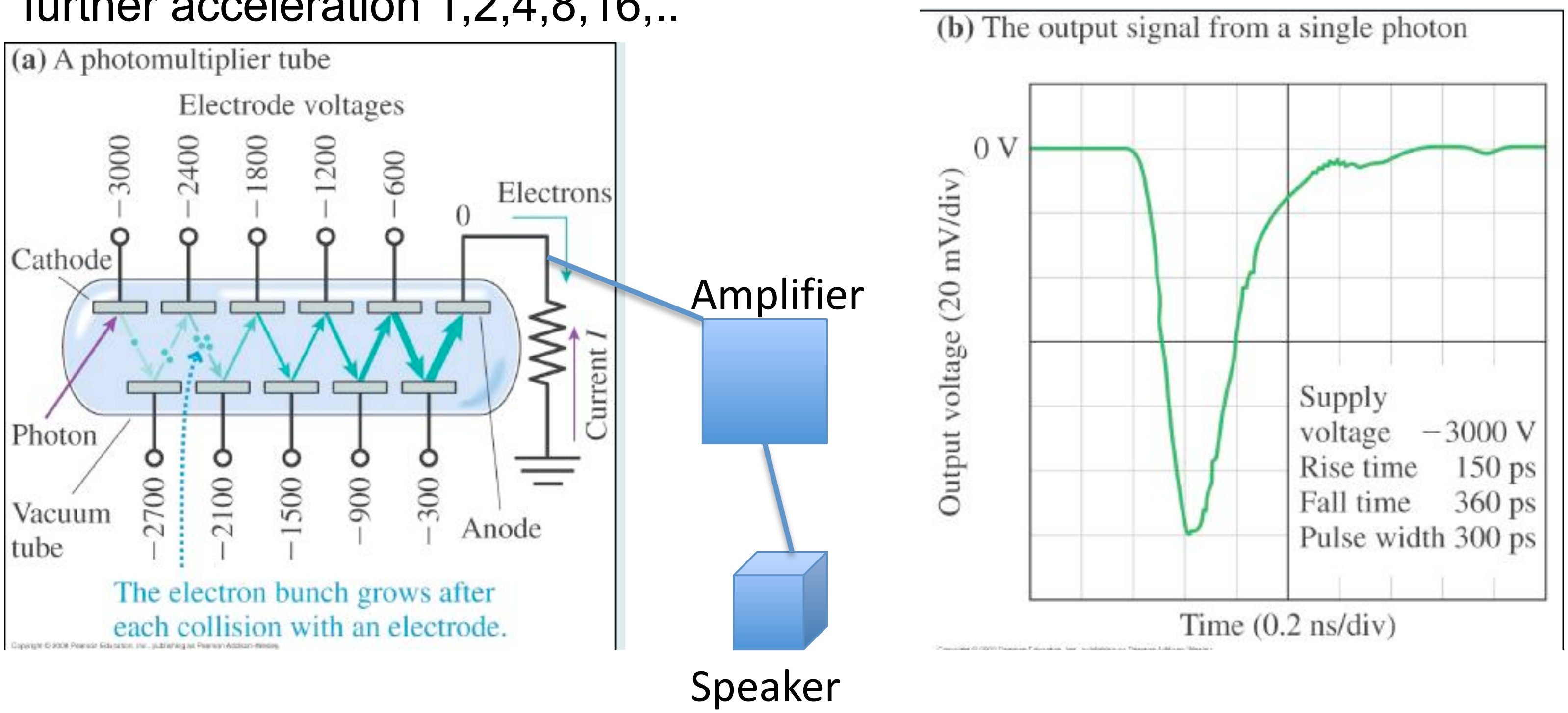
$$\Delta r = 2d\cos\theta_m = m\lambda \qquad m = 1, 2, 3, \ldots$$

The Bragg condition.

Particle properties of light

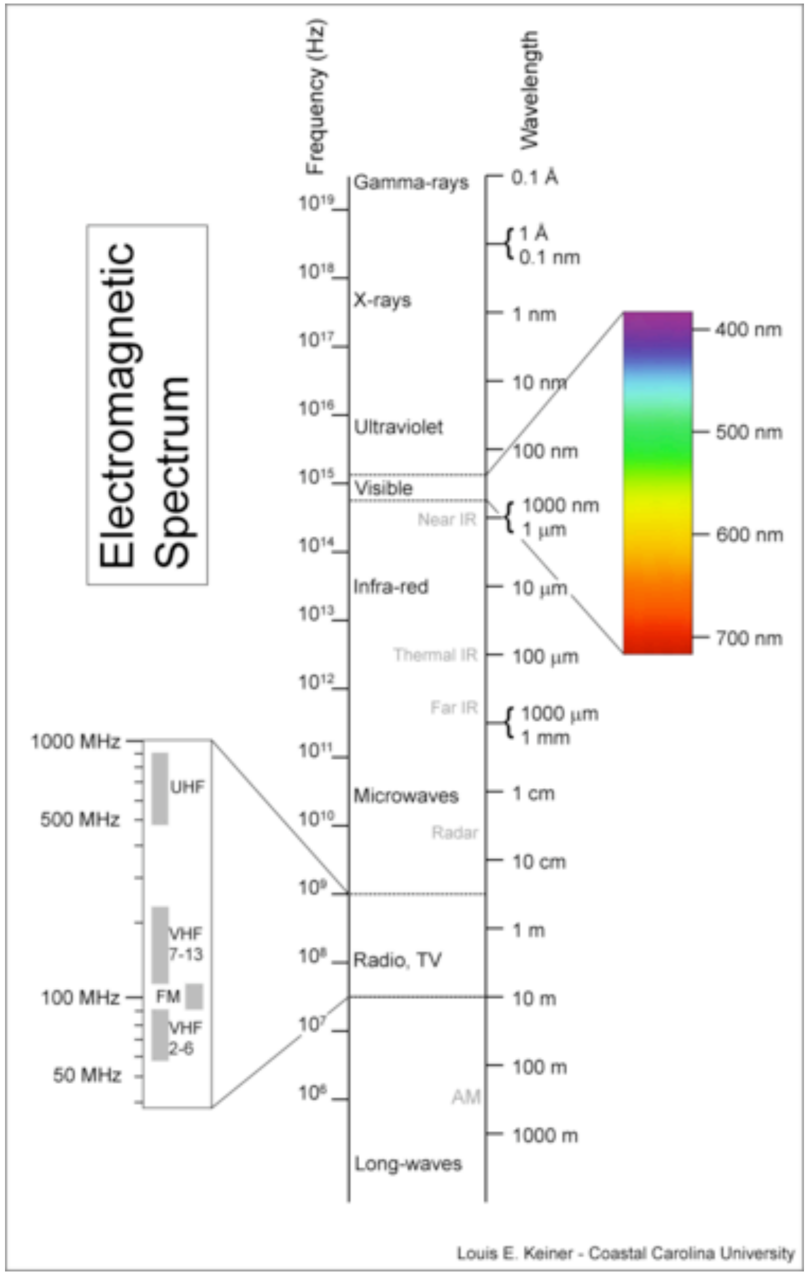
Photomultiplier

Single photon produces single electron (photoelectric effect). Electron accelerates and produces secondary emission that exponentiates with further acceleration 1,2,4,8,16,..



Low light full click no half clicks.

The unit of light is a particle PHOTON.



Red photons
Blue photons
Green photon
X-ray photons
Gamma ray photons
etc
…

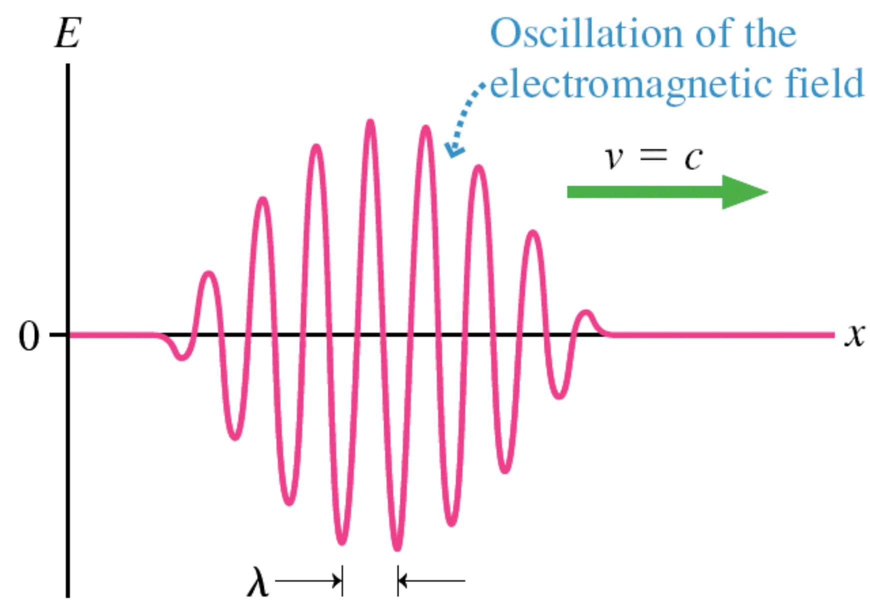
Photons

Photon Momentum

$$E = pc = hf$$
$$p = hf/c$$

Photons are sometimes visualized as **wave packets.** The electromagnetic wave shown has a wavelength and a frequency, yet it is also discrete and fairly localized.

FIGURE 39.11 A wave packet has wave-like and particle-like properties.



$$P = dE/dt = (dN/dt)hf = R = hf$$
$$E = N_T hf$$

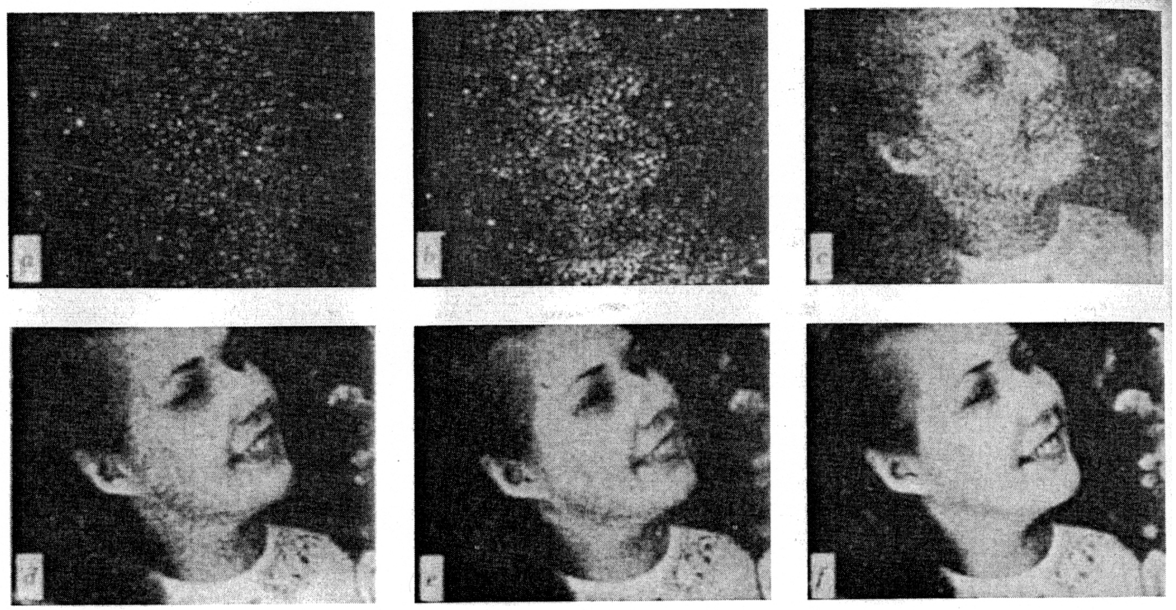
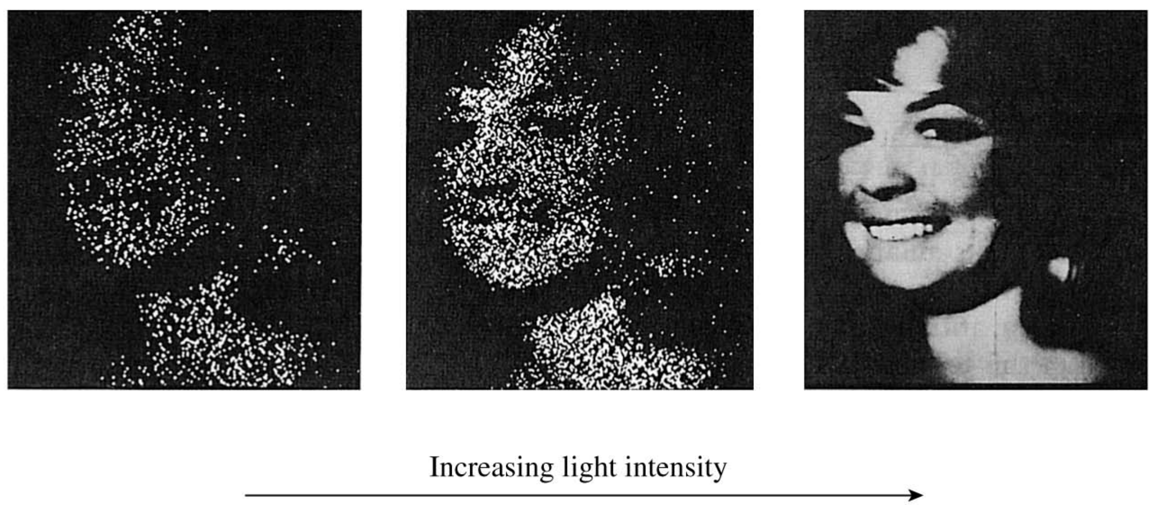


Fig. 2.6 This sequence of photographs of a girl's face shows that photography is a quantum process. The probabilistic nature of quantum effects is evident from the first photographs in which the numbers of photons are very small. As the number of photons increases the photograph becomes more and more distinct until the optimum exposure is reached. The number of photons involved in these photographs ranges from about 3000 in the lowest exposure to about 30 000 000 in the final exposure.

FIGURE 25.9 Photographs made with increasing levels of light intensity.

The photo at very low light levels shows individual points, as if particles are arriving at the detector.



The particle-like behavior is not noticeable at higher light levels.

The Photon Model of Light

The **photon model** of light consists of three basic postulates:

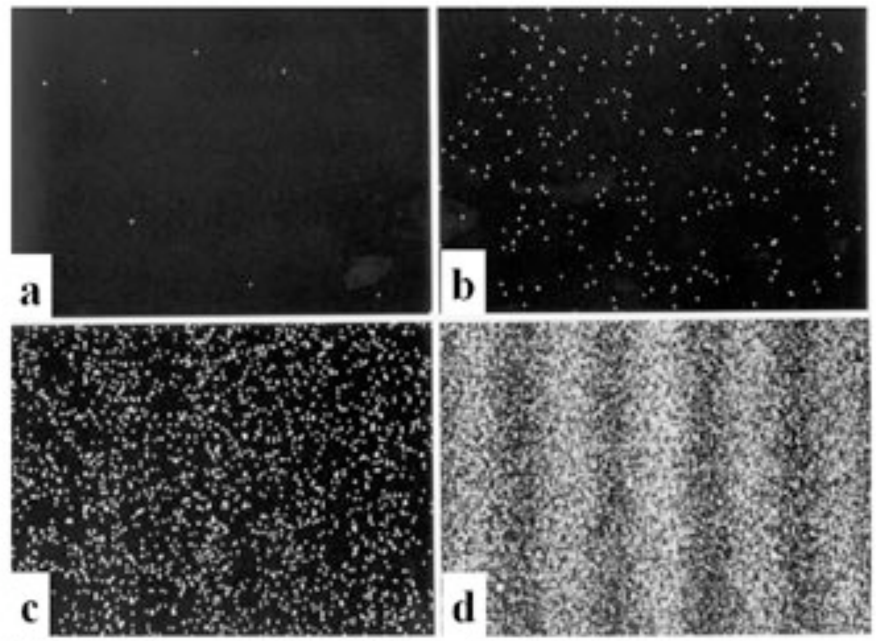
1. Light consists of discrete, massless units called photons. A photon travels in vacuum at the speed of light, 3.00×10^8 m/s.
2. Each photon has energy

$$E_{\text{photon}} = hf$$

where f is the frequency of the light and h is a *universal constant* called **Planck's constant.** The value of Planck's constant is $h = 6.63 \times 10^{-34}$ J s.

3. The superposition of a sufficiently large number of photons has the characteristics of a classical light wave.

Well if light is a particle (photon) is electron a wave?



Single-electron Build-up of Interference Pattern

Electron Interferometry

When we send many electrons through slit they show interference – wave properties
Just as it happened when we send low intensity (single photon) light

CONCEPTUAL INTERFERENCE EXPERIMENTS WITH WAVES AND PARTICLES

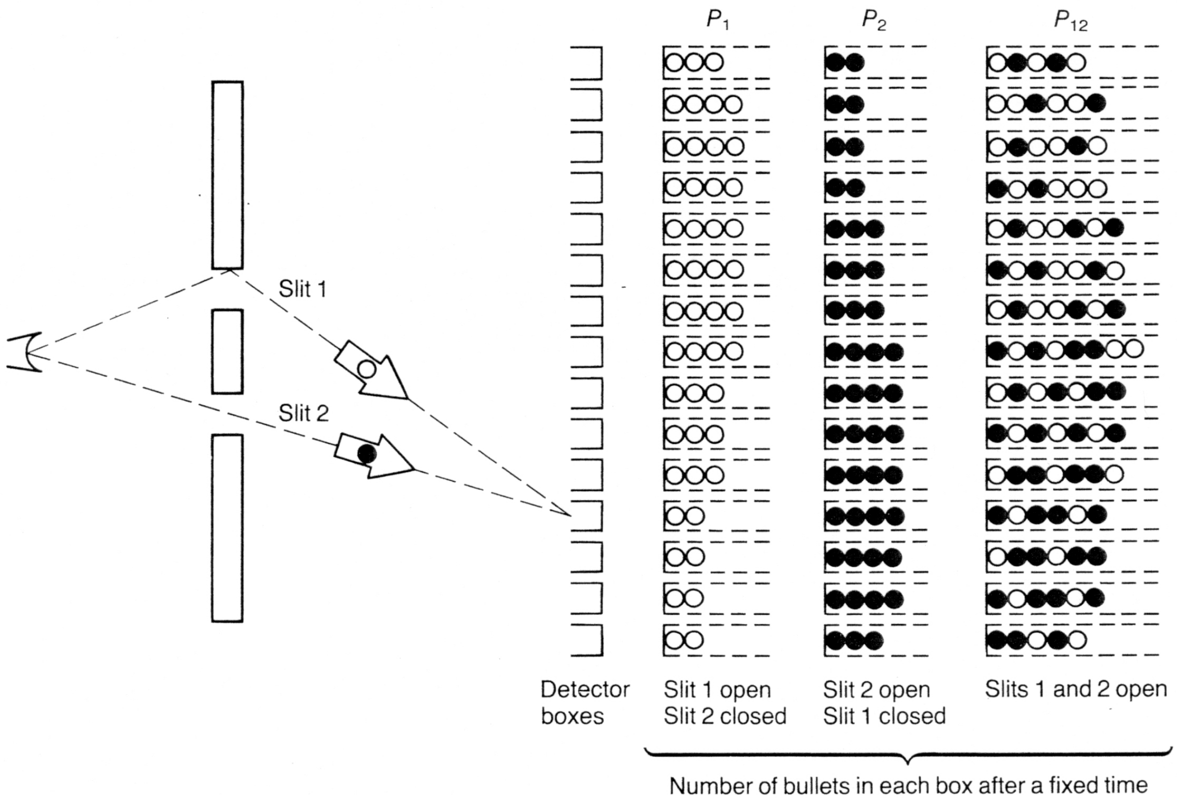


Fig. 1.7 A diagram of a double-slit experiment with bullets. The experimental set-up is shown on the left of the figure and the results of three different experiments indicated on the right. We have shown bullets that pass through slit 1 as open circles and bullets through slit 2 as black circles. The column labelled P_1 *shows the distribution of bullets arriving at the detector boxes when slit 2 is closed and only slit 1 is open. Column* P_2 *shows a similar distribution obtained with slit 1 closed and slit 2 open. As can be seen, the maximum number of bullets appears in the boxes directly in line with the slit that is left open. The result obtained with both slits open is shown in the column labelled* P_{12}*. It is now a matter of chance through which slit a bullet will come and this is shown by the scrambled mixture of black and white bullets collected in each box. The important point to notice is that the total obtained in each box when both slits are open is just the sum of the numbers obtained when only one or other of the slits is open. This is obvious in the case of bullets since we know that bullets must pass through one of the slits to reach the detector boxes.*

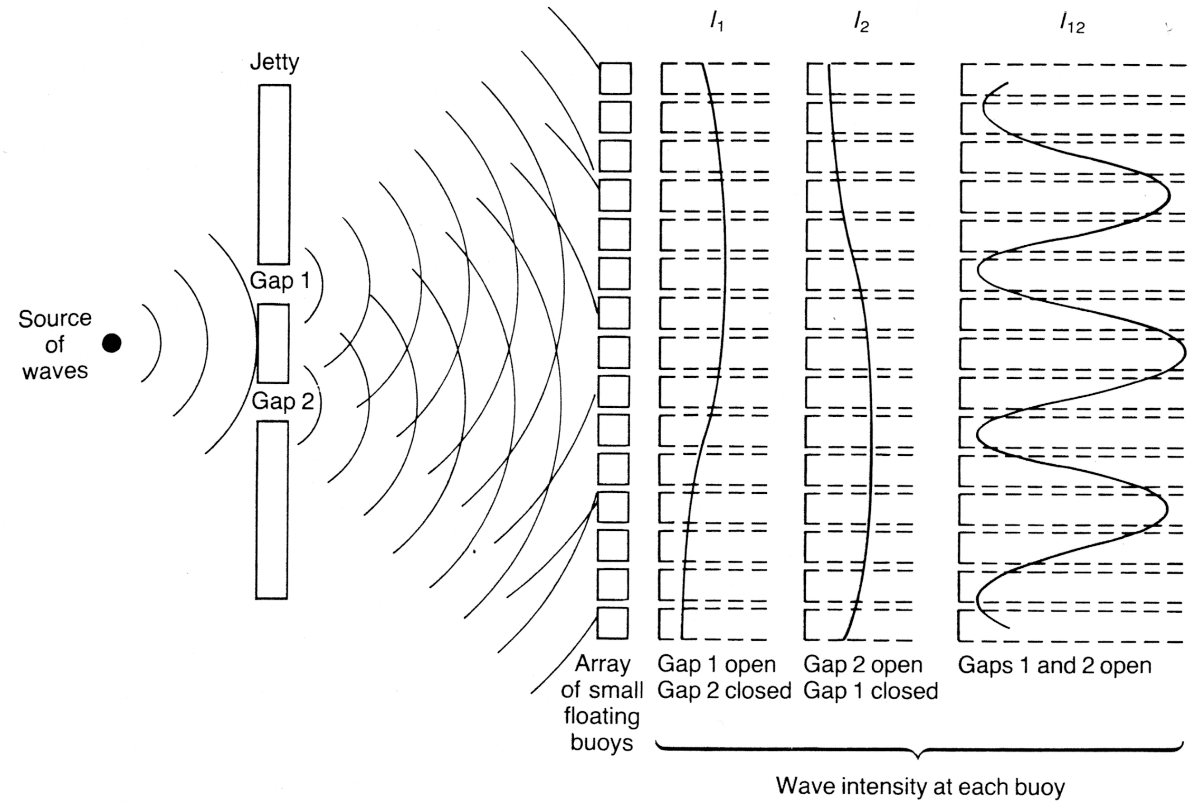


Fig. 1.9 A diagram of a double-slit experiment with water waves. The detectors are a line of small floating buoys whose jiggling up and down provides a measure of the wave energy. The wave crests spreading out from each slit are shown in the figure and can be compared with fig. 1.8. The column labelled I_1 *shows the smoothly varying wave intensity obtained when only gap 1 is open. Notice that this is very similar to the pattern* P_1 *obtained with bullets in fig. 1.7 with only slit 1 open. Again it is largest at the detector directly in line with gap 1 and the source. The second column shows that a similar pattern,* I_2*, is obtained when gap 1 is closed and gap 2 is open. The final column,* I_{12}*, shows the wave intensity pattern obtained with both slits open. It is dramatically different from the pattern obtained for bullets with both slits open. It is not equal to the sum of the patterns* I_1 *and* I_2 *obtained with one of the gaps closed. This rapidly varying intensity curve is called an interference pattern.*

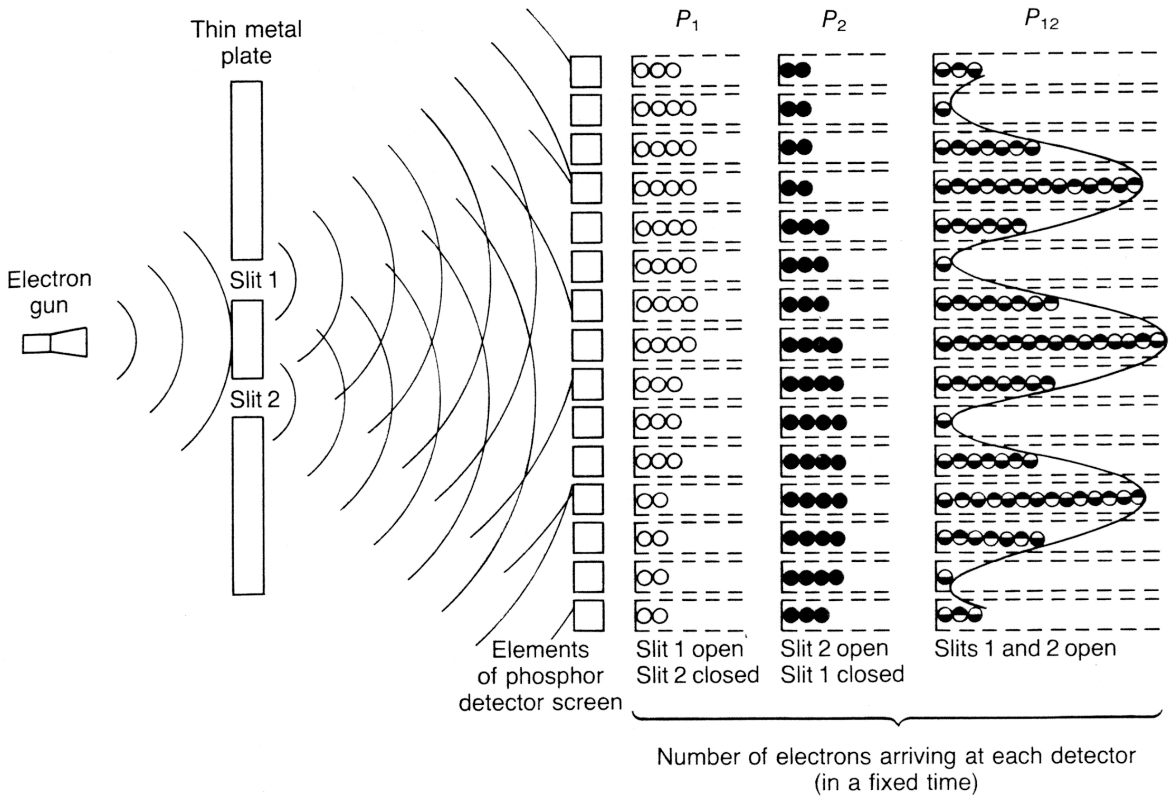


Fig. 1.11 A diagram of a double-slit experiment with electrons. Electrons always arrive with a flash at the phosphor detector at one point, in the same way that bullets always end up in just one of the detector boxes rather than the energy being spread out, as in a wave. The column marked P_1 *shows the pattern obtained with only slit 1 open. Electrons that have gone through slit 1 are represented as open circles, like the bullets of fig. 1.7. Column* P_2 *shows the same thing with only slit 2 open and the electrons that have gone through slit 2 indicated by black circles. These two patterns are exactly the same as those obtained with bullets. The difference lies in the column headed* P_{12}*, which shows the pattern obtained for electrons when both slits are open. This is just the interference pattern obtained with water waves and requires some kind of wave motion arising from each slit as indicated on the figure. It is not the sum of* P_1 *and* P_2 *and so we cannot say which slit any electron goes through. We have indicated this lack of knowledge by drawing the electrons, which still arrive like bullets, as half white and half black circles. This fact, that quantum objects such as electrons possess attributes of both wave and particle motion but behave like neither, is the central mystery of quantum mechanics.*

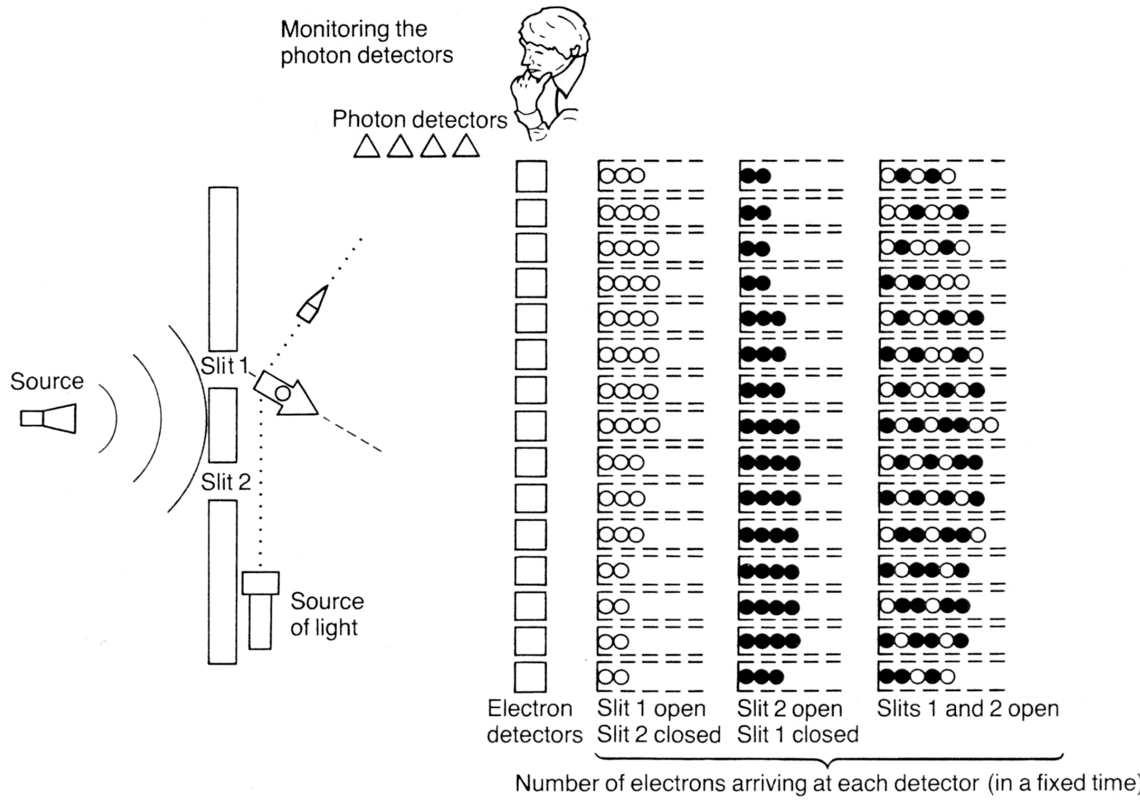


Fig. 2.3 Sketch of the experimental set-up required to observe through which slit the electron passes in a double-slit experiment. Light, in the form of photons, is directed at the slits. In the figure a photon, represented as a small bullet, has hit an electron behind slit 1. The electron is disturbed slightly in its motion and the scattered photon is observed at the photon detectors. The electron patterns obtained with only one of the slits open are almost the same as before, when we did not observe the electron behind the slits. The surprise occurs with both slits open: there is no interference pattern. The small nudges given to the electrons in their collisions with the photons are always sufficient to wash out the interference pattern completely! We can now say with certainty through which slit the electron went but now the electrons are behaving just like bullets. The observed pattern is just the sum of the patterns for slit 1 and slit 2 separately.

Matter Waves and Energy Quantization

In 1924 de Broglie postulated that *if* a material particle of momentum $p = mv$ has a wave-like nature, then its wavelength must be given by

Photons

$E = hf = pc$

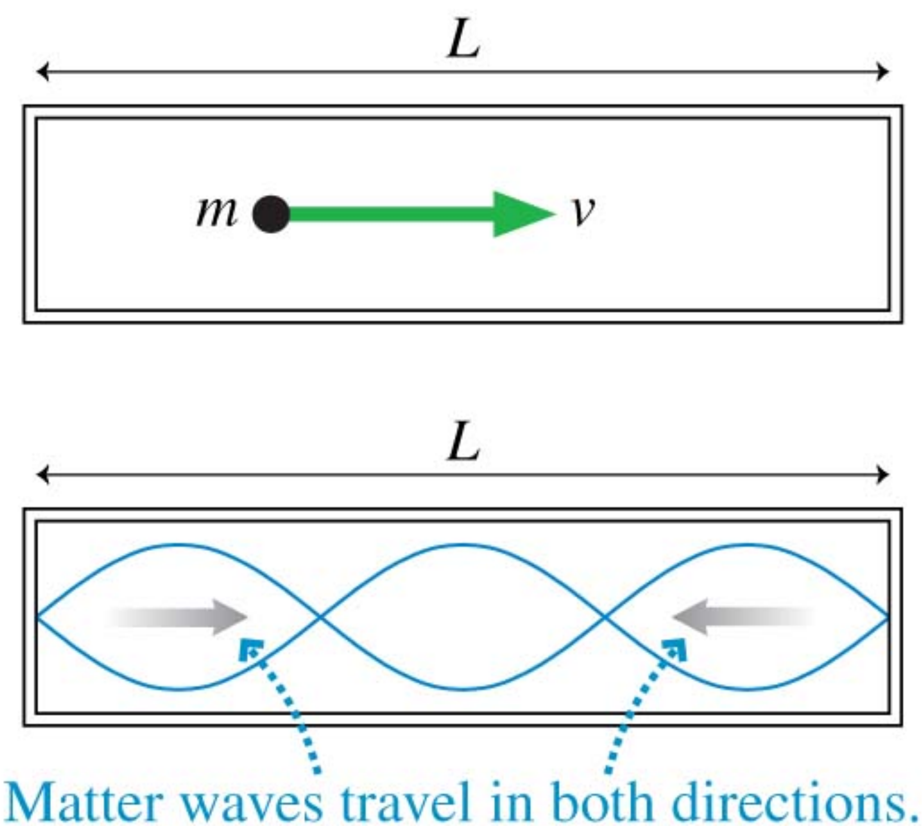
$\lambda = c/f = h/p$

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

where h is Planck's constant ($h = 6.63 \times 10^{-34}$ J s). This is called the **de Broglie wavelength.**

DB considered a matter wave to be a travelling one. Suppose however that the particle is confined in a small region and cannot escape. How do the wave properties show up?

FIGURE 39.15 A particle in a box creates a standing de Broglie wave as it reflects back and forth.



$$\lambda_n = 2L/n, n = 1,2,3,4...$$

$$\lambda = h/mv$$

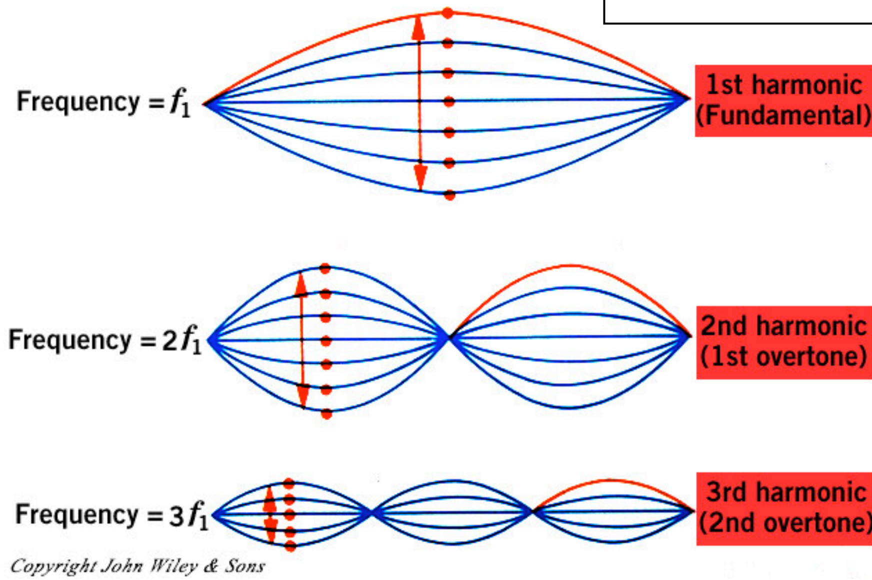
$$v_n = n(h/2Lm), n = 1,2,3$$

$$E_n = n^2(h^2/8mL^2) \equiv n^2 E_1$$

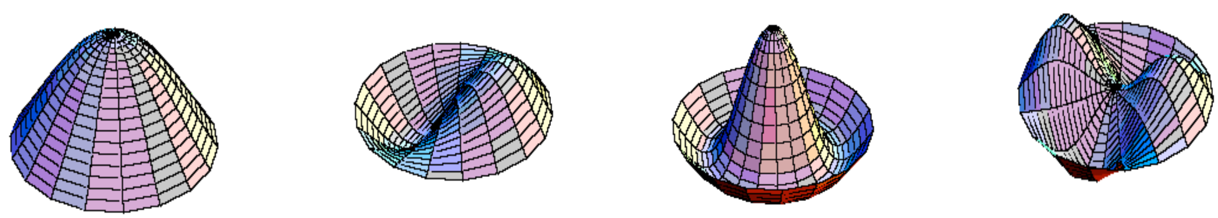
Waves with boundaries

Standing waves (harmonics)

$$y(x,t) = A \sin \omega_n t \sin \frac{n\pi x}{L}$$



Ends (or edges) must stay fixed. That's what we call a boundary condition.



This is an example of a Bessel function.

Quantization of Energy

- Consider a particle of mass *m* moving in one dimension as it bounces back and forth with speed *v* between the ends of a box of length *L*. We'll call this a *one-dimensional box;* its width isn't relevant.
- A wave, if it reflects back and forth between two fixed points, sets up a standing wave.
- A standing wave of length *L must* have a wavelength given by

$$\lambda_n = \frac{2L}{n} \qquad n = 1, 2, 3, 4, \ldots$$

Quantization of Energy

Using the de Broglie relationship $\lambda = h/mv$, a standing wave with wavelength λ_n forms when the particle has a speed

$$v_n = n\left(\frac{h}{2Lm}\right) \qquad n = 1, 2, 3, \ldots$$

Thus the particle's energy, which is purely kinetic energy, is

$$E_n = \frac{1}{2}mv_n^2 = n^2\frac{h^2}{8mL^2} \qquad n = 1, 2, 3, \ldots$$

De Broglie's hypothesis about the wave-like properties of matter leads us to the remarkable conclusion that **the energy of a confined particle is quantized.**