PHYS 270 – SUPPL. #16
Ray Optics II

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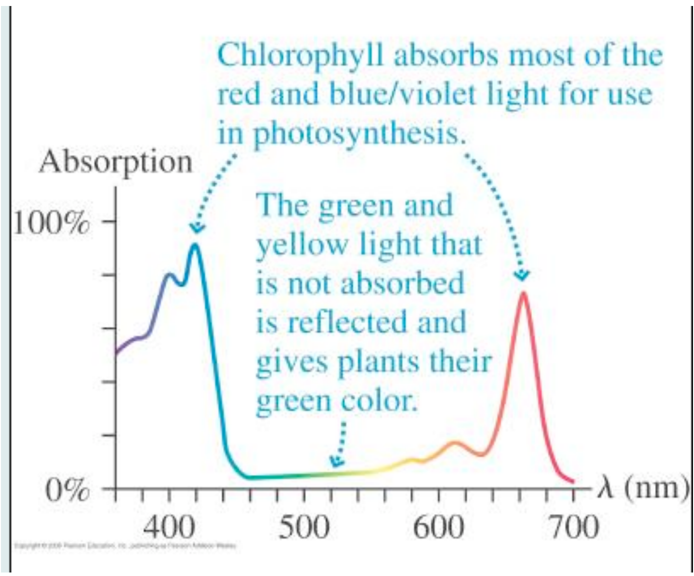
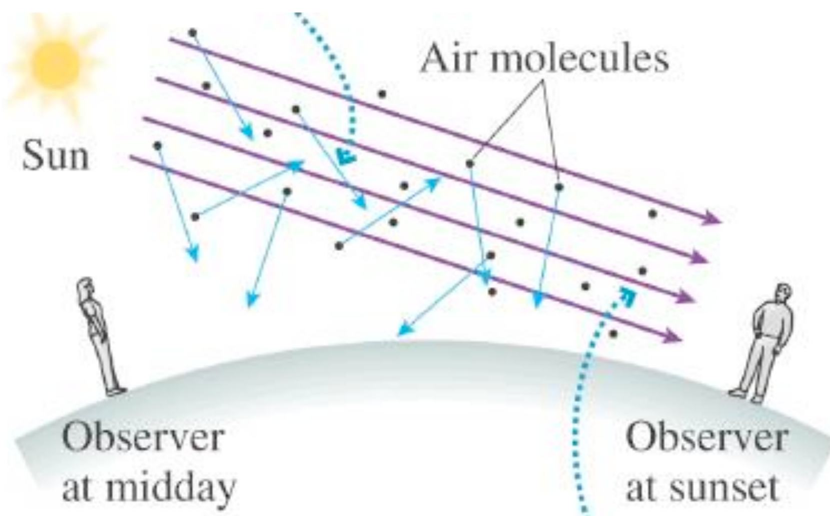
MARCH 31, 2011

Why are plants green?

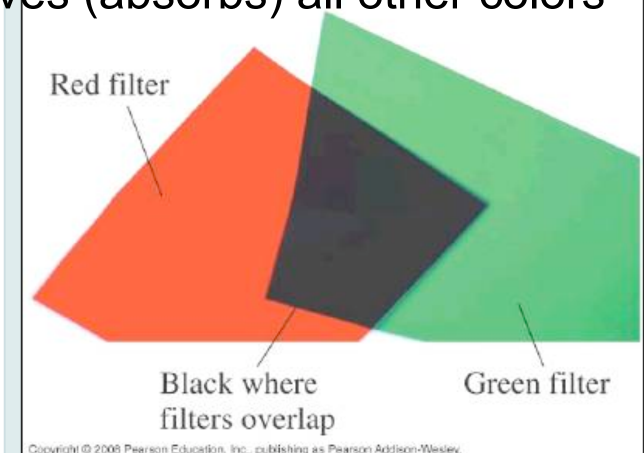


Why are sunsets red –Why is the sky blue



$I_{sc} \sim (1/\lambda)^4$
Raleigh scattering
Blue scatters 16 times more than red

Green glass green because it removes (absorbs) all other colors



Look at sky with these filters

Reflection and Refraction

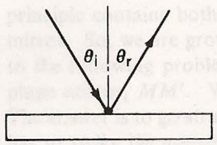


Fig. 26–1. The angle of incidence is equal to the angle of reflection.

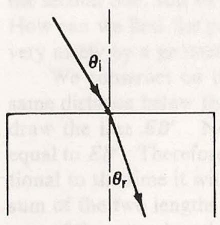


Fig. 26–2. A light ray is refracted when it passes from one medium into another.

Table 26–1

Angle in air	Angle in water
10°	8°
20°	15-1/2°
30°	22-1/2°
40°	29°
50°	35°
60°	40-1/2°
70°	45-1/2°
80°	50°

$$\theta_i = \theta_r$$

Ptolemy 140 AD

Snell 1621

$\sin\theta_i = n \sin\theta_r$

Table 26–2

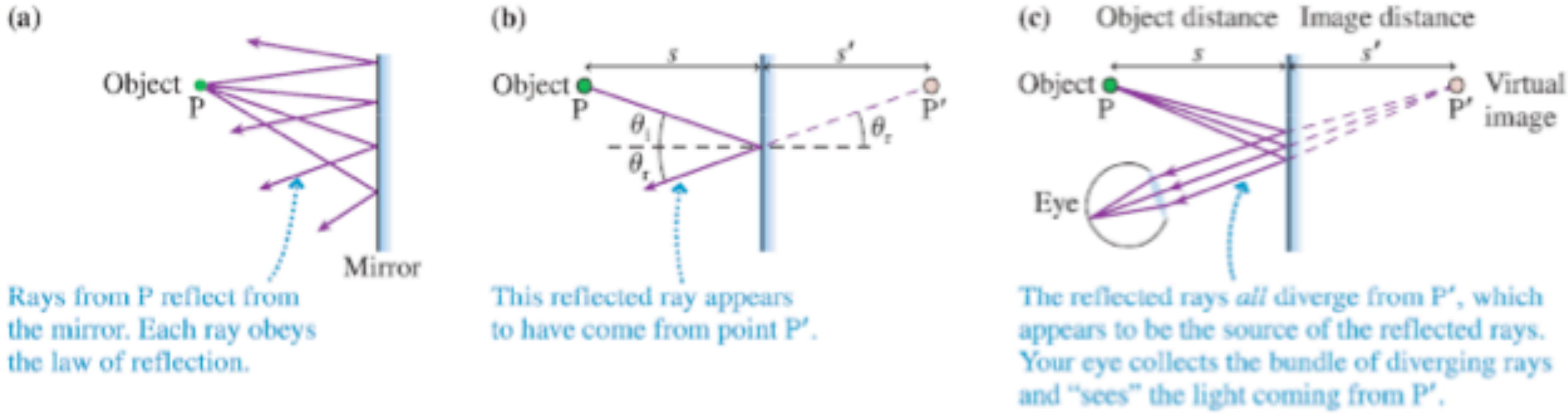
Angle in air	Angle in water
10°	7-1/2°
20°	15°
30°	22°
40°	29°
50°	35°
60°	40-1/2°
70°	45°
80°	48°

Snell's law allow us to predict how light is going to bend in going fro air to water n=1.5

Is there a way of thinking that makes this evident and proves its generality -> Fermat

Law of reflection

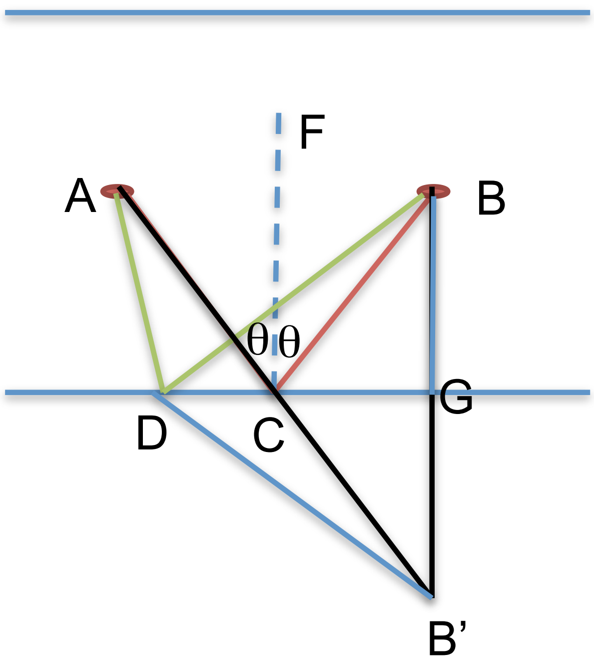
FIGURE 23.10 The light rays reflecting from a plane mirror.



$$s' = s \qquad \text{(plane mirror)}$$

Fermat’s Principle of Least Time

Of all possible paths that light takes to get from one point to another light takes the path that corresponds to **the shortest time**



If Angle ACF=Angle FCB

Then ACB is shortest path between A and B after hitting the mirror once. Prove that ACB<ADB.
BG=GB’

ACB=ACB’ and ADB=ADB’
But ACB’<ADB’ -> ACB<ADB QED

Fermat's Principle of Least Time

Of all possible paths that light takes to get from one point to another light takes the path that corresponds to **the shortest time**

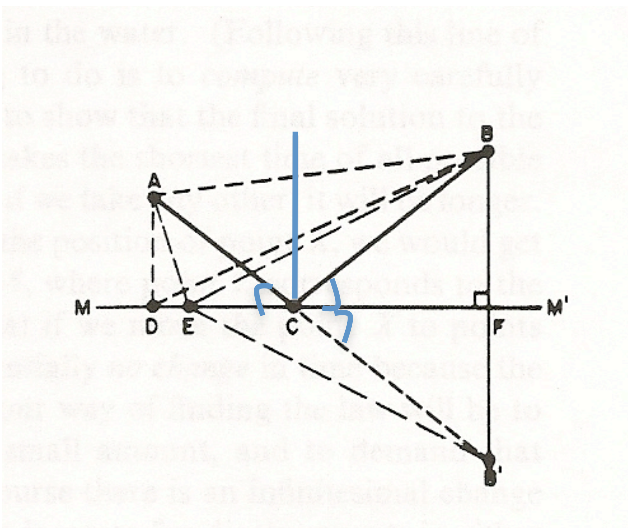


Fig. 26–3. Illustration of the principle of least time.

AEB=AEB'
Shortest length from A to B after hitting the mirror is the one that goes as a straight line from A to B', i.e. ACB'.
Notice that ACB has equal incidence and reflection angles

Angle BCF=Angle B'CF= Angle DCA

Fermat’s Principle for Refraction

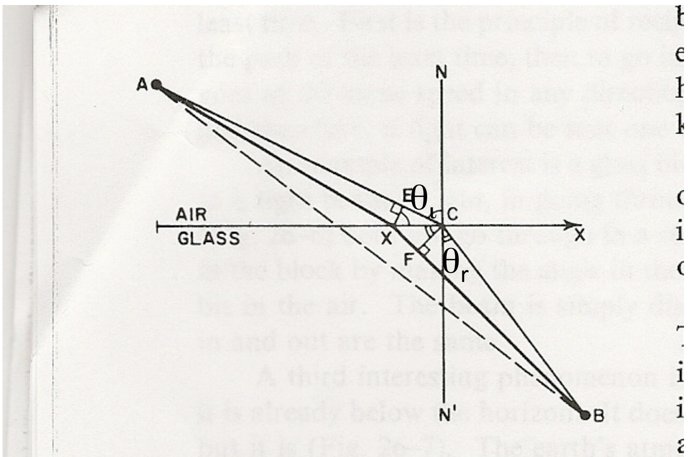


Fig. 26–4. Illustration of Fermat's principle for refraction.

$$v = c / n$$

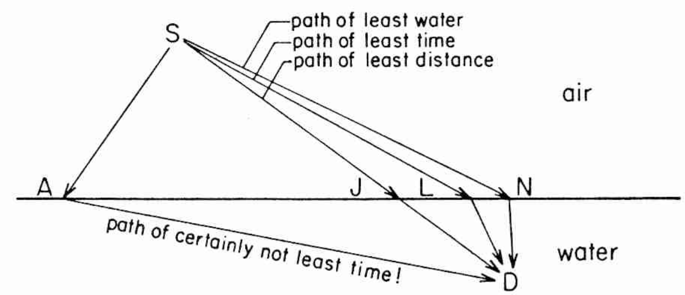


FIGURE 30. *Finding the path of least time for light is like finding the path of least time for a lifeguard running and then swimming to rescue a drowning victim: the path of least distance has too much water in it; the path of least water has too much land in it; the path of least time is a compromise between the two.*

$$v \sin\theta_i = c \sin\theta_r$$
$$\sin\theta_i = n \sin\theta_r$$

Snell’s law

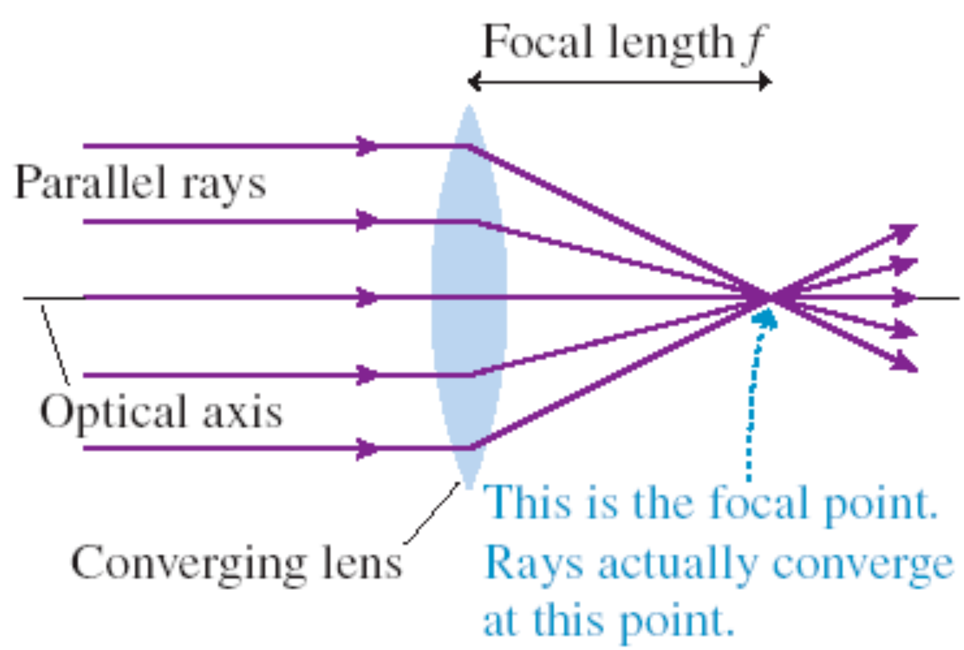
The slower the medium the smaller the refraction angle

What is a lens?

Device that provides a capability to create bright and well focused images. How?

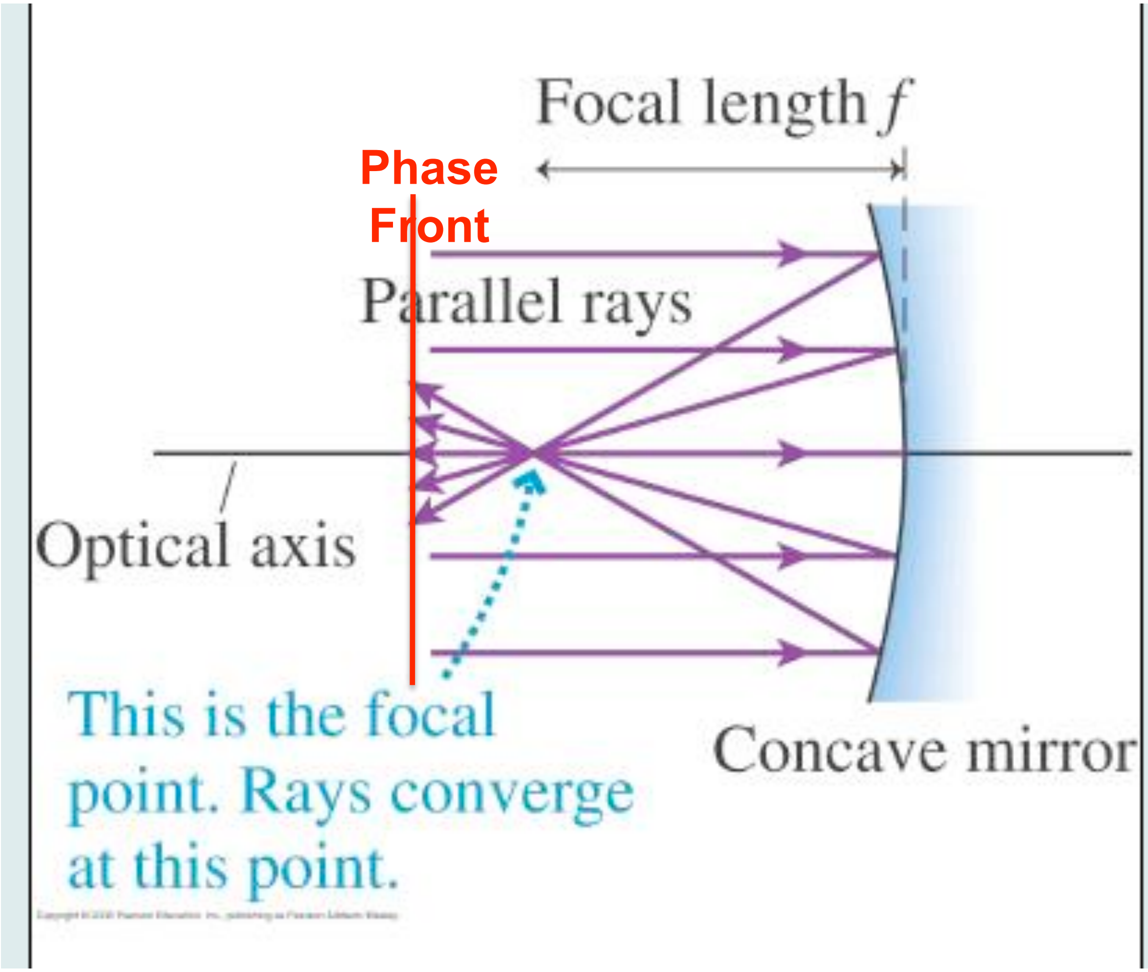
Use Refraction to create images out of divergent light rays

- Focal point
- Focal length
    - Property of lens no matter how it is used
    - Distance from lens that paraxial rays converge
    - Reversibility



What is a Concave Mirror

or parabolic mirror and parabolic dishes



- Use reflection to achieve the same results as a lens
- Focal point
- Focal length: Distance from mirror that paraxial rays converge
- Reversibility

Parabola: locus of all points equidistant from a point and a line

$$w = 2.4\lambda L / D = 2.4\lambda f / D$$

IA=constant

How Images are formed

Tactics: Ray tracing for a converging lens

TACTICS BOX 23.2 **Ray tracing for a converging lens**

MP

1. **Draw an optical axis.** Use graph paper or a ruler! Establish an appropriate scale.
2. **Center the lens on the axis.** Mark and label the focal points at distance f on either side.
3. **Represent the object with an upright arrow at distance s.** It's usually best to place the base of the arrow on the axis and to draw the arrow about half the radius of the lens.

Tactics: Ray tracing for a converging lens

④ **Draw the three "special rays" from the tip of the arrow.** Use a straight-edge.

a. A ray parallel to the axis refracts through the far focal point.
b. A ray that enters the lens along a line through the near focal point emerges parallel to the axis.
c. A ray through the center of the lens does not bend.

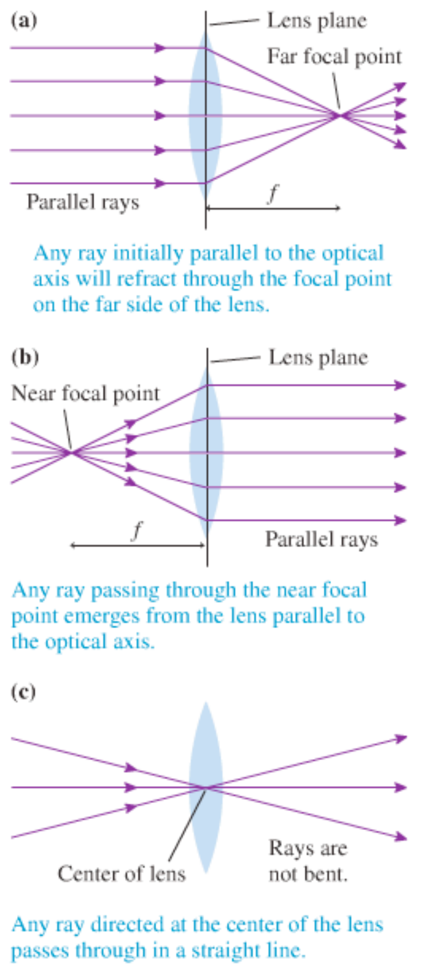
Tactics: Ray tracing for a converging lens

5. **Extend the rays until they converge.** This is the image point. Draw the rest of the image in the image plane. If the base of the object is on the axis, then the base of the image will also be on the axis.
6. **Measure the image distance s'.** Also, if needed, measure the image height relative to the object height.

s’ negative for virtual image. Image formed by dotted lines (extension of real rays)

Thin Lenses – Real Image –Special Rays

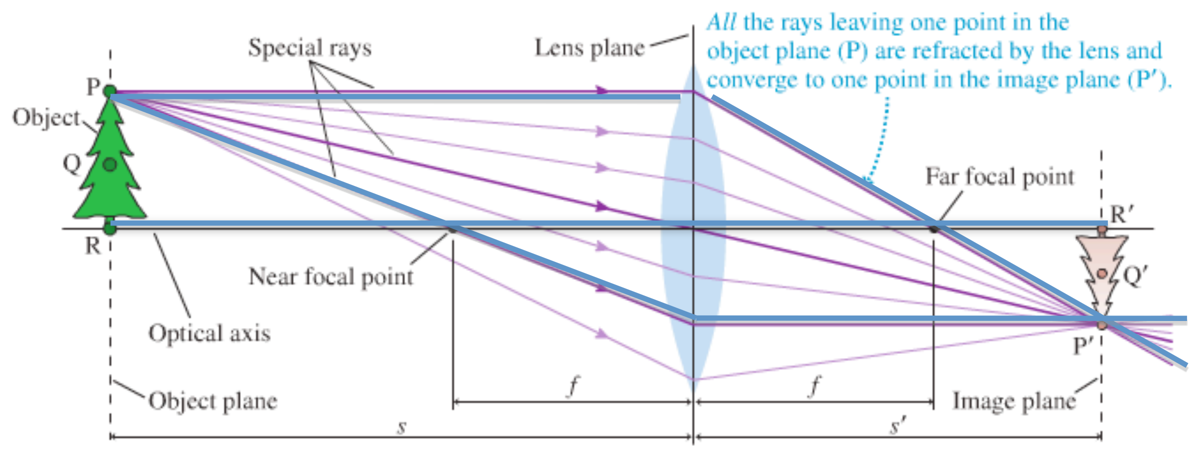
FIGURE 23.35 Three important sets of rays passing through a thin converging lens.



Thin lens-> d<<f,s,s'

Object plane distance s and Image plane distance s'

FIGURE 23.36 Rays from an object point P are refracted by the lens and converge to a real image at point P′.

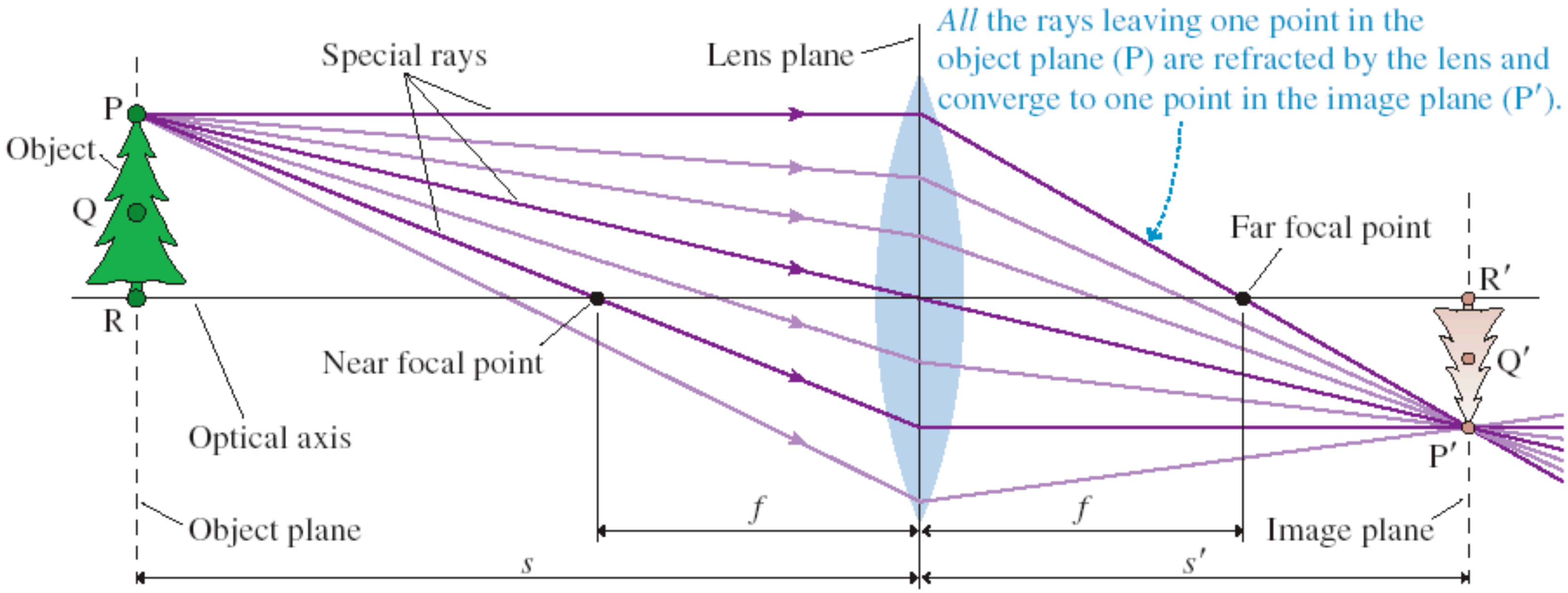


$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

Inverted Image

Thin Lenses: Ray Tracing

FIGURE 23.36 Rays from an object point P are refracted by the lens and converge to a real image at point P′.



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad \text{(thin-lens equation)}$$

Lateral Magnification

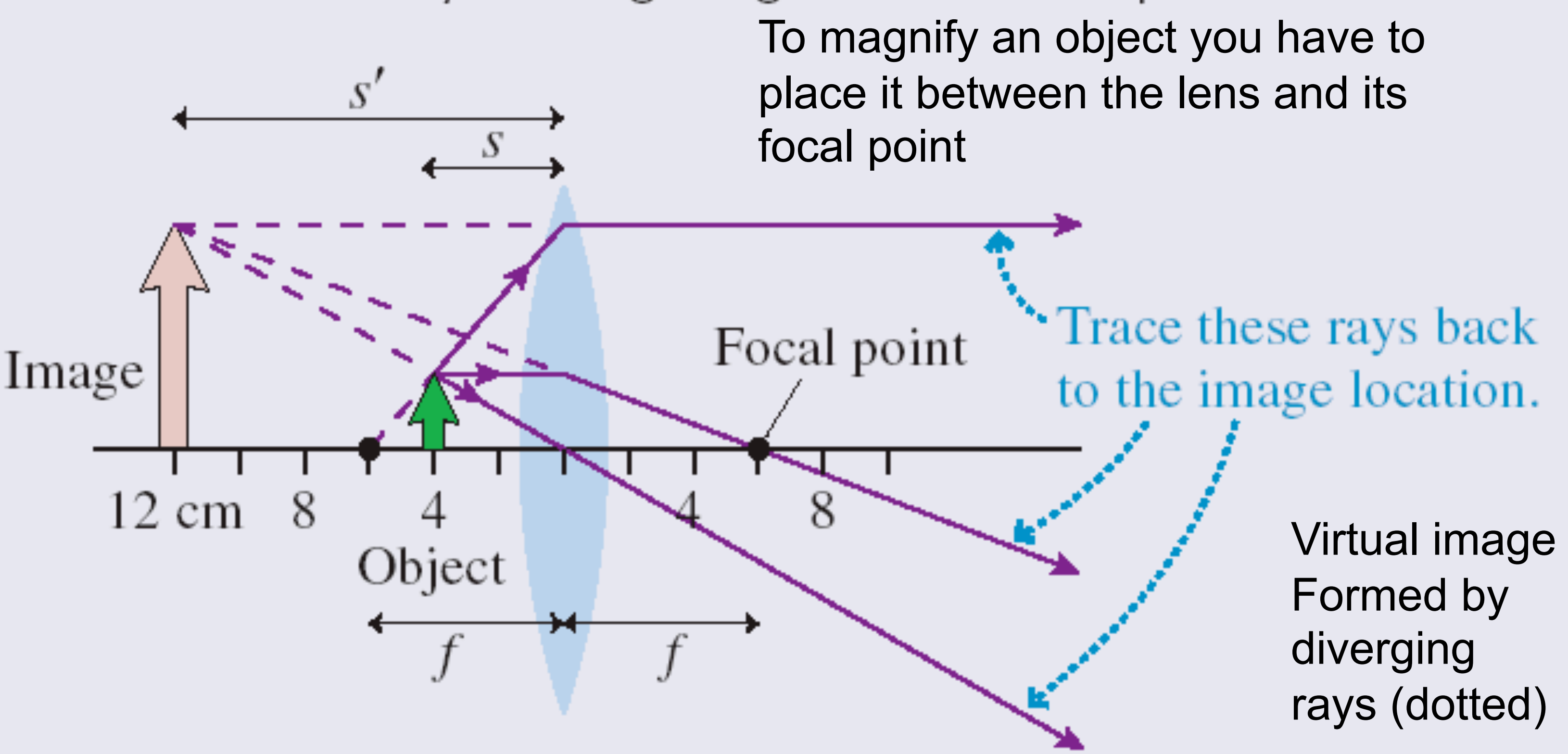
The image can be either larger or smaller than the object, depending on the location and focal length of the lens. The **lateral magnification** *m* is defined as

$$m = -\frac{s'}{s}$$

1. A positive value of *m* indicates that the image is upright relative to the object. A negative value of *m* indicates that the image is inverted relative to the object.
2. The absolute value of *m* gives the size ratio of the image and object: $h'/h = |m|$.

EXAMPLE 23.10 Magnifying a flower

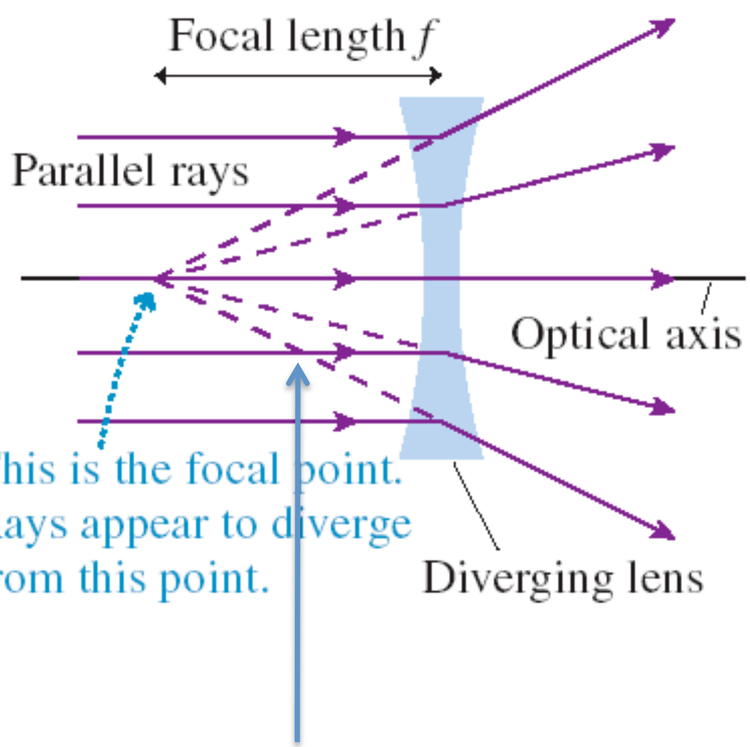
FIGURE 23.41 Ray-tracing diagram for Example 23.10.



$$m = -s'/s = -12cm/4cm = 3$$

Thin Lenses: Ray Tracing

FIGURE 23.34 The focal point and focal length of [illegible] diverging lenses.



Virtual point

Tactics: Ray tracing for a diverging lens

TACTICS BOX 23.3 **Ray tracing for a diverging lens**

MP

❶–❸ **Follow steps 1 through 3 of Tactics Box 23.2.**

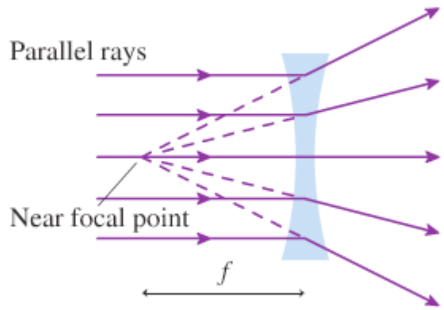
❹ **Draw the three "special rays" from the tip of the arrow.** Use a straight-edge.

- **a.** A ray parallel to the axis diverges along a line through the near focal point.
- **b.** A ray along a line toward the far focal point emerges parallel to the axis.
- **c.** A ray through the center of the lens does not bend.

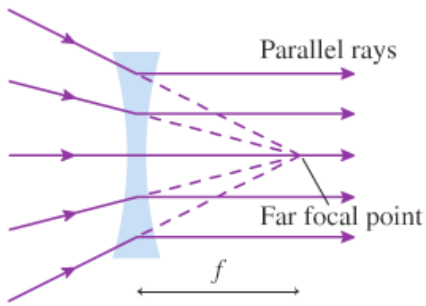
❺ **Trace the diverging rays backward.** The point from which they are diverging is the image point, which is always a virtual image.

❻ **Measure the image distance s'.** This will be a negative number.

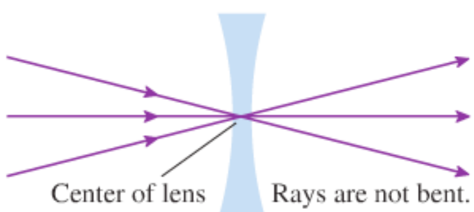
Exercise 28



Any ray initially parallel to the optical axis diverges along a line through the near focal point.

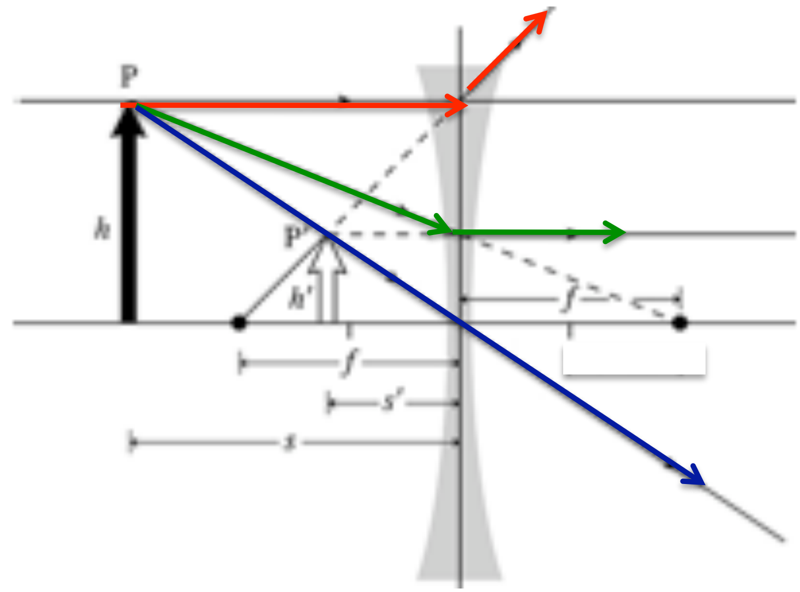


Any ray directed along a line toward the far focal point emerges from the lens parallel to the optical axis.



Any ray directed at the center of the lens passes through in a straight line.

1. Draw the optical axis
2. Center the lens on the axis
3. Represent the object with an upright arrow at distance s.
4. Draw the three special rays



5. Extend the rays until they converge (dotted lines)
6. Draw the image.

$$m = -s'/s$$

$$|m| = s'/s$$

The Thin Lens Equation

The object distance s is related to the image distance s' by

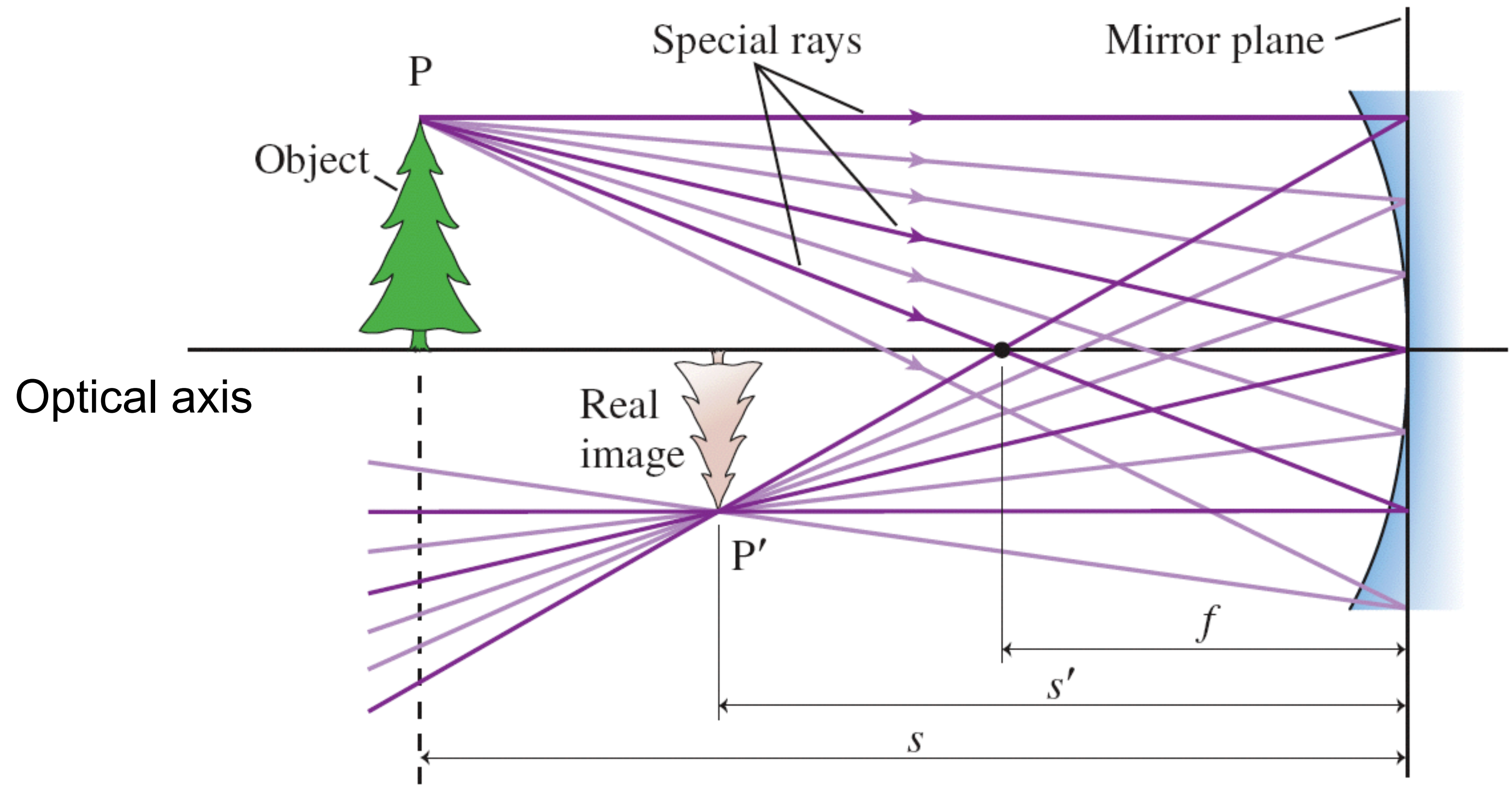
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad \text{(thin-lens equation)}$$

where f is the focal length of the lens, which can be found from

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{(lens maker's equation)}$$

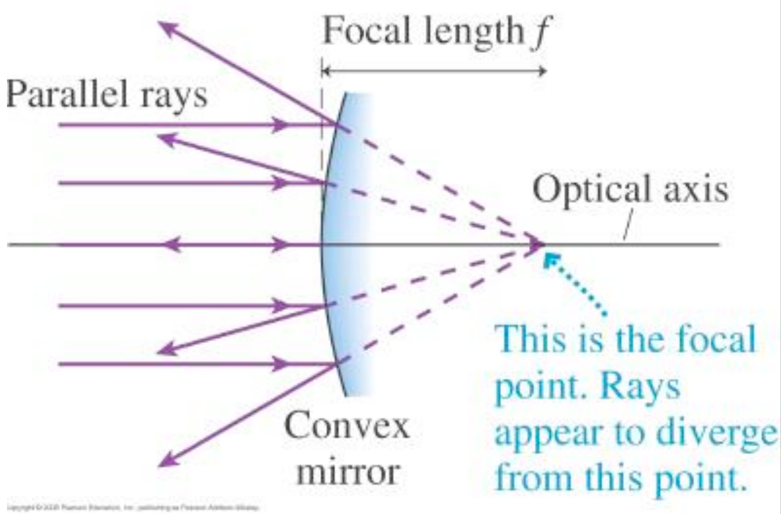
where R_1 is the radius of curvature of the first surface, and R_2 is the radius of curvature of the second surface, and the material surrounding the lens has $n = 1$.

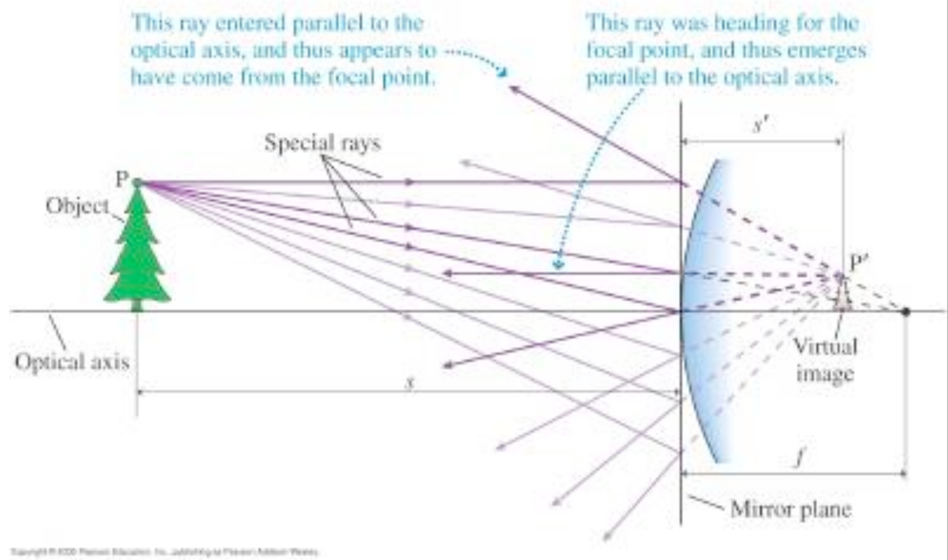
FIGURE 23.52 A real image formed by a concave mirror.



1. A parallel ray reflects through focal point
2. A ray through the focal point reflects parallel to the axis
3. A ray striking the mirror center reflects at an equal angle on the opposite side of the axis

Convex Mirror





Virtual Image – Rays Diverge

The Mirror Equation

For a spherical mirror with negligible thickness, the object and image distances are related by

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad \text{(thin-mirror equation)}$$

where the focal length f is related to the mirror's radius of curvature by

$$f = \frac{R}{2}$$

TABLE 23.5 Sign convention for spherical mirrors

	Positive	Negative
R and f	Concave toward the object	Convex toward the object
s'	Real image, same side as object	Virtual image, opposite side from object

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Fermat’s Principle for Refraction

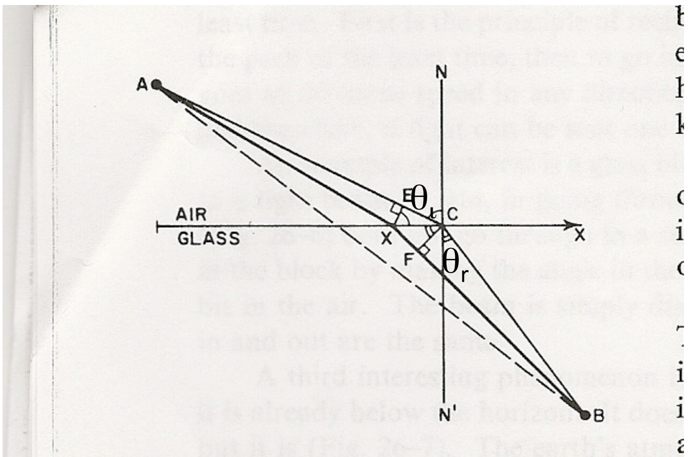


Fig. 26–4. Illustration of Fermat's principle for refraction.

$$v = c / n$$

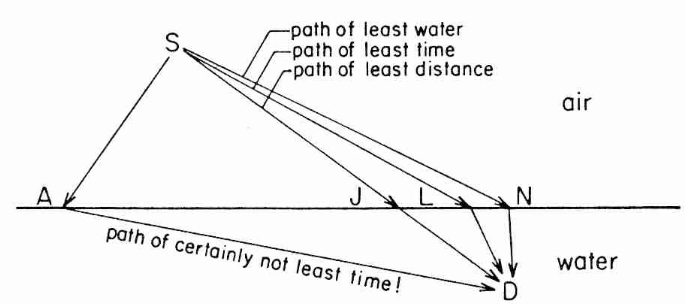


FIGURE 30. *Finding the path of least time for light is like finding the path of least time for a lifeguard running and then swimming to rescue a drowning victim: the path of least distance has too much water in it; the path of least water has too much land in it; the path of least time is a compromise between the two.*

$$v \sin\theta_i = c \sin\theta_r$$
$$\sin\theta_i = n \sin\theta_r$$

Snell’s law

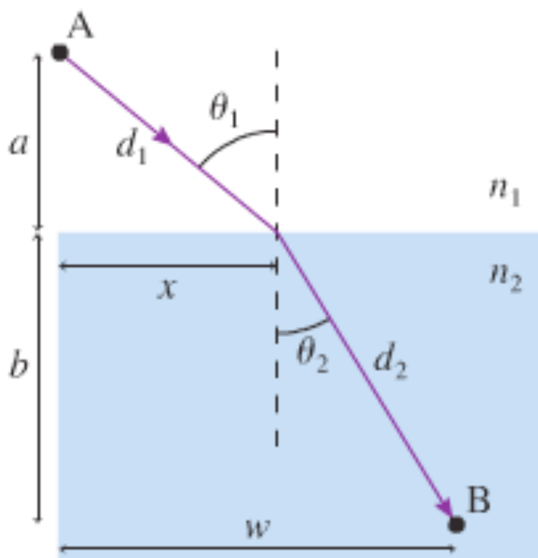
The slower the medium the smaller the refraction angle

1. Find travel time from A to B

$$t = t_1 + t_2 = \frac{d_1}{v_1} + \frac{d_2}{v_2} = \frac{d_1}{c/n_1} + \frac{d_2}{c/n_2} = \frac{n_1 d_1}{c} + \frac{n_2 d_2}{c} = \frac{n_1}{c}\sqrt{x^2 + a^2} + \frac{n_2}{c}\sqrt{(w-x)^2 + b^2}$$

2. Minimize this expression with respect to time

$$\frac{dt}{dx} = 0 = \frac{n_1 x}{c\sqrt{x^2 + a^2}} - \frac{n_2 (w-x)}{c\sqrt{(w-x)^2 + b^2}}$$



3. Solve for x. Easier to

$$\frac{x}{\sqrt{x^2 + a^2}} = \frac{x}{d_1} = \sin\theta_1 \qquad \frac{w-x}{\sqrt{(w-x)^2 + b^2}} = \frac{w-x}{d_2} = \sin\theta_2$$

$$\frac{n_1}{c}\sin\theta_1 - \frac{n_2}{c}\sin\theta_2 = 0 \Rightarrow n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Fermat’s Applications

$$r_1 + r_2 = const$$

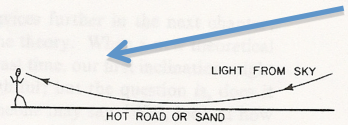


Fig. 26–8. A mirage.

water

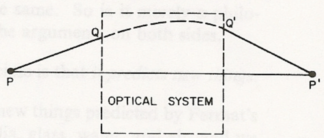


Fig. 26–9. An optical “black box.”

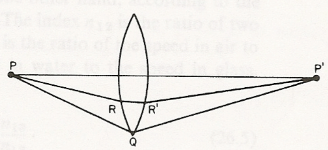


Fig. 26–10. A focusing optical system.

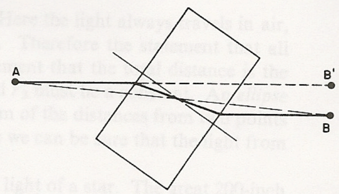


Fig. 26–6. A beam of light is offset as it passes through a transparent block.

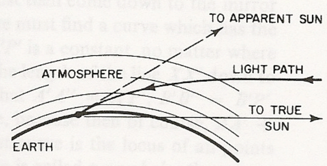


Fig. 26–7. Near the horizon, the apparent sun is higher than the true sun by about 1/2 degree.

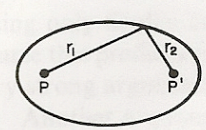


Fig. 26–11. An ellipsoidal mirror.

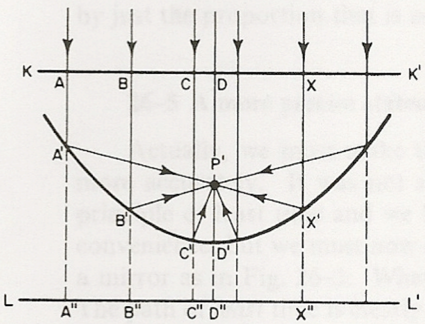


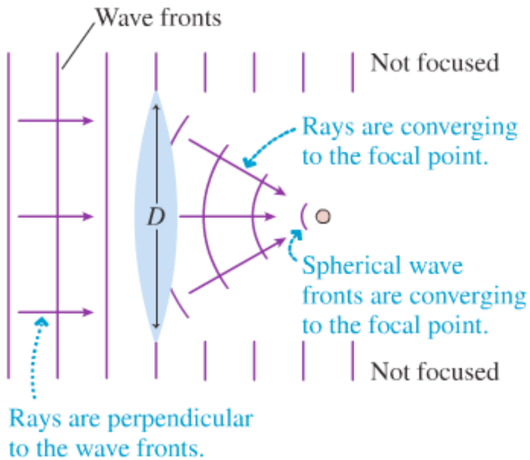
Fig. 26–12. A paraboloidal mirror.

Make a lot of rays go from P to P’

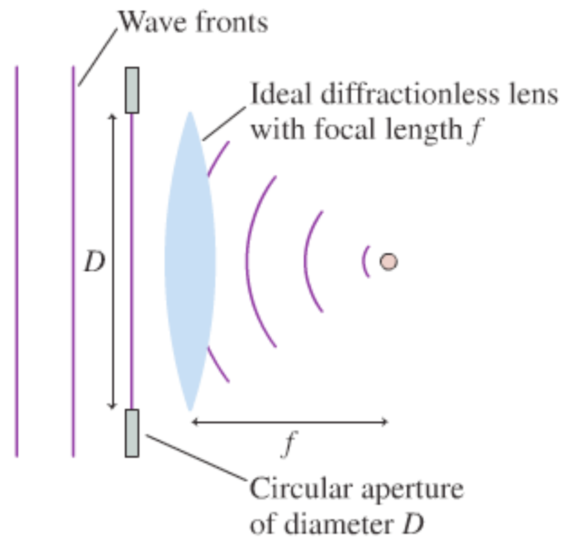
AA’P’=BB’P’=…DD’P’=…

FIGURE 24.18 A lens both focuses and diffracts the light passing through.

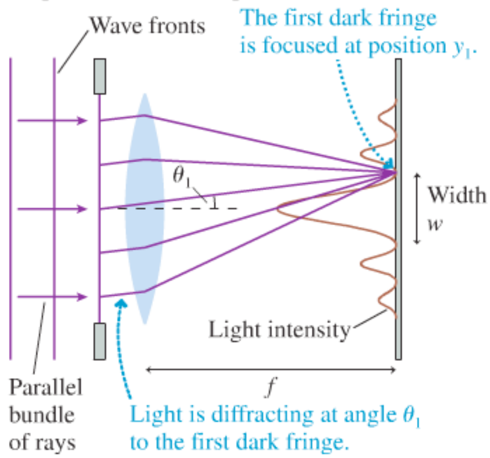
(a) A lens acts as a circular aperture.



(b) The aperture and focusing effects can be separated.



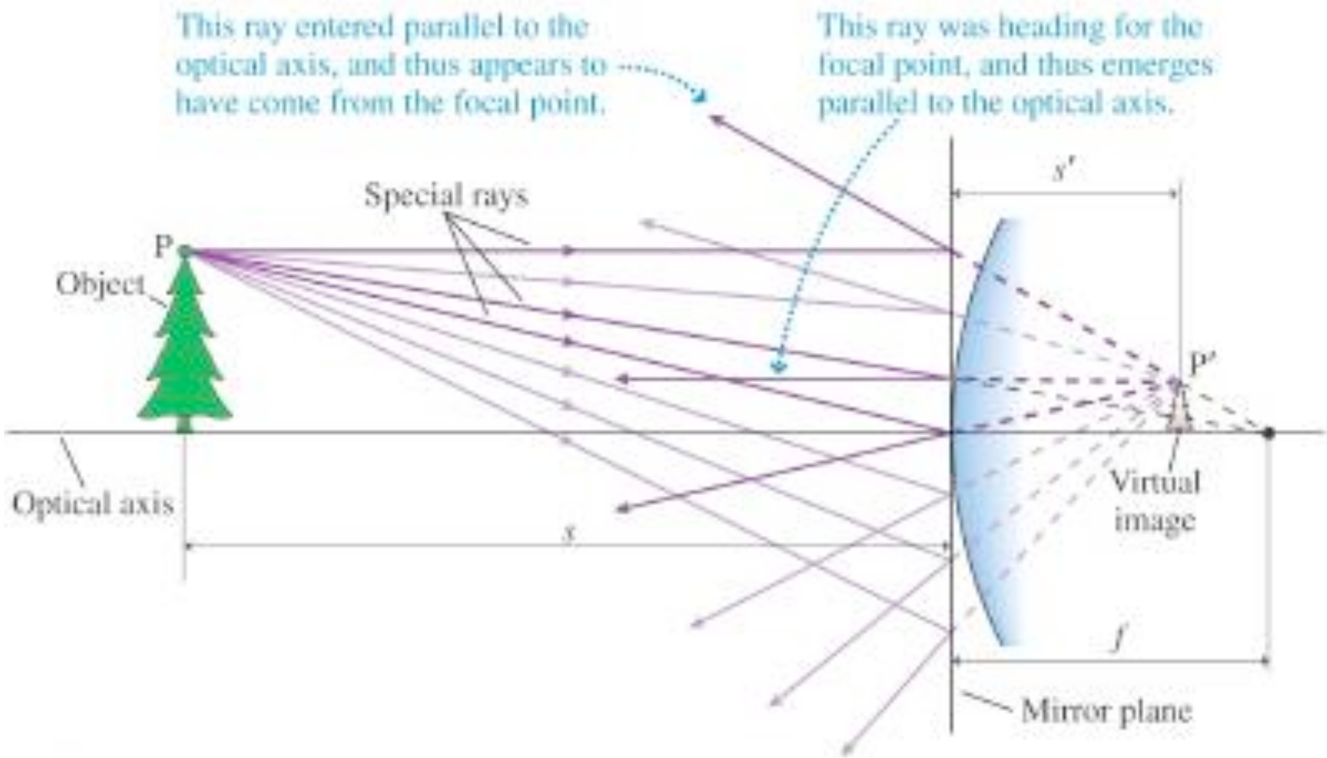
(c) The lens focuses the diffraction pattern in the focal plane.



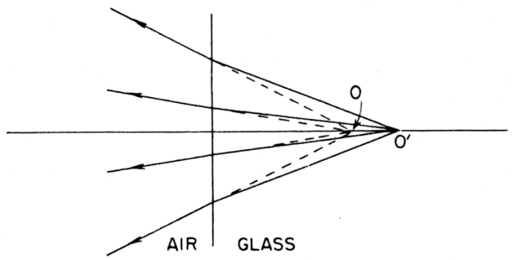
$$\theta_1 = 1.22\lambda / D$$

$$y_1 \approx f\theta_1$$

$$w \approx 2f\theta_1 = 2.44\lambda f / D$$


This ray entered parallel to the optical axis, and thus appears to have come from the focal point.
This ray was heading for the focal point, and thus emerges parallel to the optical axis.
Special rays
P
Object
P'
s'
Virtual image
Optical axis
s
f
Mirror plane

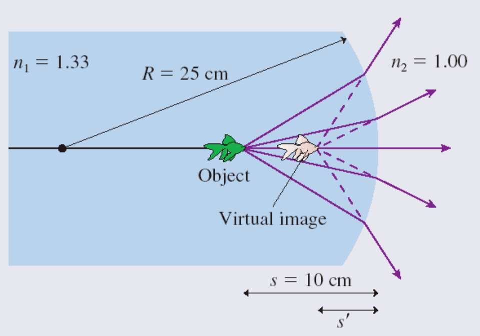
Flat Surface Images



A plane surface reimages light from O' to O

Look from a rare medium to a dense medium with a plane surface at an object that is at a certain distance the object will appear as though the light is coming not as far back. Look at a the bottom of a swimming pool from above it looks about ¾ shallower (i.e. 1/n shallower)

FIGURE 23.46 The curved surface of a fish bowl produces a virtual image of the fish.



$$f = \frac{R}{n-1}$$

$$\frac{1}{s} + \frac{n}{s'} = \frac{1}{f}$$

$$R \rightarrow \infty$$

$$\frac{1}{s} + \frac{n}{s'} = 0$$

$$s' = -ns$$

Notice again that negative s' means image on the same side with the object

Focal Length

$$\frac{1}{s} + \frac{n}{s'} = \frac{n-1}{R}$$

$$s \to \infty$$

$$\frac{n}{f'} = \frac{n-1}{R}$$

$$\Rightarrow \quad f' = \frac{n}{n-1} R$$

$$s' \to \infty$$

$$\Rightarrow \quad f = \frac{R}{n-1}$$

$$\frac{1}{s} + \frac{n}{s'} = \frac{1}{f}$$

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$

The form of equation $\frac{1}{s} + \frac{n}{s'} = \frac{1}{f}$ is much more useful than the one involving R and n, because it easier to measure f than n and R.

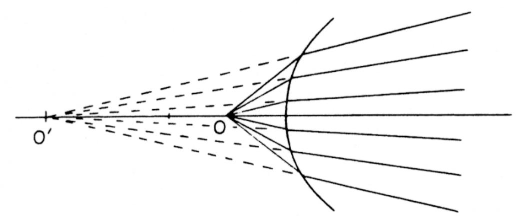
Check what happens if s<f, then s’ negative. Namely it will focus in negative values. What is the meaning?

$$\frac{n}{s'} = \frac{1}{f} - \frac{1}{s}$$

$$s < f,$$

$$1/f < 1/s$$

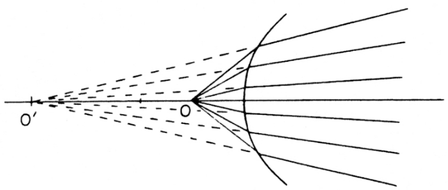
$$s' < 0$$



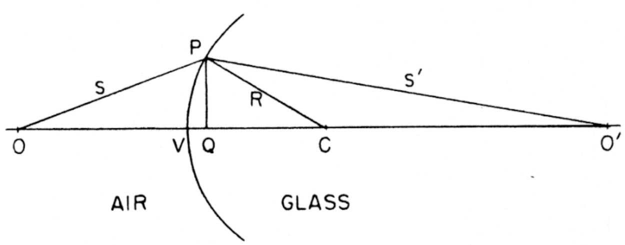
Virtual Image

$$\frac{1}{s}+\frac{n}{s'}=\frac{1}{f}$$

If we draw the rays that diverge from O they will bent at the surface and will not come to focus, because O is so close that they are “beyond parallel”. However the diverge as coming from a point O’ outside the glass. This should be contrasted to the image O’ found when s>f. This image is called areal image while the first a virtual image. When s’ is negative it means that O’ is on the other side of the surface and everything is OK.



Virtual image



Real image