Lecture 14
Wave Optics-2 Chapter 22

PHYSICS 270

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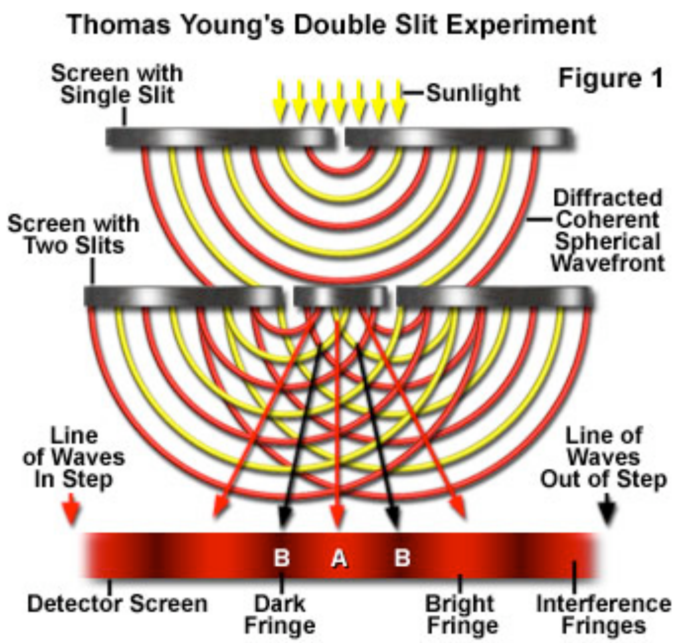
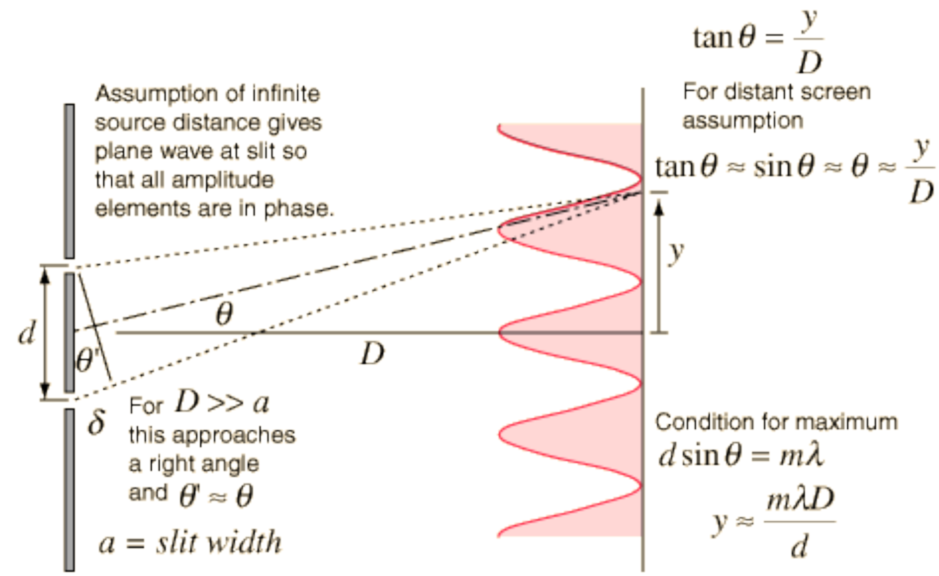
March 17, 2011

Double Slit experiment for light





Minima

$$d\sin\theta_m = (m + 1/2)\lambda,\ \mathrm{m} = 0,1,2.,\ \mathrm{dark}$$

$$\theta_m \approx (m + 1/2)\lambda$$

Maxima

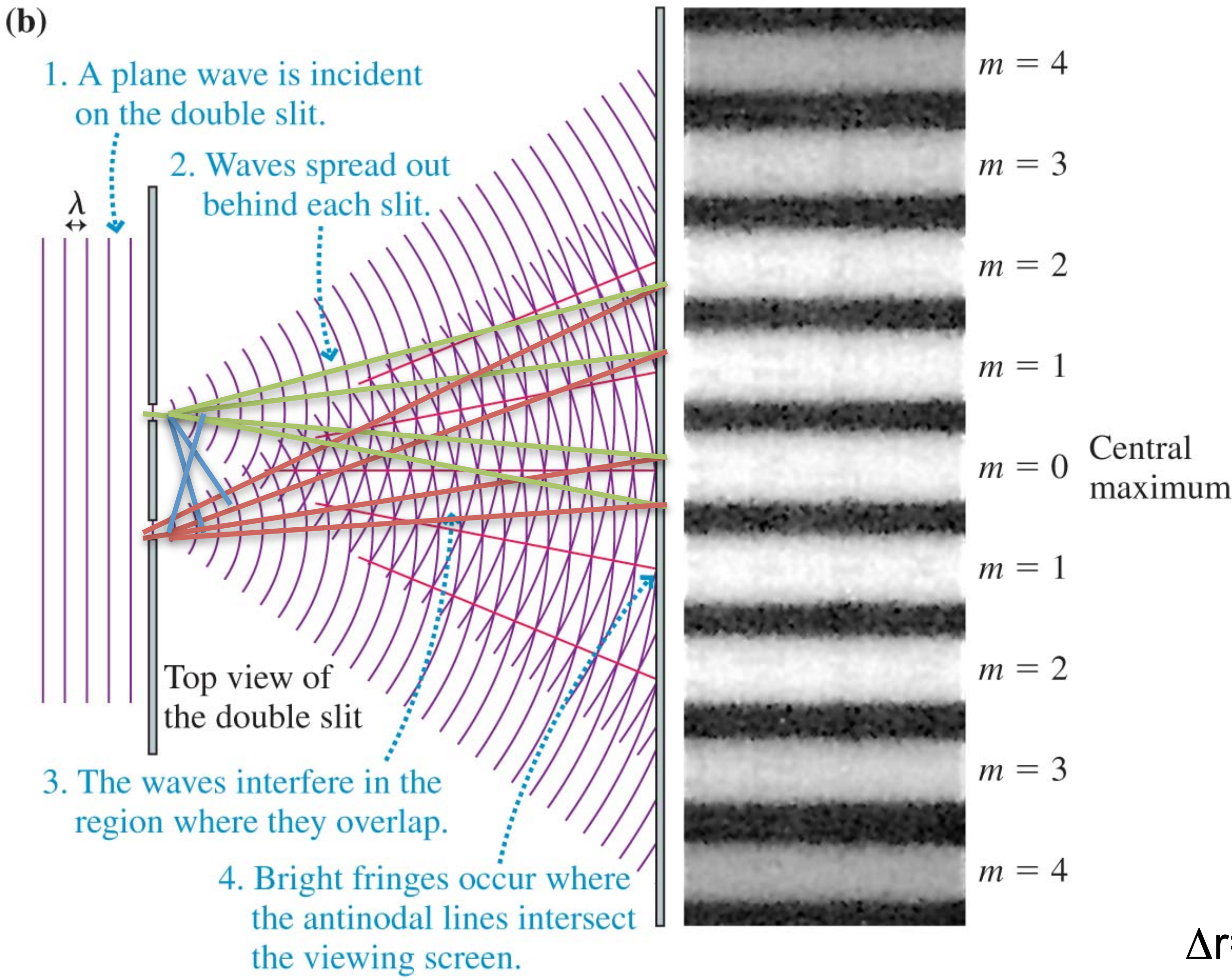
$$d\sin\theta_m = m\lambda,\ \ m = 0,1,2,3,..,bright$$

$$\theta_m = m\lambda / d$$

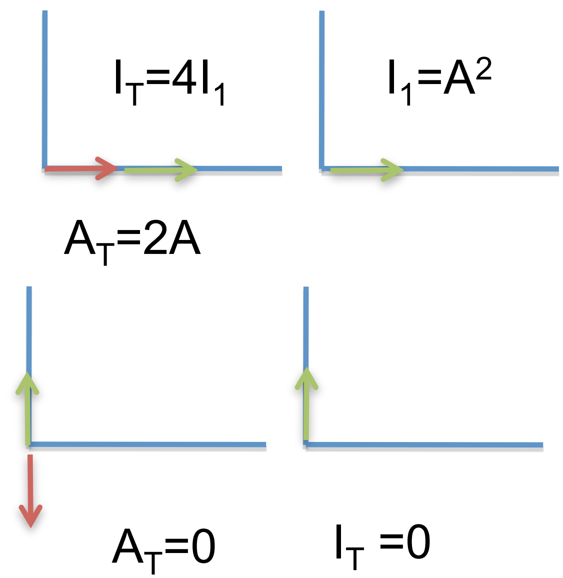
Dark fringes exactly midway of bright ones

How to measure the wavelength of light ?

FIGURE 22.3 A double-slit interference experiment.



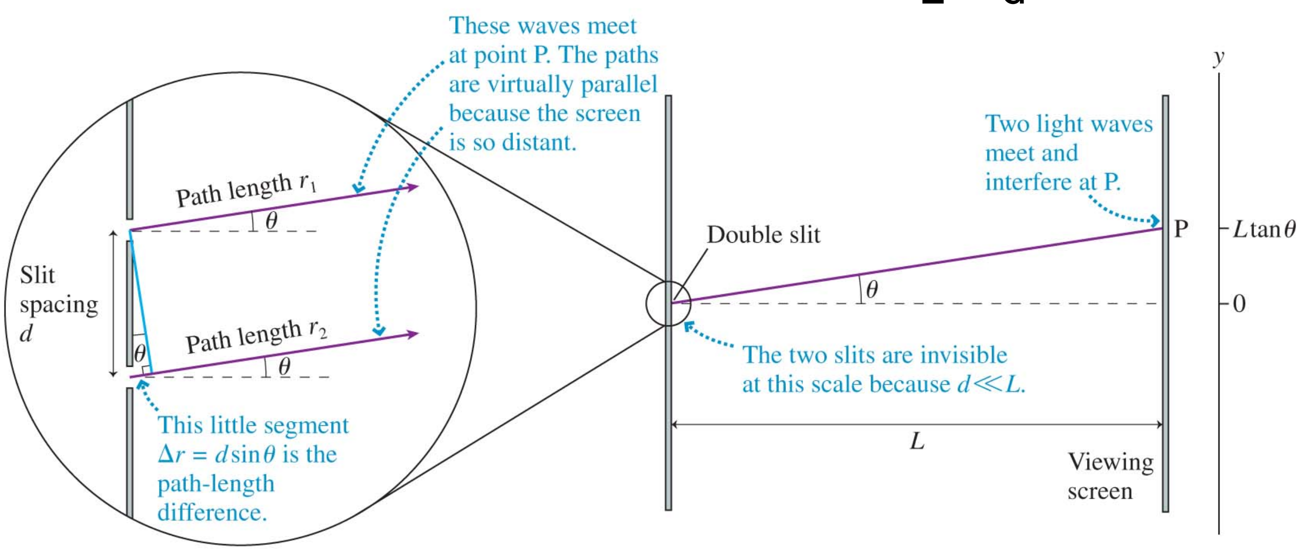
$\Delta r = 0, \lambda, 2\lambda, 3\lambda ..., m\lambda$



$\Delta r = \lambda/2, 3\lambda/2, 5\lambda/2..., (m+1/2)\lambda$

FIGURE 22.4 Geometry of the double-slit experiment.

L>>d



$$y_m^b = m\frac{\lambda L}{d}, m = 0,1,2,$$

$$\theta_m^b \approx m\frac{\lambda}{d}, m = 0,1,2..$$

$$\Delta y = y_{m+1} - y_m = [(m+1) - m]\frac{\lambda L}{d} = \frac{\lambda L}{d}$$

Independent of m - Same spacing

$$.\theta_m^d \approx (m + 1/2)\frac{\lambda}{d}, m = 0,1,2,..$$

$$y_m^d \approx (m + 1/2)\frac{\lambda L}{d}, m = 0,1,2$$

Bright Fringe	Δr	θ	Δy
Central	0	0	0
m=1	λ	λ/d	Lλ/d
m=2	2 λ	2 λ/d	2 Lλ/d
m=3	3 λ	3 λ/d	3 Lλ/d

Dark Fringe	Δr	θ	Δy
m=0	λ/2	λ/2d	Lλ/2d
m=1	3 λ/2	3 λ/2d	3 Lλ/2d
m=2	5 λ/2	5 λ/2d	5 Lλ/2d
m=3	7 λ/2	7 λ/2d	7 Lλ/2d

Analyzing Double-Slit Interference

The mth bright fringe emerging from the double slit is at an angle

$$\theta_m = m\frac{\lambda}{d} \qquad m = 0, 1, 2, 3, \ldots \qquad \text{(angles of bright fringes)}$$

where θ_m is in radians, and we have used the small-angle approximation. The y-position on the screen of the mth fringe is

$$y_m = \frac{m\lambda L}{d} \qquad m = 0, 1, 2, 3, \ldots \qquad \text{(positions of bright fringes)}$$

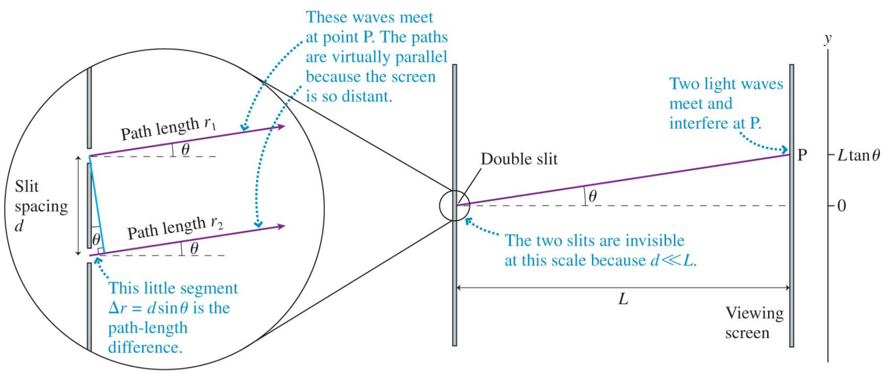
while dark fringes are located at positions

$$y'_m = \left(m + \frac{1}{2}\right)\frac{\lambda L}{d} \qquad m = 0, 1, 2, \ldots$$

(positions of dark fringes)

Double slit intensity pattern

FIGURE 22.4 Geometry of the double-slit experiment.



If there was no interference intensity for two slits → $2I_1$

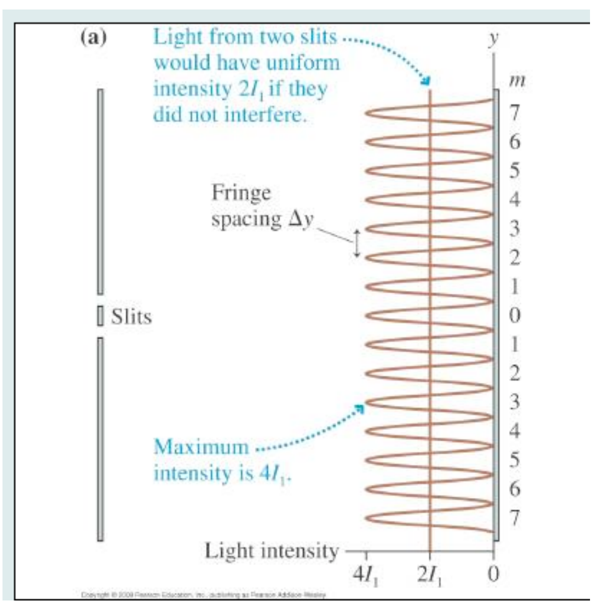
$$\Delta\phi = k(r_2 - r_1) = k\Delta r = kd\sin\theta = 2\pi\frac{d}{\lambda}\sin\theta \approx 2\pi\frac{d}{\lambda}\tan\theta = 2\pi\frac{d}{\lambda L}y$$

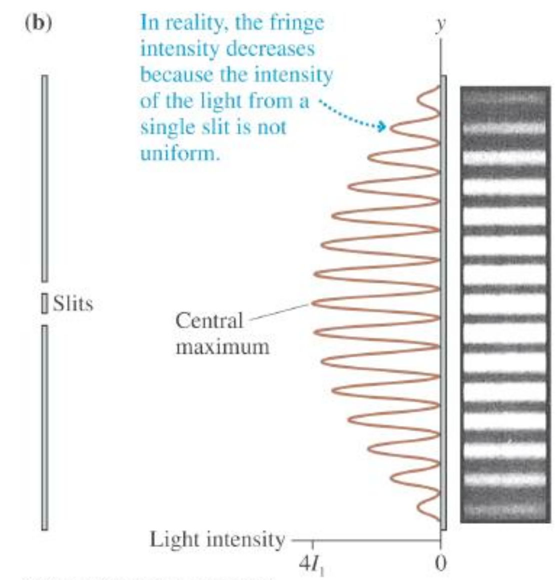
$$A = \left|2a\cos(\frac{\Delta\phi}{2})\right| = \left|2a\cos(\frac{\pi d}{\lambda L}y)\right|$$

$$I = cA^2 = 4ca^2\cos^2(\frac{\pi d}{\lambda L}y)$$

$$I_2 = 4I_1\cos^2(\frac{\pi d}{\lambda L}y)$$

I_1 light intensity of single slit





Interference of N overlapped waves

The Diffraction Grating

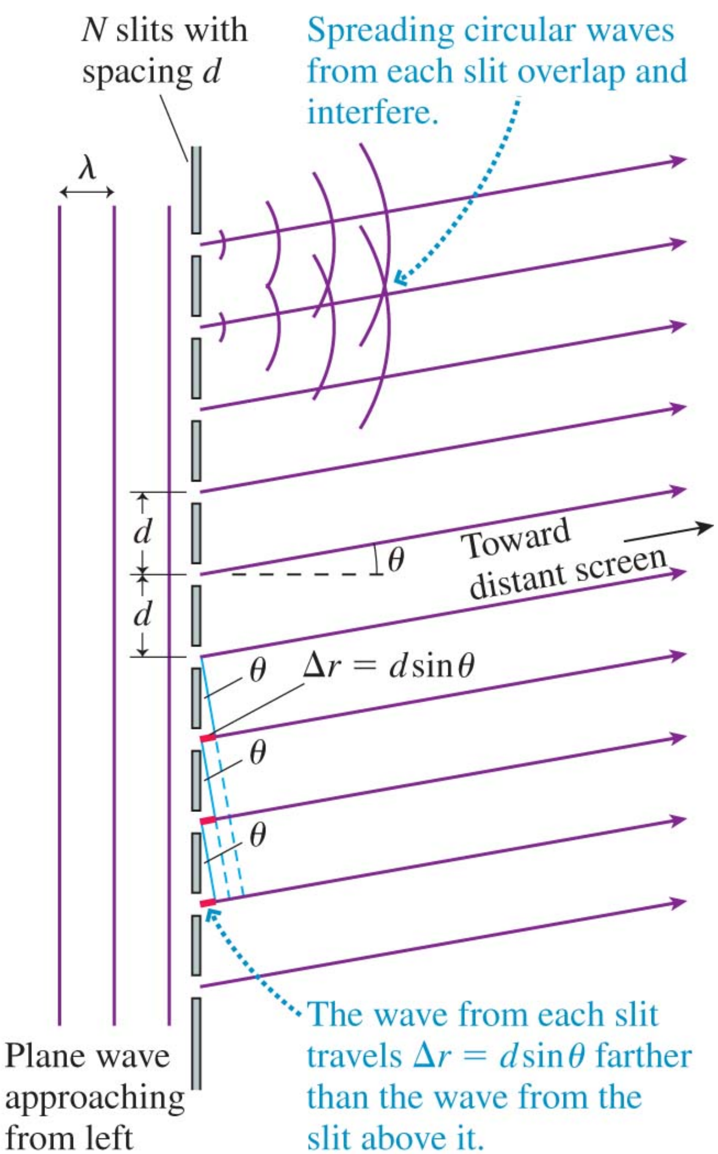
Suppose we were to replace the double slit with an opaque screen that has *N* closely spaced slits. When illuminated from one side, each of these slits becomes the source of a light wave that diffracts, or spreads out, behind the slit. Such a multi-slit device is called a **diffraction grating.** Bright fringes will occur at angles ϑ_m, such that

$$d \sin\theta_m = m\lambda \qquad m = 0, 1, 2, 3, \ldots$$

The *y*-positions of these fringes will occur at

$$y_m = L \tan\theta_m \qquad \text{(positions of bright fringes)}$$

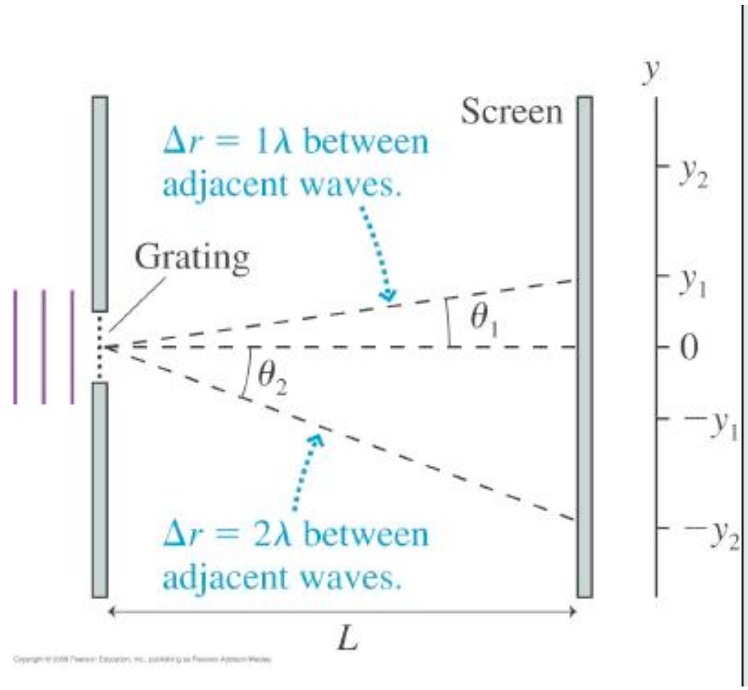
FIGURE 22.6 Top view of a diffraction grating with $N = 10$ slits.



$$d \sin\theta_m = m\lambda \qquad m = 0, 1, 2, 3, \ldots$$

$$y_m = L \tan\theta_m \approx L\theta_m$$

m is the order of diffraction

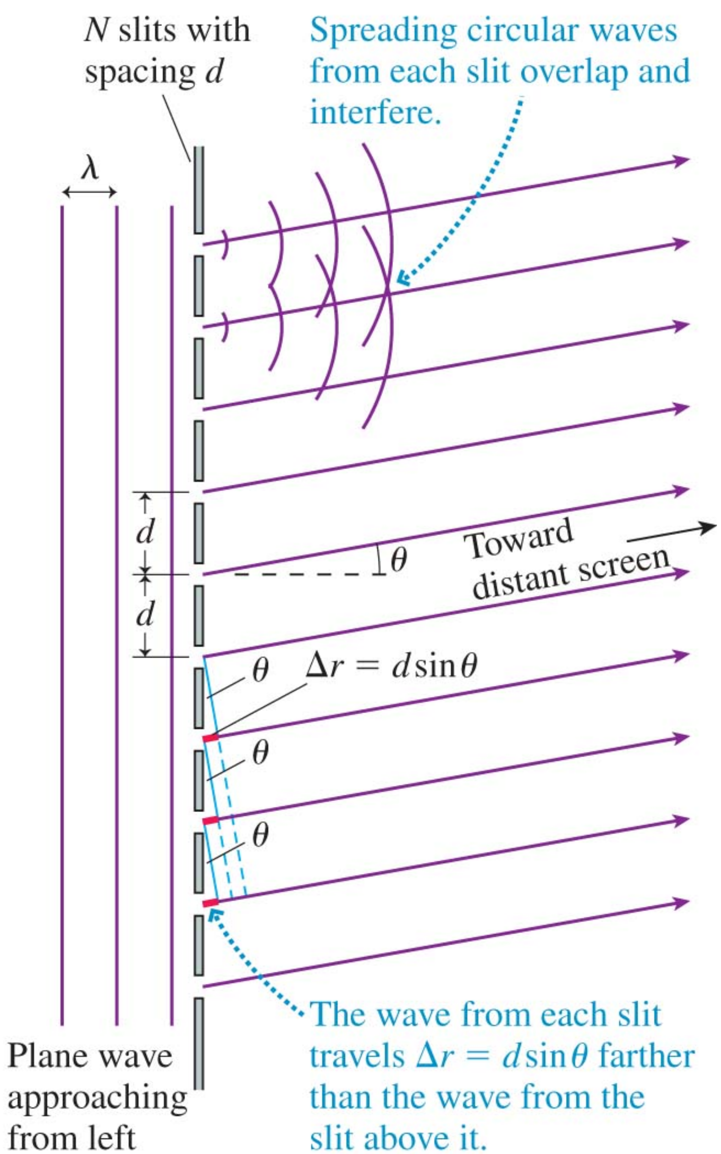


$$I \propto A_R^2 = 4A^2 \cos^2(\Delta\phi/2)$$

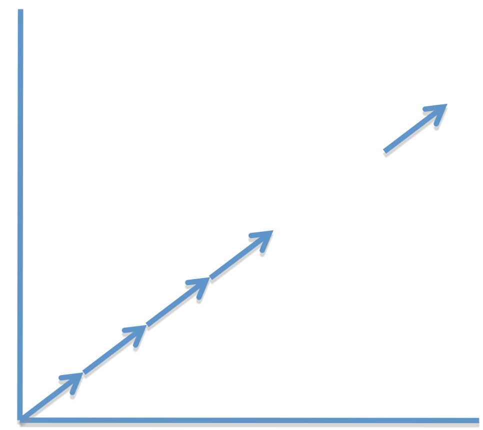
$$\Delta\phi = 2\pi \frac{\Delta r}{\lambda} = 2\pi \frac{d \sin\theta}{\lambda} \approx 2\pi \frac{d \tan\theta}{\lambda}$$

$$I_N = N^2 I_1 \cos^2\left(\frac{\pi d}{\lambda L} y\right)$$

FIGURE 22.6 Top view of a diffraction grating with $N = 10$ slits.



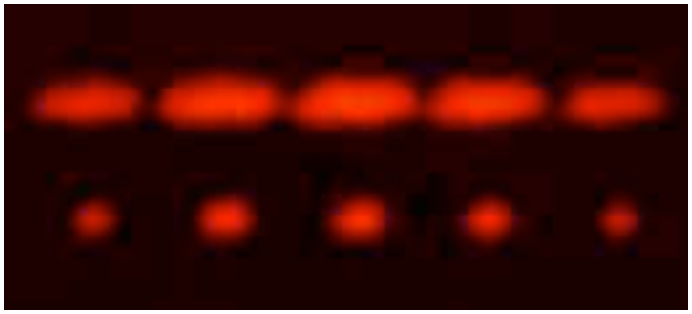
$$d \sin\theta_m = m\lambda \qquad m = 0, 1, 2, 3, \ldots$$



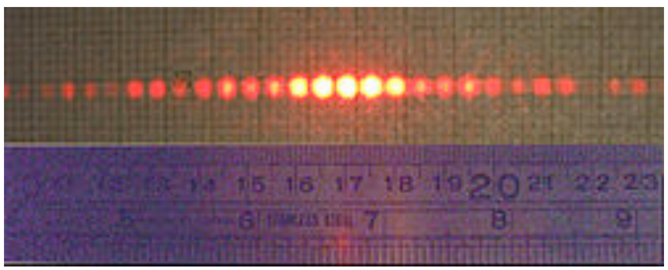
$A_{total} = NA = 10A$

$I_{tot} = (NA)^2 = N^2 I_1 = 100 I_1$

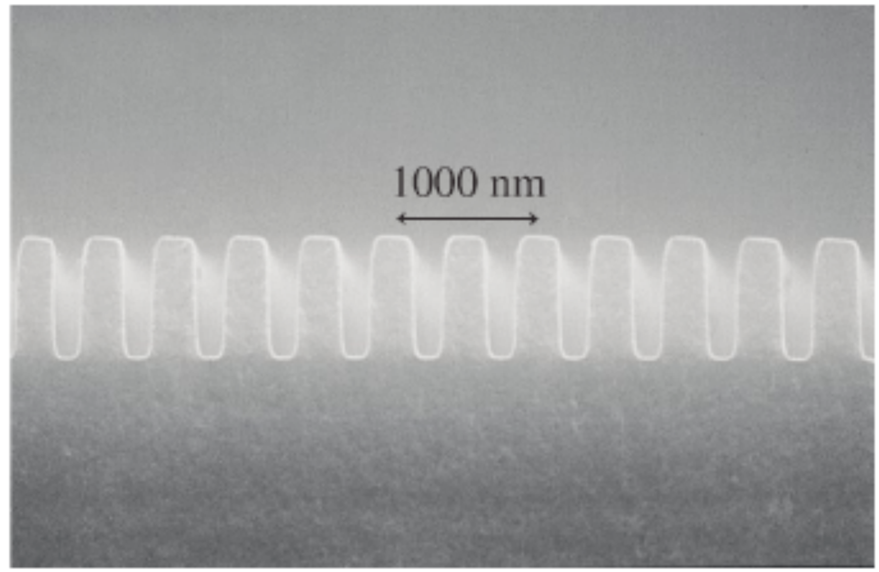
Key dependence N^2



Two slit and five slit diffraction

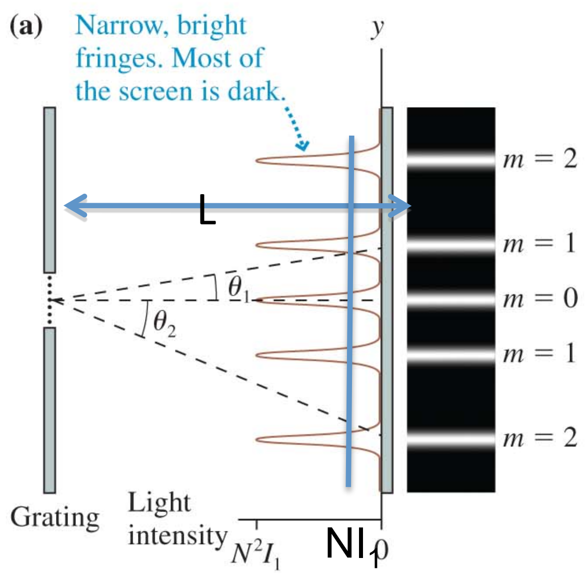


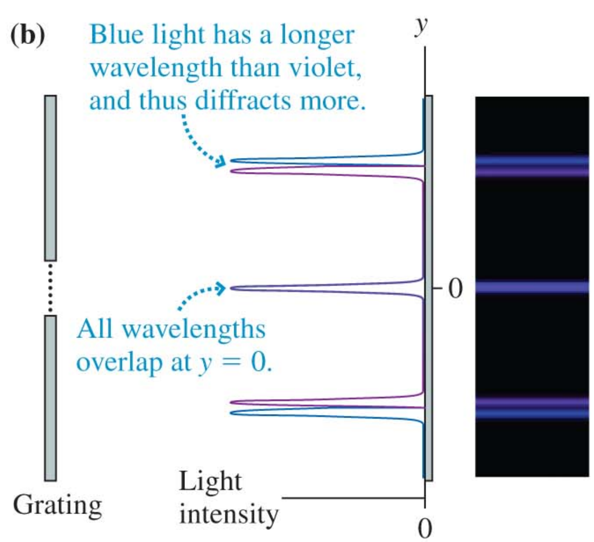
653 nm with 150 slits



A microscopic side-on look at a diffraction grating.

FIGURE 22.8 The interference pattern behind a diffraction grating.



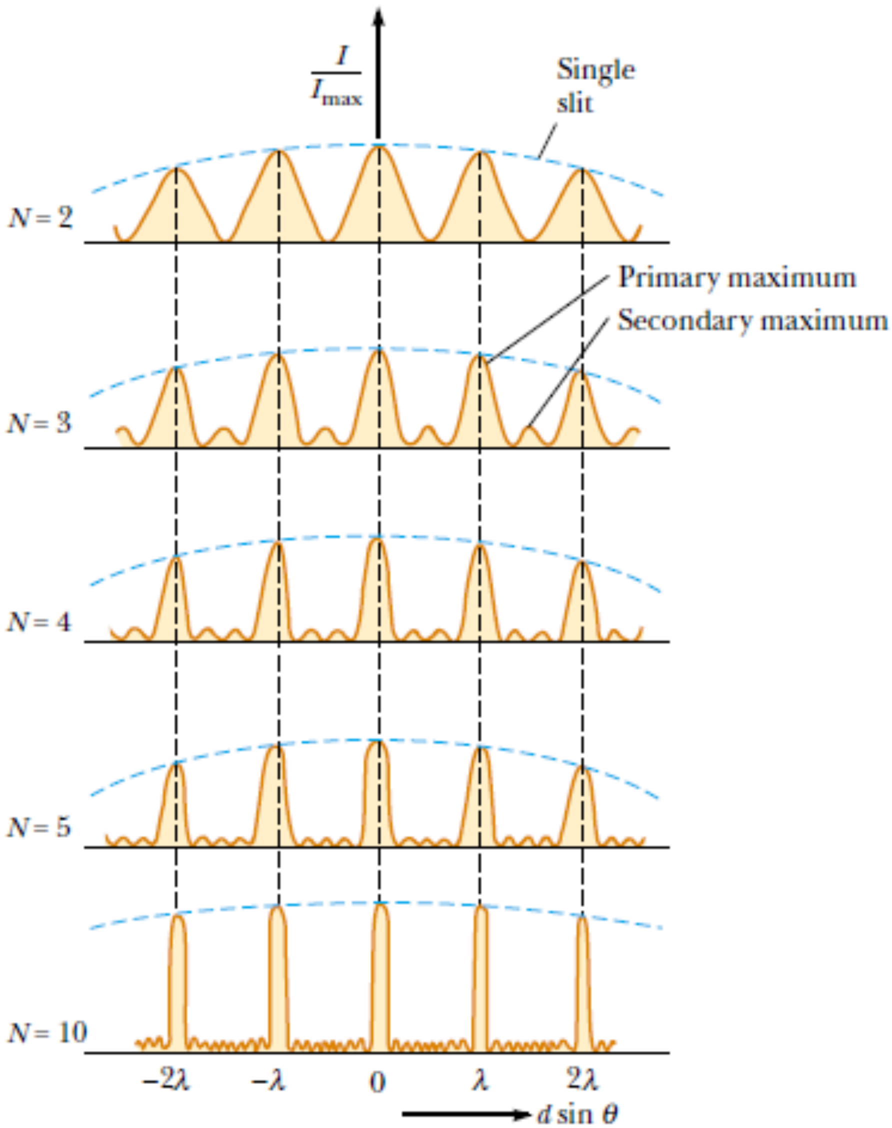


Interference pattern is mostly dark but with a small number of very bright and very narrow lines. Why ?

Conservation of energy. No interference $E_T = NI_1\Delta_1$
Interference $E_T = N^2 I_1 \Delta_N$
Therefore $\Delta_N/\Delta_1 = 1/N$

$$y_m = m\lambda L / d$$

Dispersive effect – Light with longer wavelength diffracts more



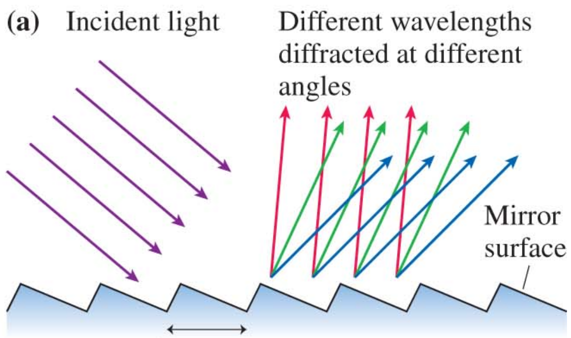
$\frac{I}{I_{max}}$
Single slit
$N = 2$
Primary maximum
Secondary maximum
$N = 3$
$N = 4$
$N = 5$
$N = 10$
-2λ
$-\lambda$
0
λ
2λ
$d \sin \theta$

Spectral analysis or
Spectroscopy

Transmission Grating ->
Many parallel slits

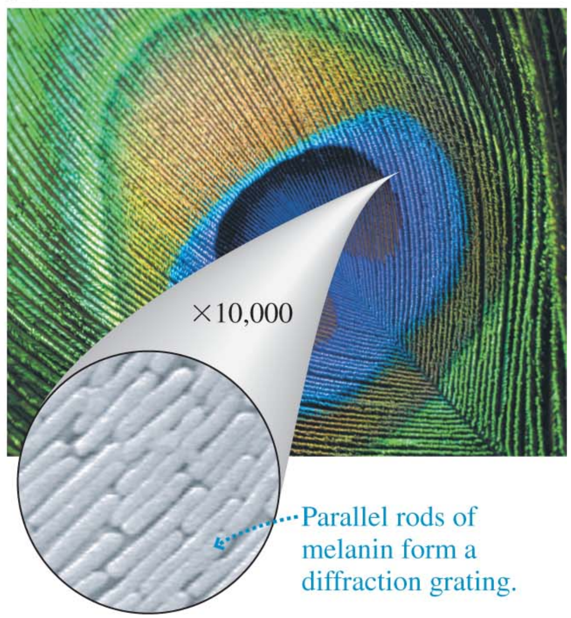
FIGURE 22.9 Reflection gratings.

(a)



A reflection grating can be made by cutting parallel grooves in a mirror surface. These can be very precise, for scientific use, or mass produced in plastic.

(b)



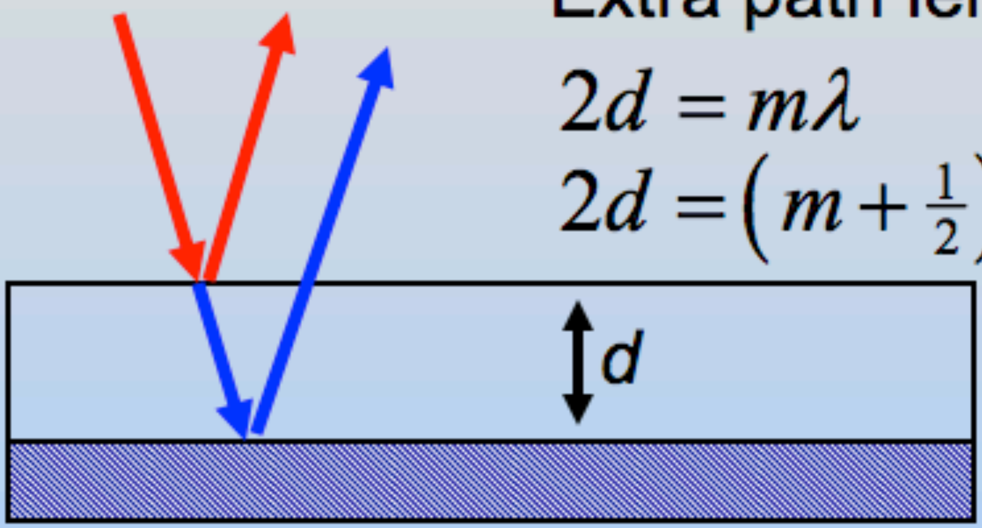
Parallel rods of melanin form a diffraction grating.

Thin Film: Extra Path

Extra path length ~ 2*d*

$$2d = m\lambda \quad \Rightarrow \text{Constructive}$$

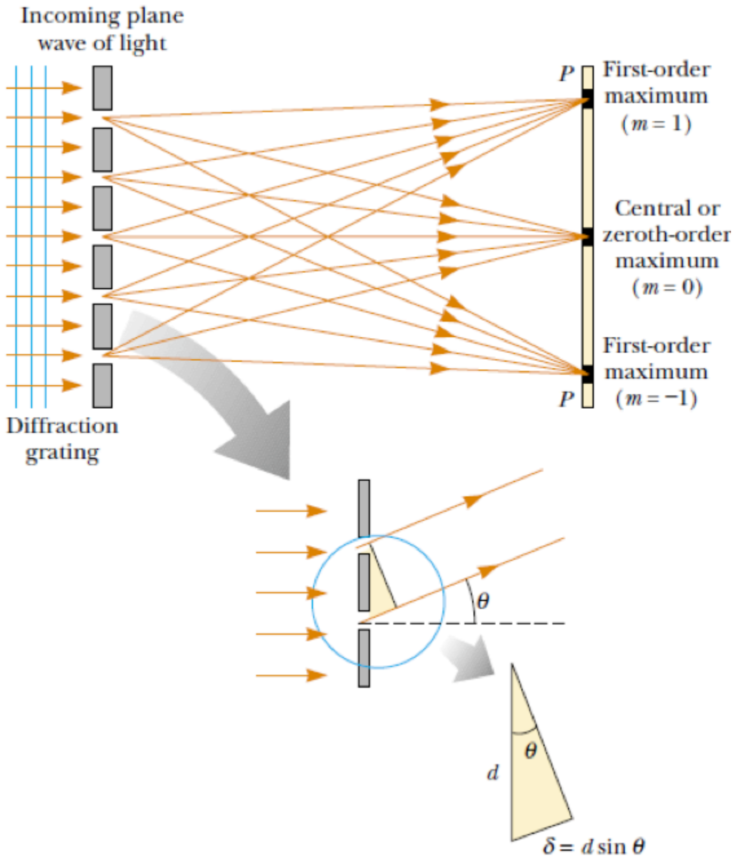
$$2d = \left(m + \tfrac{1}{2}\right)\lambda \Rightarrow \text{Destructive}$$



Oil on concrete, non-reflective coating on glass, etc.

$$R = A[\cos\omega t + \cos(\omega t + \phi) + \cos(\omega t + 2\phi) + \cos(\omega t + 3\phi) + ...]$$

$$\phi = 2\pi \frac{d\sin\theta}{\lambda}$$



$$O\hat{Q}S = \phi$$

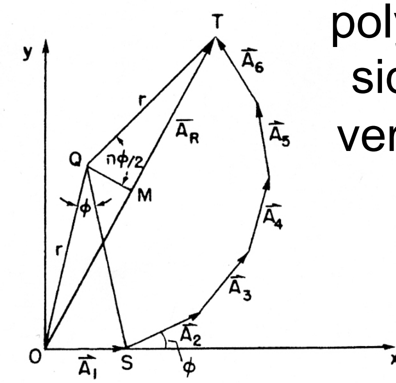
$$A = 2r\sin(\phi/2)$$

$$O\hat{Q}T = n\phi$$

$$A_R = 2r\sin(n\phi/2) = A\frac{\sin(n\phi/2)}{\sin(\phi/2)}$$

$$I = I_o \frac{\sin^2(n\phi/2)}{\sin^2(\phi/2)}$$

$$I_o \equiv A^2$$



Equilateral polygon with 6 sides whose vertices lie on a circle

Fig. 30–1. The resultant amplitude of $n = 6$ equally spaced sources with net successive phase differences ϕ.

Angle OQS=$\pi-2\theta=$
$=\pi-(\pi-\phi)=\phi$

$\sin(\phi/2)=(A/2)/r$

$r=(A/2)\sin(\phi/2)$

From triangle QMT

$r=(A_R/2)\sin(N\phi/2)$

$$A = 2r\sin(\phi/2)$$

$$A_R = 2r\sin(N\phi/2) = A\frac{\sin(N\phi/2)}{\sin(\phi/2)}$$

$$I = I_o\frac{\sin^2(N\phi/2)}{\sin^2(\phi/2)}$$

$$I_o \equiv A^2$$

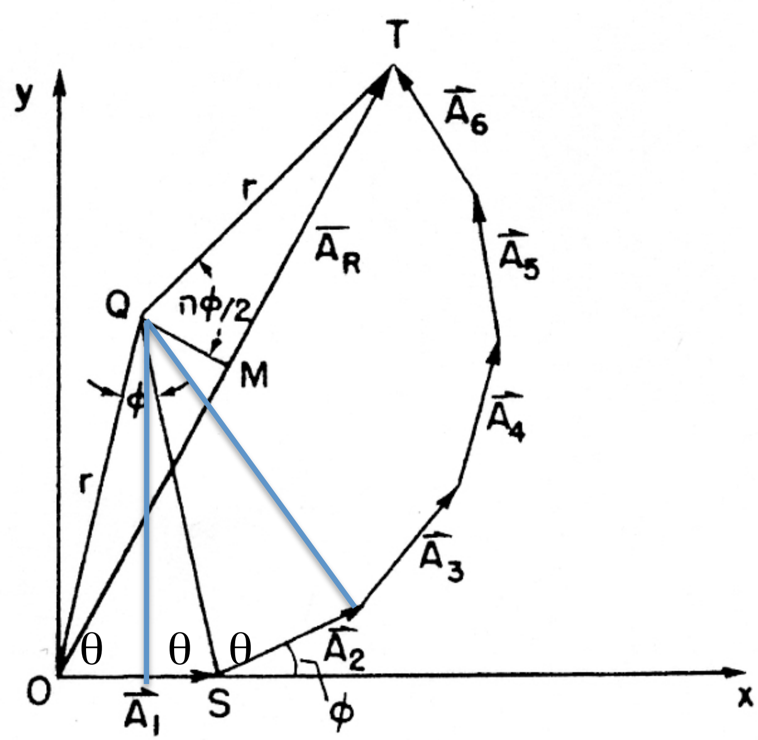
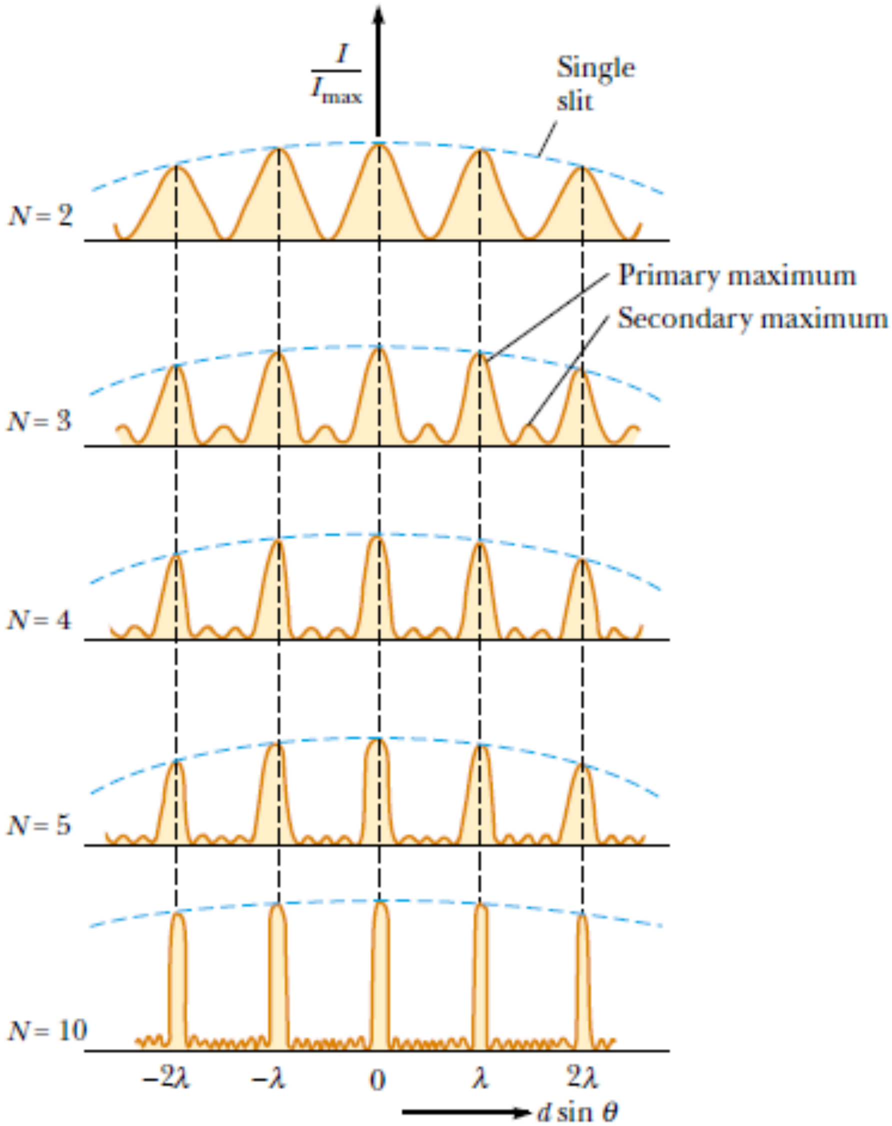


Fig. 30–1. The resultant amplitude of $n = 6$ equally spaced sources with net successive phase differences ϕ.

Gratings



$$I = I_o \frac{\sin^2(N\phi/2)}{\sin^2(\phi/2)}$$

$$\phi = 2\pi \frac{d\sin\theta}{\lambda}$$

Phased Arrays

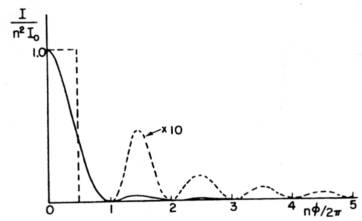


Fig. 30–2. The intensity as a function of phase angle for a large number of oscillators of equal strength.

Addition of several sources with fixed but controlled phases To get much stronger intensity on a preferred direction and the nearby maxima present in two sources will be reduced in strength. Take N large and plot region near $\phi=0$, you get $N^2 I_o$. Increase ϕ and the first time it reaches zero is when $N\phi/2=\pi$ etc.

$$I = I_o \frac{\sin^2(N\phi/2)}{\sin^2(\phi/2)}$$

$$\phi = 2\pi \frac{d \sin\theta}{\lambda}$$

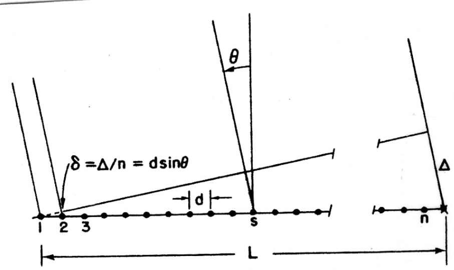


Fig. 30–3. A linear array of n equal oscillators, driven with phases $\alpha_s = s\alpha$.

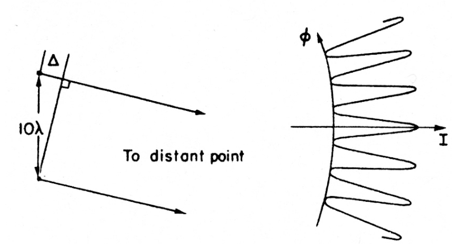


Fig. 29–7. The intensity pattern for two dipoles separated by 10λ.

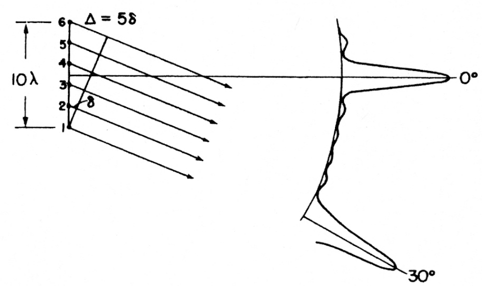
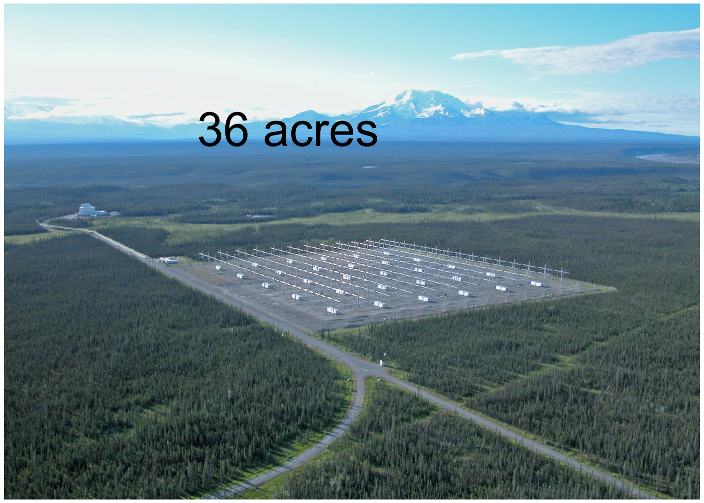


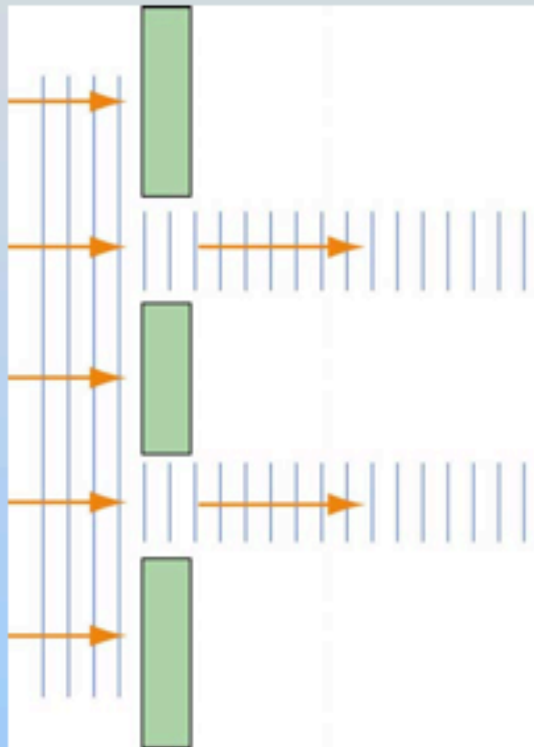
Fig. 29–8. A six-dipole antenna array and part of its intensity pattern.

HAARP



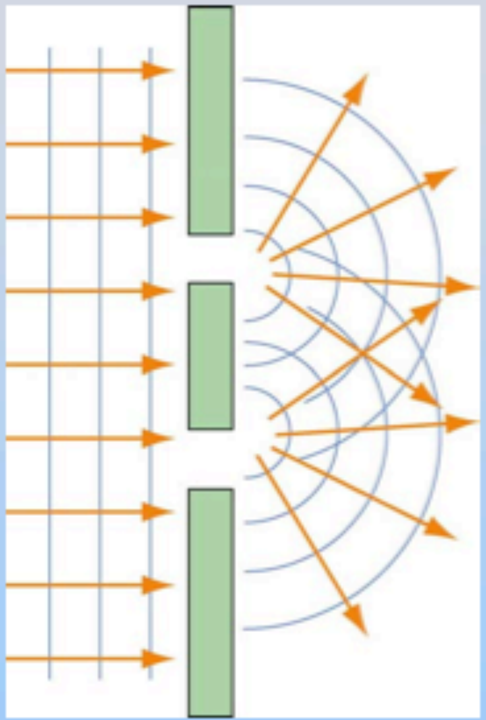
Diffraction

Diffraction: The bending of waves as they pass by certain obstacles



No Diffraction

No spreading after passing though slits



Diffraction

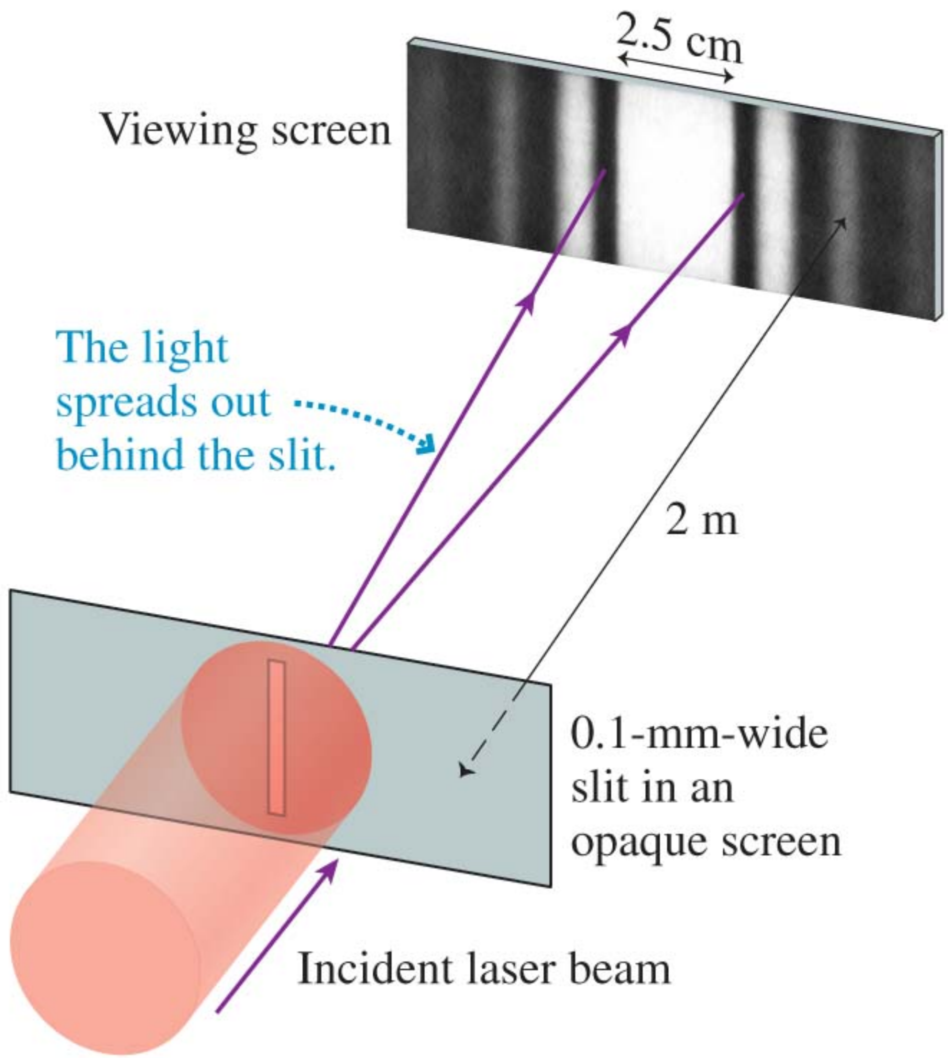
Spreading after passing though slits

SINGLE SLIT DIFFRACTION

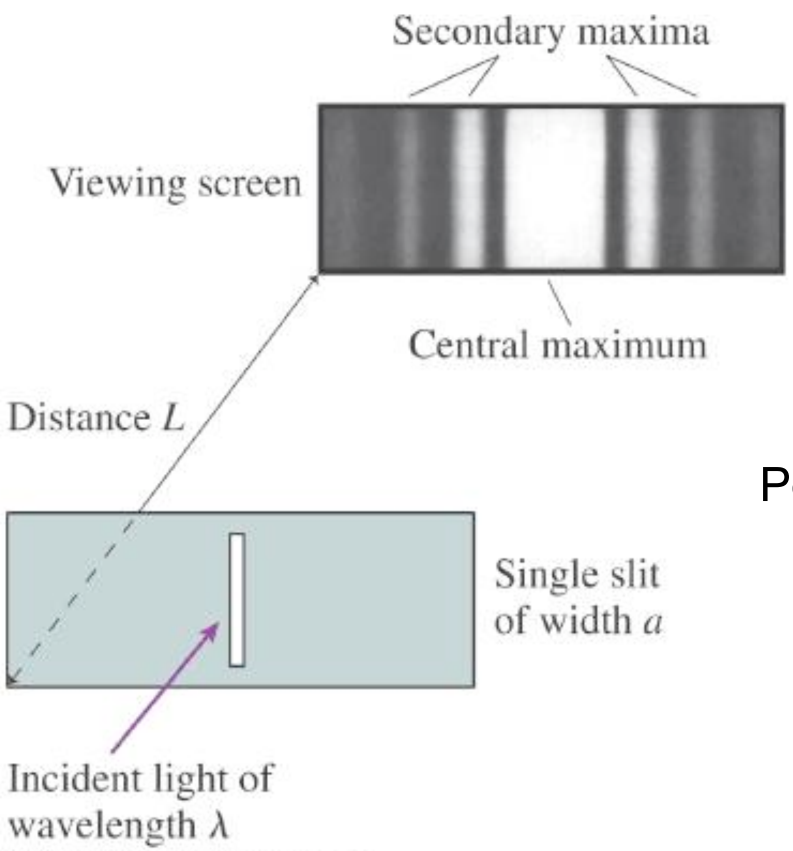
At a beach in Tel Aviv, Israel, plane water waves pass through two openings in a breakwall. Notice the diffraction effect—the waves exit the openings with circular wave fronts, as in Figure 37.1b. Notice also how the beach has been shaped by the circular wave fronts.

FIGURE 22.2 Light, just like a water wave, does spread out behind a hole *if* the hole is sufficiently small.

Diffracts



Single Slit Diffraction



Like water wave passing through a barrier

Point sources vs. extended sources

↓

Huygen's Principle

Every unobstructed point on a wavefront will act a source of secondary spherical waves. The new wavefront is the surface tangent to all the secondary spherical waves.

Figure 14.4.1 illustrates the propagation of the wave based on Huygens's principle.

Figure 14.4.1 Propagation of wave based on Huygens's principle.

According to Huygens's principle, light waves incident on two slits will spread out and exhibit an interference pattern in the region beyond (Figure 14.4.2a). The pattern is called a diffraction pattern. On the other hand, if no bending occurs and the light wave continue to travel in straight lines, then no diffraction pattern would be observed (Figure 14.4.2b).

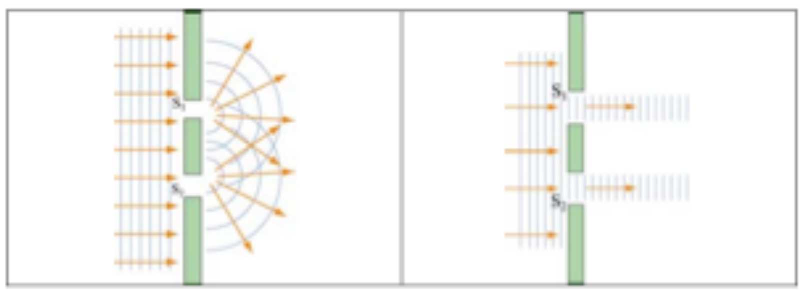


Figure 14.4.2 (a) Spreading of light leading to a diffraction pattern. (b) Absence of diffraction pattern if the paths of the light wave are straight lines.

FIGURE 22.11 Huygens' principle applied to the propagation of plane waves and spherical waves.

1. Each point of a wavefront is the source of a spherical *wavelet* that spreads out at the wave speed
2. At a later time the shape of the waveform is the tangent to all the *wavelets*

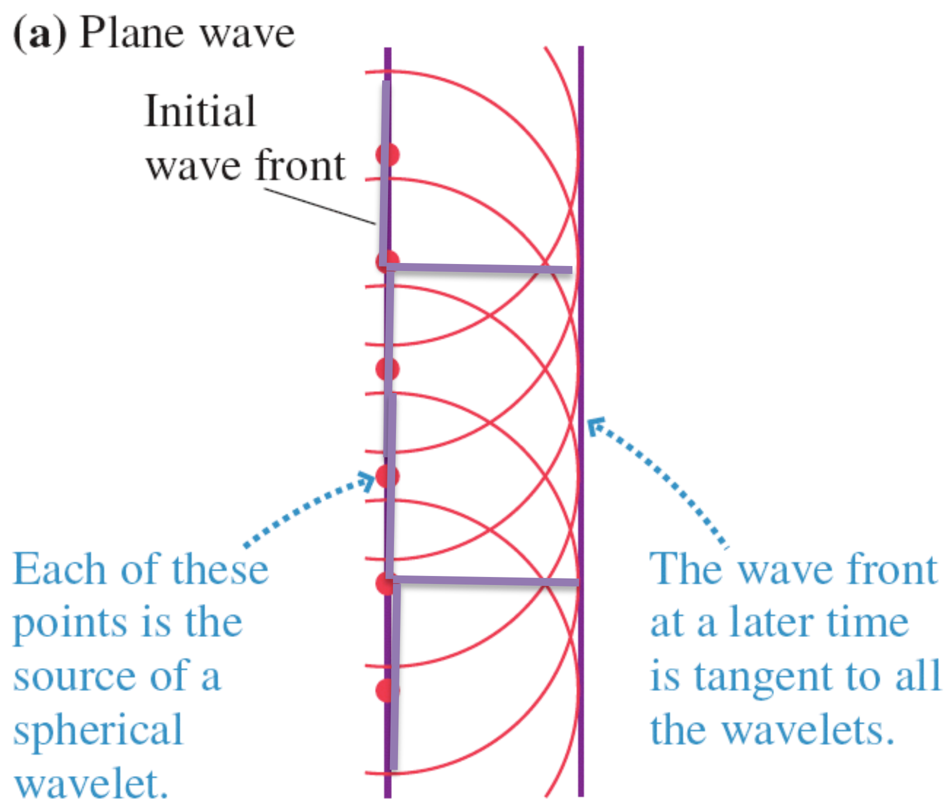
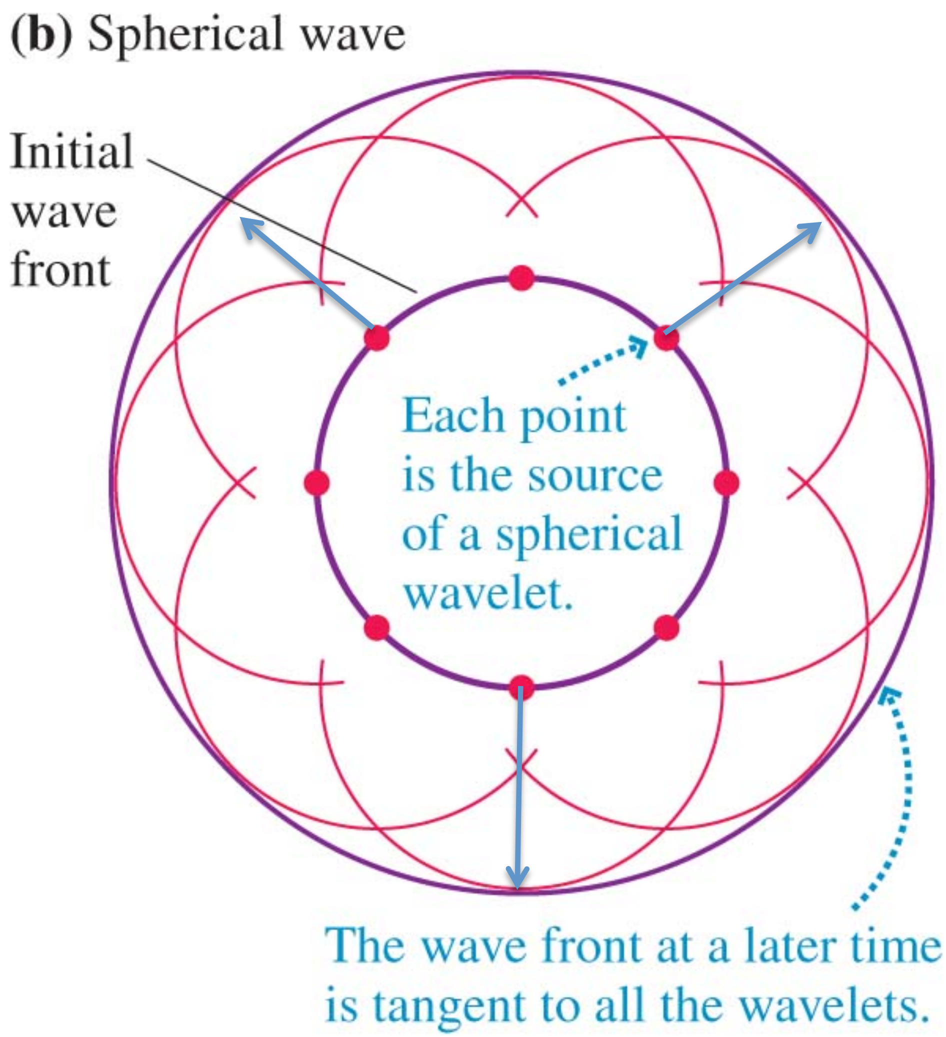
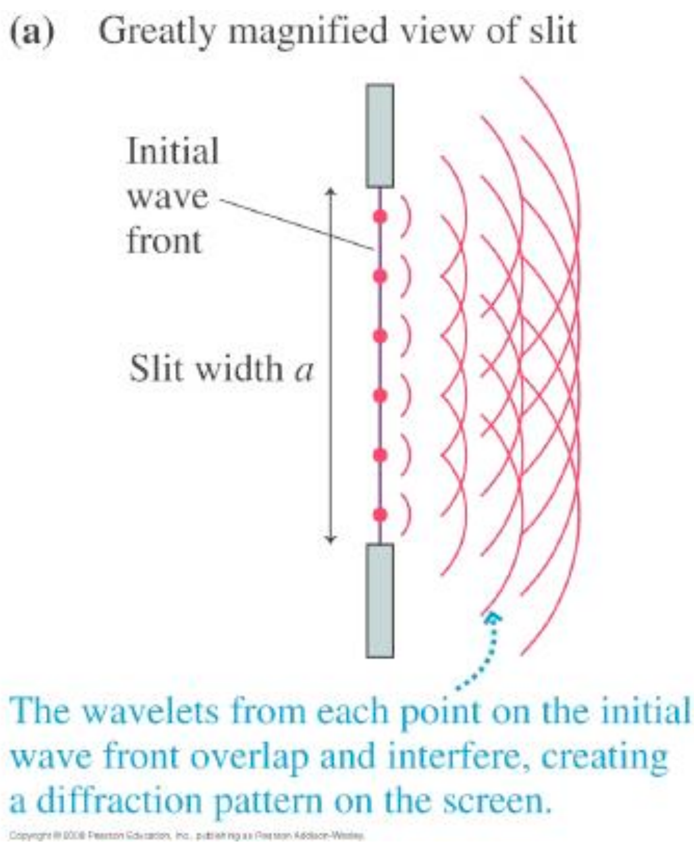
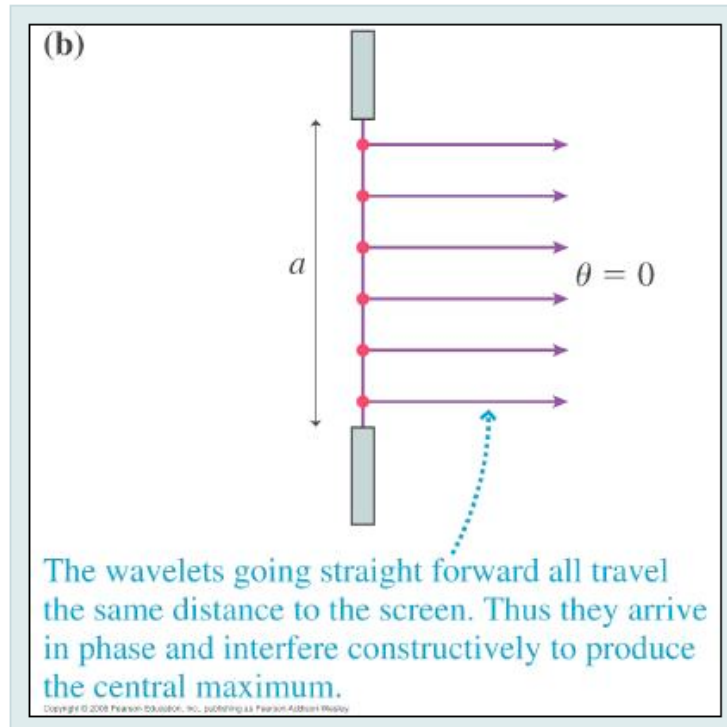


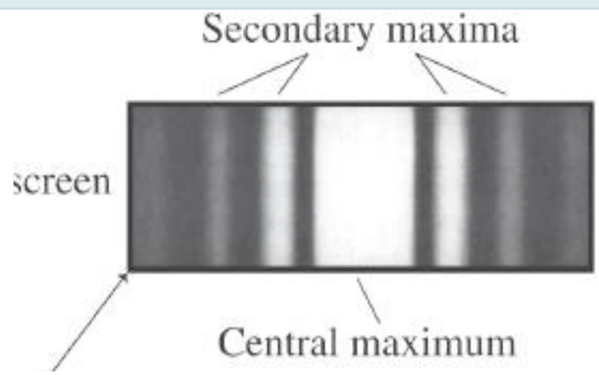
FIGURE 22.11 Huygens' principle applied to the propagation of plane waves and spherical waves.



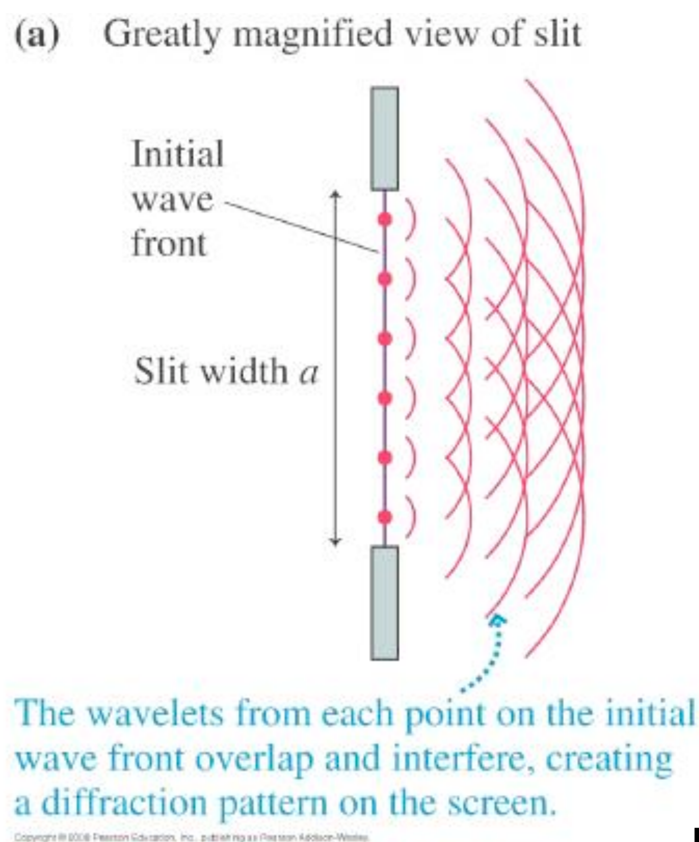
Single Slit Diffraction

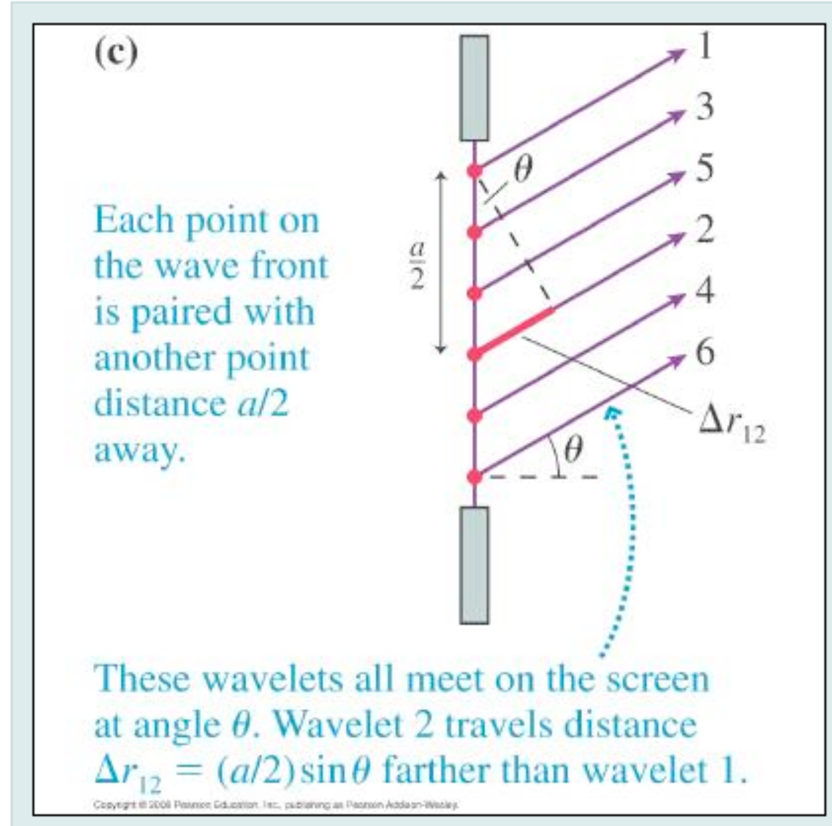






a<λ

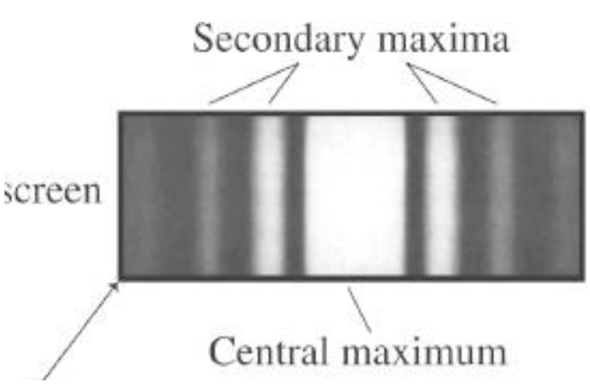




Pair wavelets with extra distance traveled $\lambda/2$

$$\Delta r_{12} = \frac{a}{2}\sin\theta_1 = \lambda/2$$

Same for 3 - 4 and 5 - 6 destructive interference



Pair wavelets with extra distance traveled $\lambda/4$

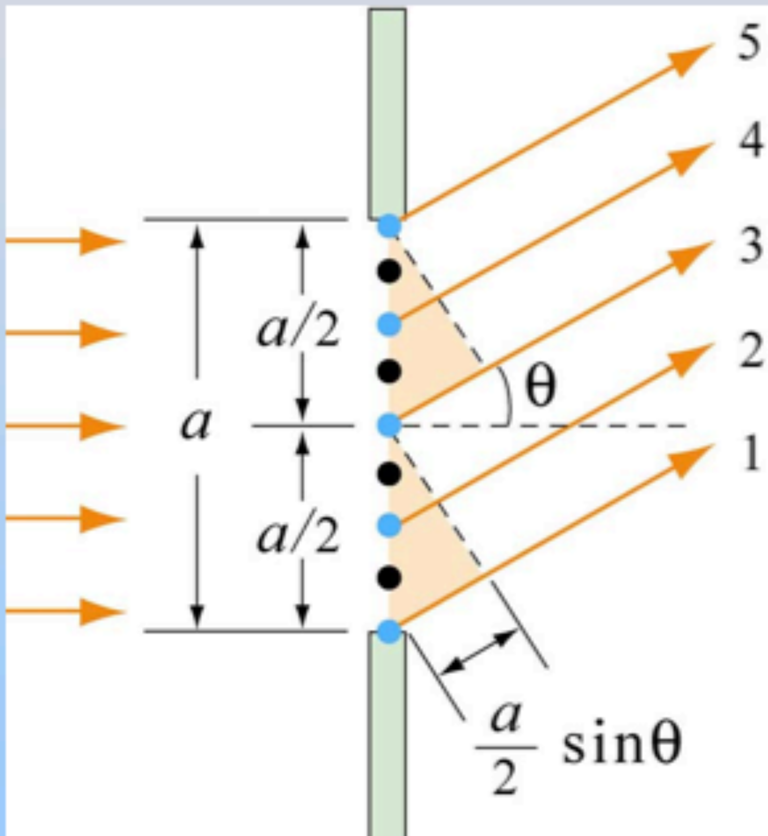
$$a\sin\theta_p = p\lambda, p = 1,2,..$$ etc

$$\theta_p \approx p(\lambda/a)$$

Single-Slit Diffraction

"Derivation" (Motivation) by Division:

In two slit we assumed slit width smaller than lamda



Divide slit into two portions:

$$\delta = r_1 - r_3 = r_2 - r_4 = \frac{a}{2}\sin\theta$$

Destructive interference:

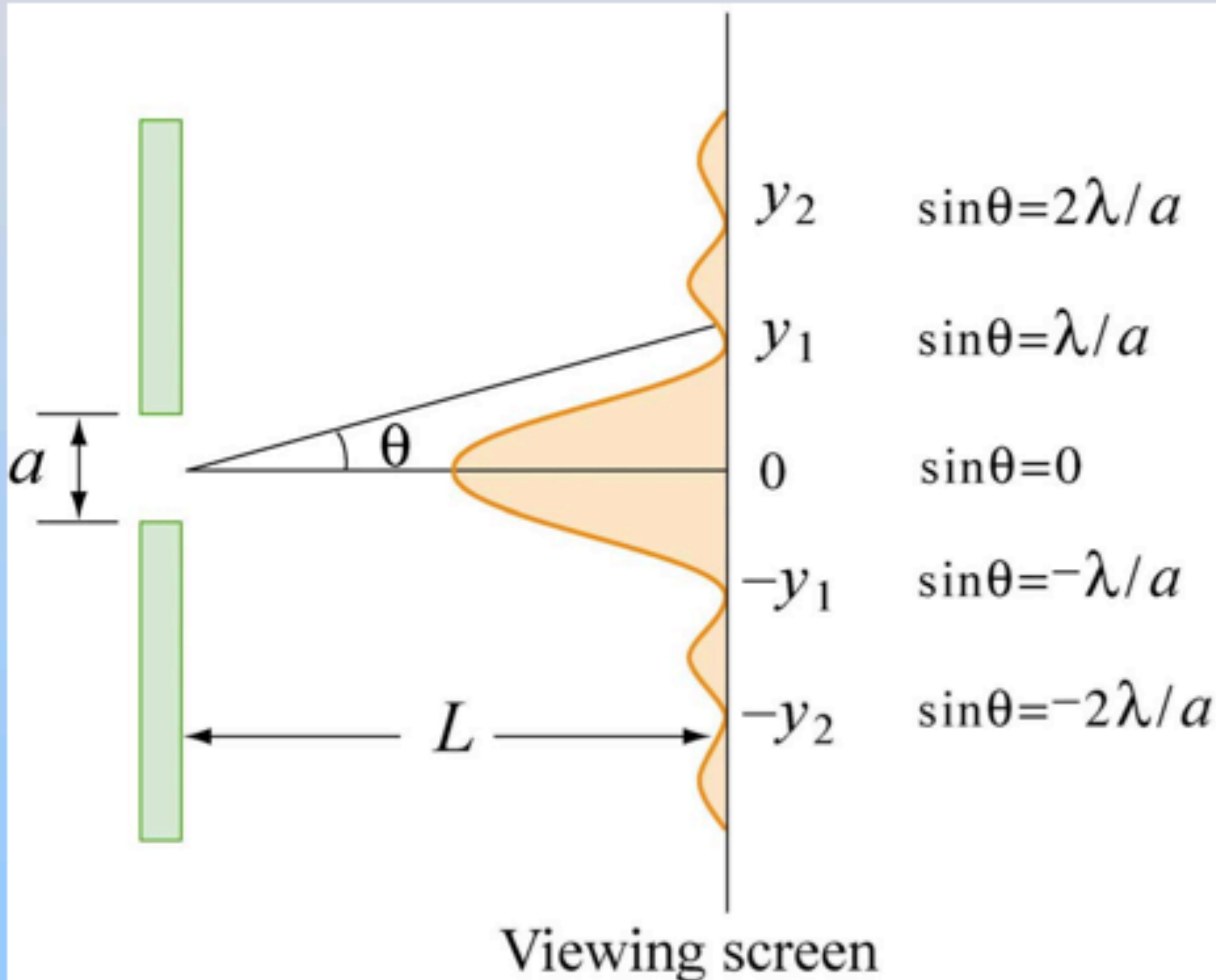
$$\delta = \frac{a}{2}\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda$$

$$a\sin\theta = m\lambda \qquad m = \pm 1, \pm 2, \ldots$$

Don't get confused – this is DESTRUCTIVE!

Intensity Distribution

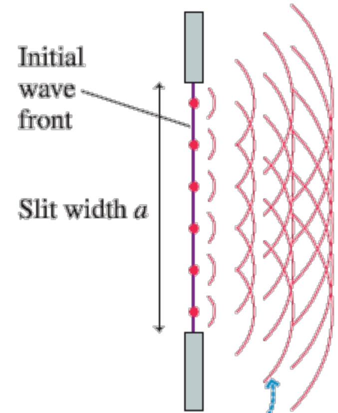
Destructive Interference: $a\sin\theta = m\lambda \qquad m = \pm 1, \pm 2, \ldots$



What happens when slit is too large to be considered a point source ?
Huygens principle replace wave-front by a continuous series of point sources

$$I = I_o \frac{\sin^2(n\phi/2)}{\sin^2(\phi/2)}$$

$$\phi = 2\pi \frac{d \sin\theta}{\lambda}$$



The wavelets from each point on the initial wave front overlap and interfere, creating a diffraction pattern on the screen.

$d \rightarrow 0; \phi \rightarrow 0$

Take $n\phi$ the difference from one end to the other constant, say $n\phi = \Phi$

but send ϕ to zero. $\sin\phi \approx \phi$

$$I = n^2 I_o \frac{\sin^2(\Phi/2)}{\Phi^2} = I_{max} \frac{\sin^2(\Phi/2)}{\Phi^2}$$

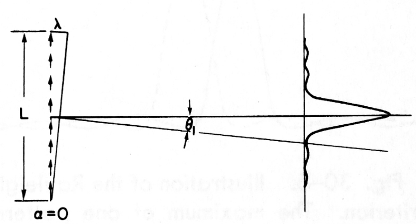
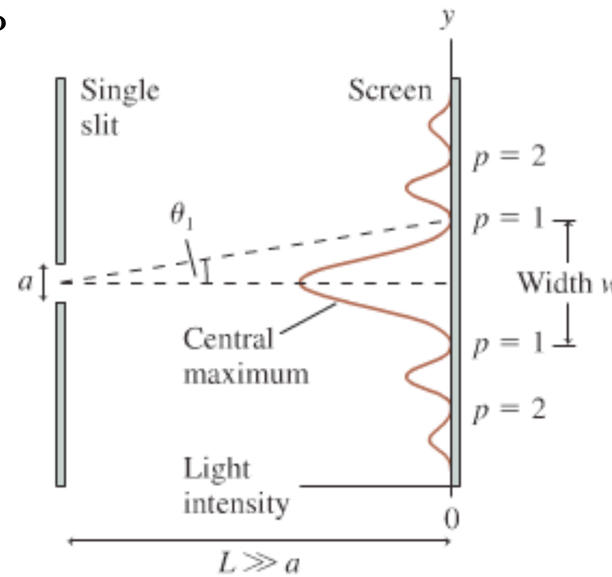
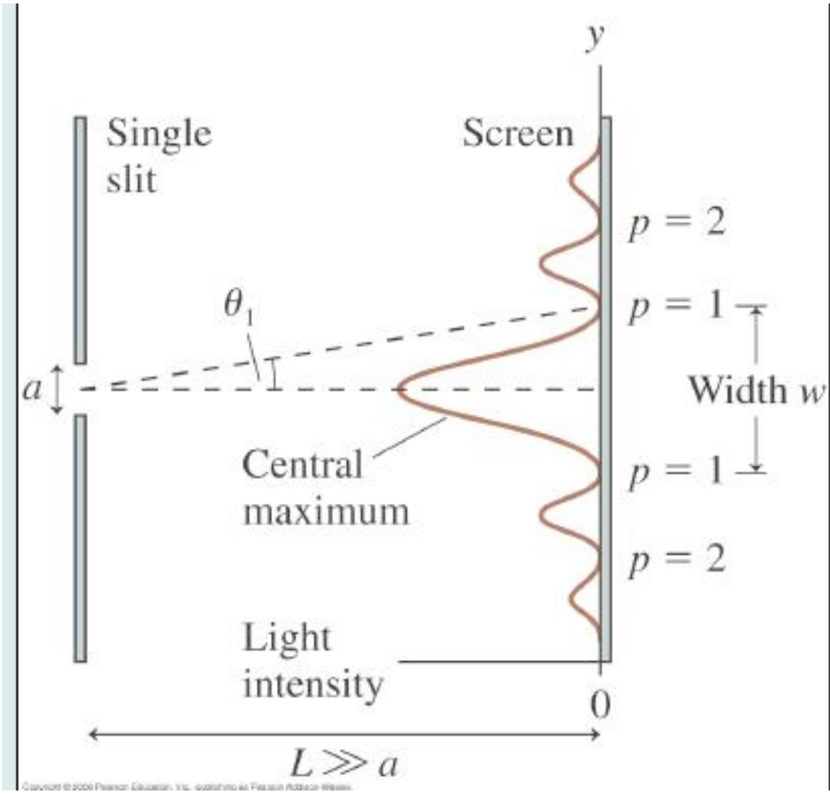
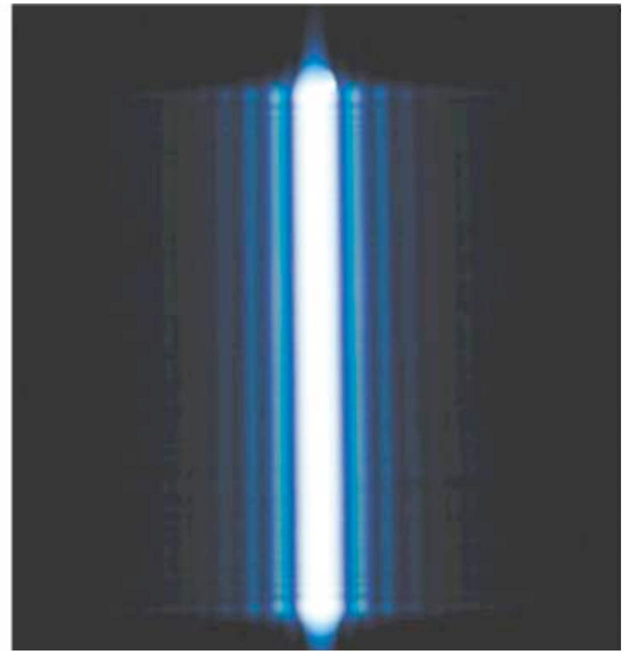


Fig. 30–5. The intensity pattern of a continuous line of oscillators has a single strong maximum and many weak "side lobes."




y
Single slit
Screen
p = 2
θ1
p = 1
a
Width w
Central maximum
p = 1
p = 2
Light intensity
0
L ≫ a



Actual double slit interference pattern (a<wavelength and a> wavelength) –
Convolution of ideal double slit and single slit patterns