Lecture 10
AC CIRCUITS
CHAPTER 36

PHYSICS 270

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Chapter 36. AC Circuits

Topics:

- AC Sources and Phasors
- Capacitor Circuits (Capacitive Reactance)
- *RC* Filter Circuits
- Inductor Circuits (Inductive Reactance)
- The Series *RLC* Circuit

10.4 Generators

One of the most important applications of Faraday's law of induction is to generators and motors. A generator converts mechanical energy into electric energy, while a motor converts electrical energy into mechanical energy.

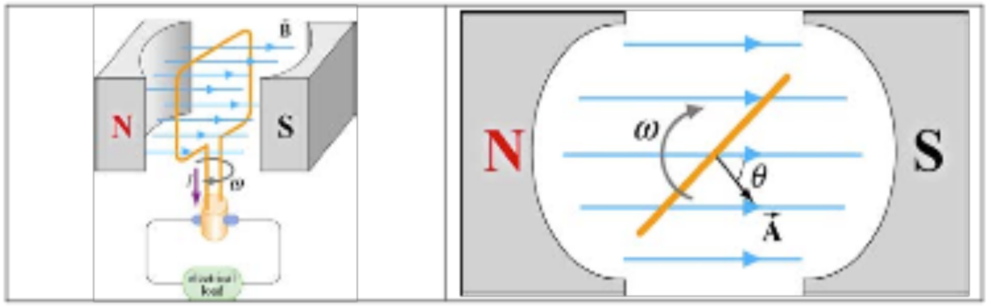


Figure 10.4.1 (a) A simple generator. (b) The rotating loop as seen from above.

Figure 10.4.1(a) is a simple illustration of a generator. It consists of an N-turn loop rotating in a magnetic field which is assumed to be uniform. The magnetic flux varies with time, thereby inducing an emf. From Figure 10.4.1(b), we see that the magnetic flux through the loop may be written as

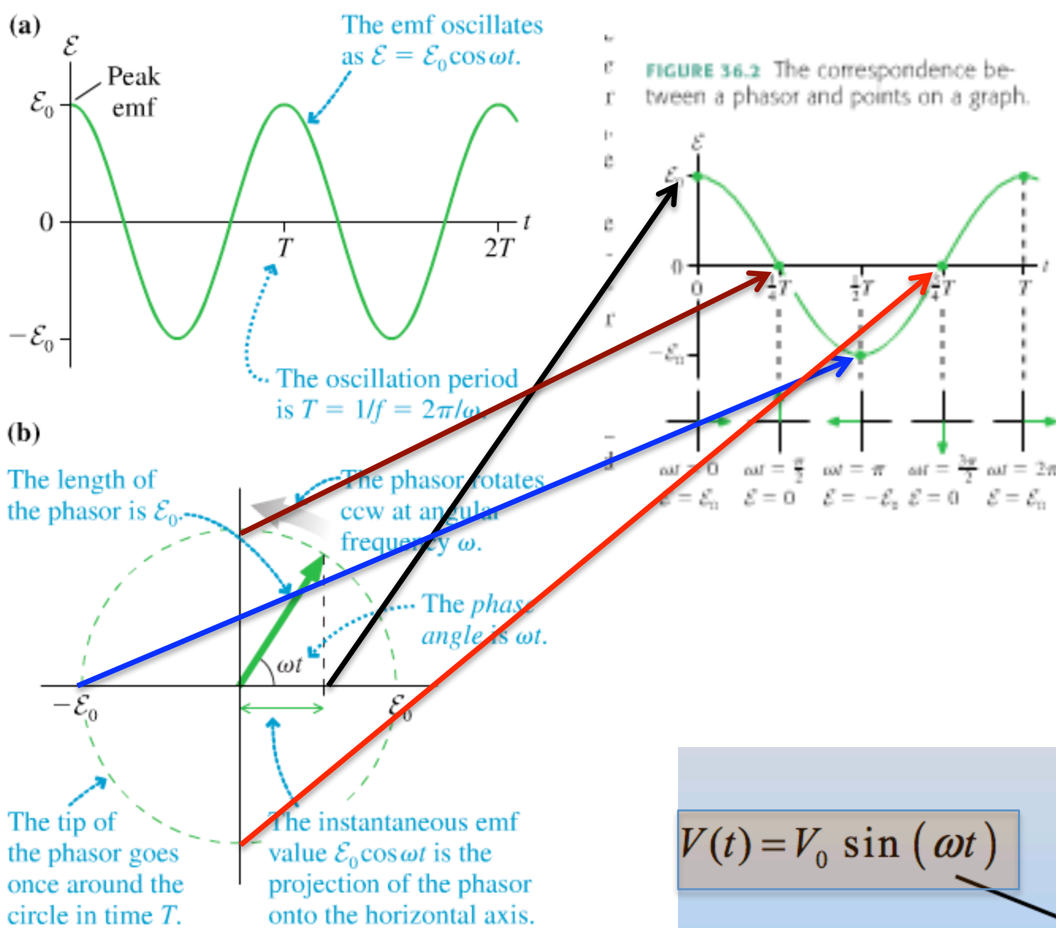
$$\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = BA\cos\theta = BA\cos\omega t \quad (10.4.1)$$

The rate of change of magnetic flux is

Creates a emf whose amplitude oscillates with time at frequency ω

Edison (GE) DC voltage and current – battery. Tesla (Westinghouse) AC (Alternating) voltage and current. The war of currents won by AC due to step-up step down potential.

FIGURE 36.1 An oscillating emf can be represented as a graph or as a phasor diagram.



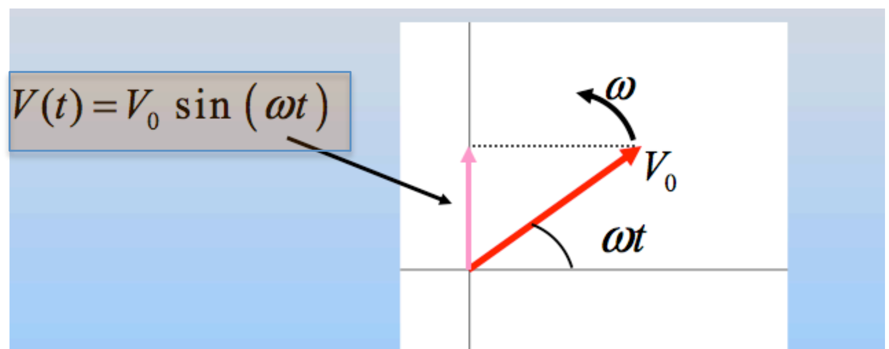
Phasor is a rotating vector whose:

1.Length corresponds to amplitude

2. Rotates ccw with angular velocity ω.

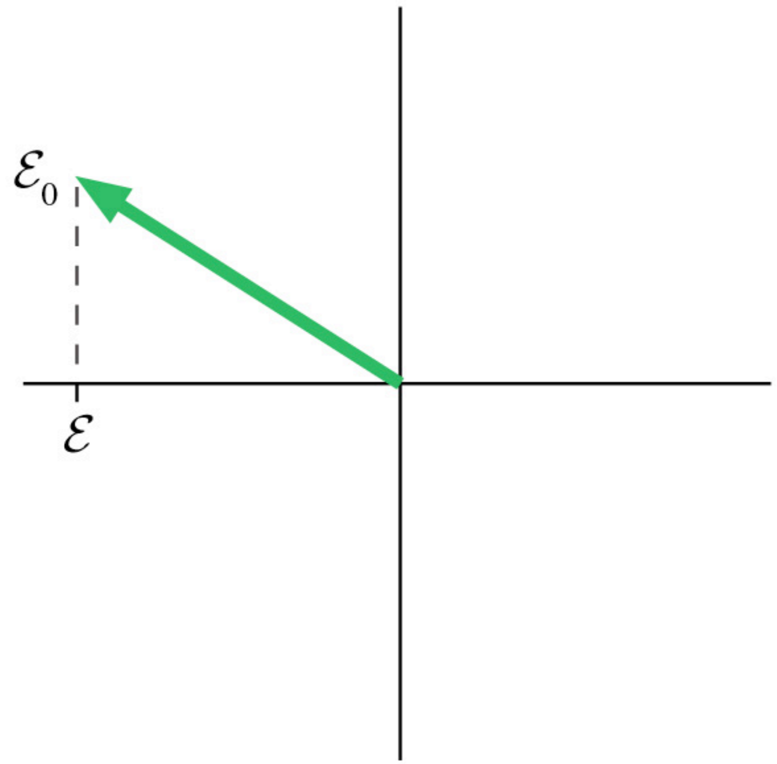
3. Projection on horizontal (for cos) or vertical (for sin) gives the instantaneous value of the alternating quantity at time t.

Cos(ωt)



Notes: (1) As the phasor (red vector) rotates, the projection (pink vector) oscillates

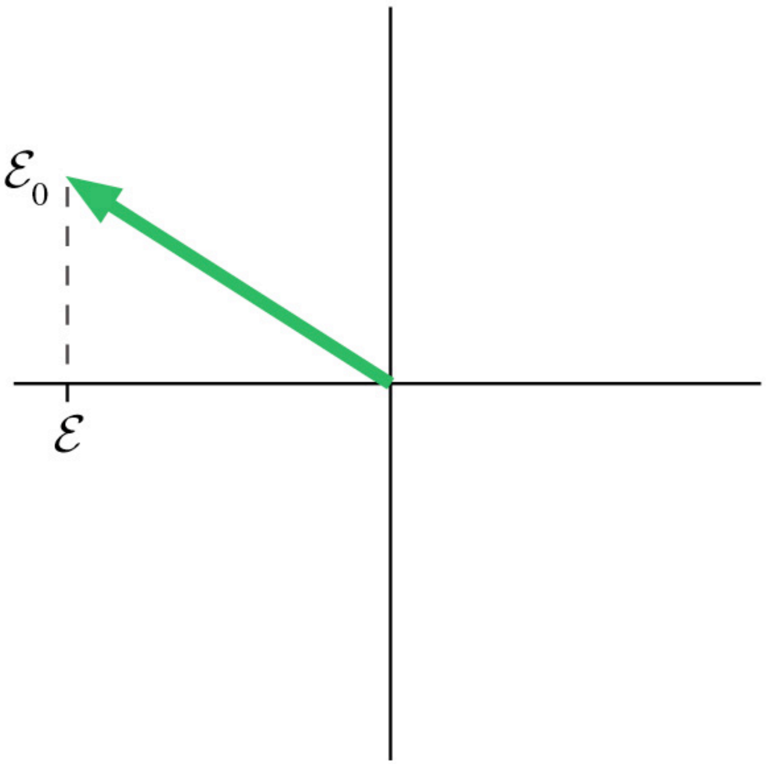
The magnitude of the instantaneous value of the emf represented by this phasor is



A. constant.
B. increasing.
C. decreasing.
D. It's not possible to tell without knowing *t*.

E. Other

The magnitude of the instantaneous value of the emf represented by this phasor is



A. constant.
B. increasing. (for cos)
C. decreasing.(for sin)
D. It's not possible to tell without knowing *t*.

AC Resistor circuit

FIGURE 36.3 Instantaneous current i_R through a resistor.

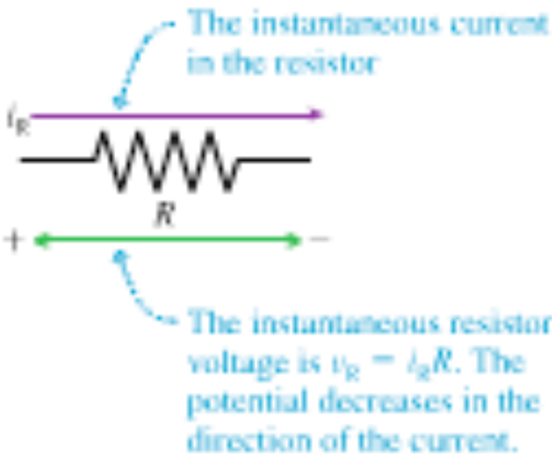


FIGURE 36.4 An AC resistor circuit.

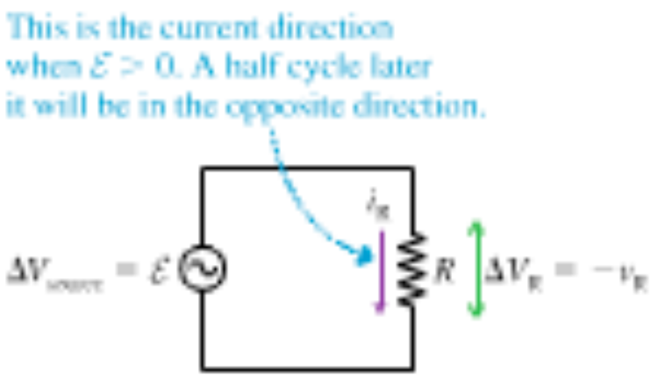
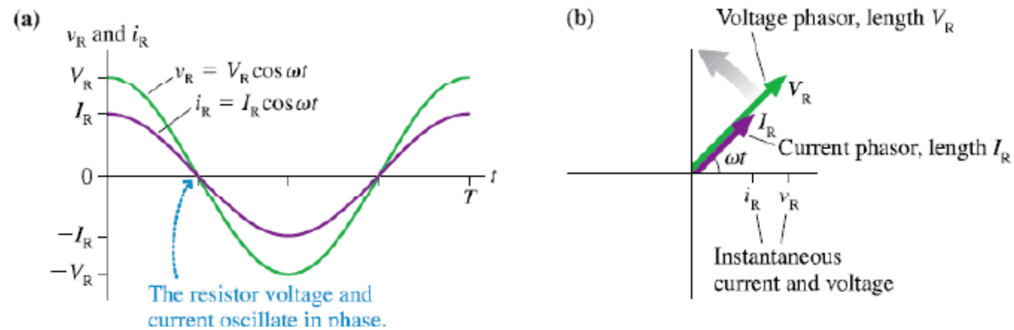
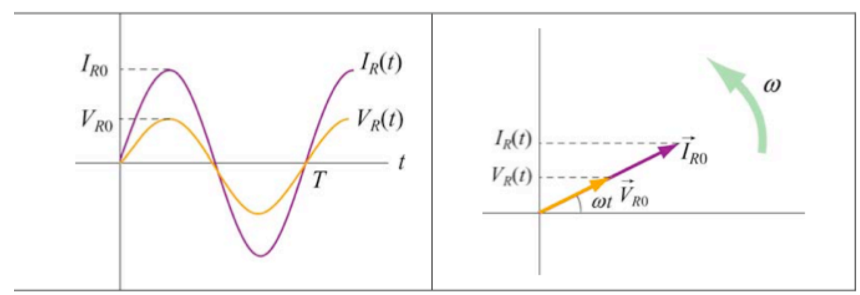


FIGURE 36.5 Graph and phasor diagram of the resistor current and voltage. The current and voltage are in phase.

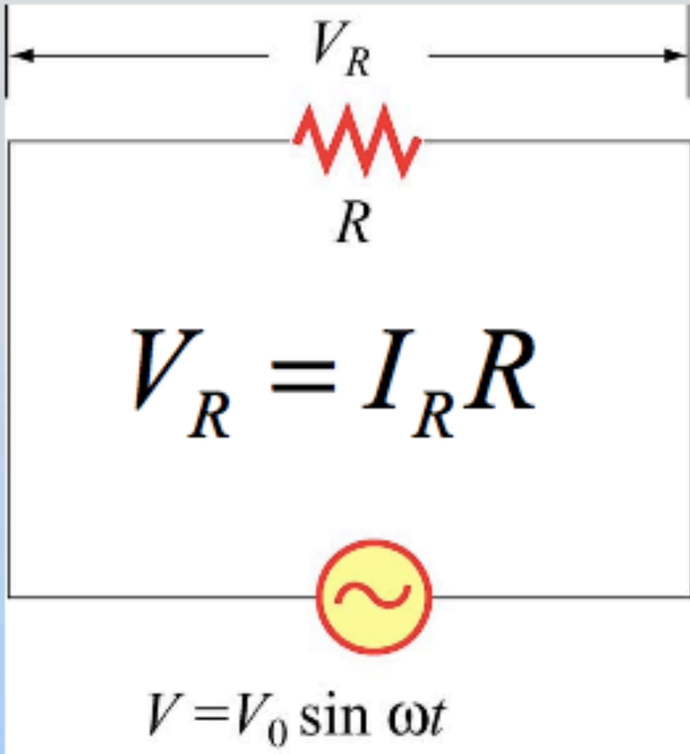


Cos input



Sin input

AC Circuit: Resistors



$$V_R = I_R R$$

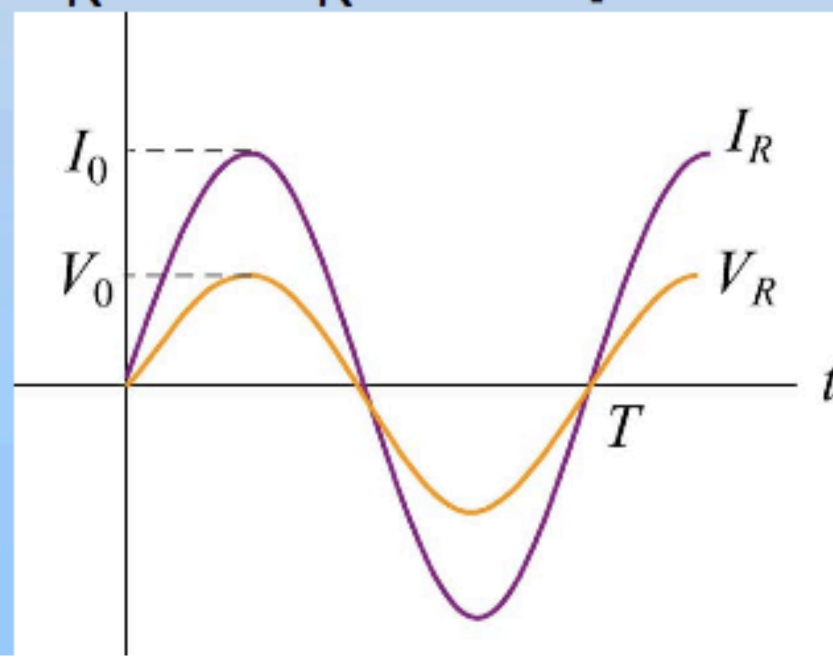
$$I_R = \frac{V_R}{R} = \frac{V_0}{R}\sin \omega t$$

$$= I_0 \sin(\omega t - 0)$$

$$I_0 = \frac{V_0}{R}$$

$$\varphi = 0$$

I_R and V_R are **in phase**



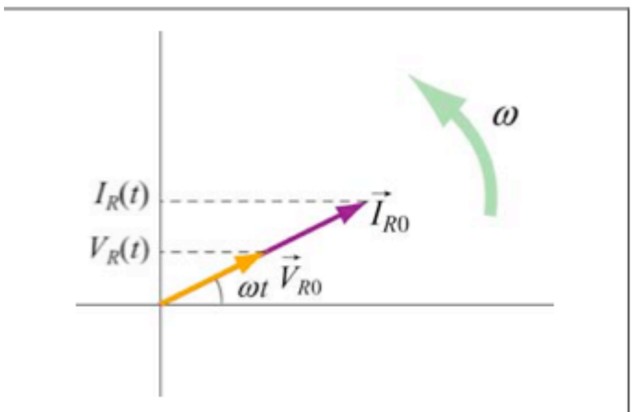
The average value of current over one period can be obtained as:

$$\langle I_R(t)\rangle = \frac{1}{T}\int_0^T I_R(t)dt = \frac{1}{T}\int_0^T I_{R0}\sin\omega t\ dt = \frac{I_{R0}}{T}\int_0^T \sin\frac{2\pi t}{T}\ dt = 0 \qquad (12.2.3)$$

This average vanishes because

$$\langle \sin\omega t\rangle = \frac{1}{T}\int_0^T \sin\omega t\ dt = 0 \qquad (12.2.4)$$

Similarly, one may find the following relations useful when averaging over one period:



$$\langle \cos\omega t\rangle = \frac{1}{T}\int_0^T \cos\omega t\ dt = 0$$

$$\langle \sin\omega t\cos\omega t\rangle = \frac{1}{T}\int_0^T \sin\omega t\cos\omega t\ dt = 0$$

$$\langle \sin^2\omega t\rangle = \frac{1}{T}\int_0^T \sin^2\omega t\ dt = \frac{1}{T}\int_0^T \sin^2\left(\frac{2\pi t}{T}\right) dt = \frac{1}{2}$$

$$\langle \cos^2\omega t\rangle = \frac{1}{T}\int_0^T \cos^2\omega t\ dt = \frac{1}{T}\int_0^T \cos^2\left(\frac{2\pi t}{T}\right) dt = \frac{1}{2}$$

Whether we take sin or cos is a matter of choice. Result the same. $< >=0$, $<{}^2>=1/2$

From the above, we see that the average of the square of the current is non-vanishing:

$$\langle I_R^2(t)\rangle = \frac{1}{T}\int_0^T I_R^2(t)dt = \frac{1}{2}I_{R0}^2$$

It is convenient to define the root-mean-square (rms) current as

$$I_{\text{rms}} = \sqrt{\langle I_R^2(t)\rangle} = \frac{I_{R0}}{\sqrt{2}} \tag{12.2.7}$$

In a similar manner, the rms voltage can be defined as

$$V_{\text{rms}} = \sqrt{\langle V_R^2(t)\rangle} = \frac{V_{R0}}{\sqrt{2}} \tag{12.2.8}$$

The rms voltage supplied to the domestic wall outlets in the United States is $V_{\text{rms}} = 120\text{ V}$ at a frequency $f = 60\text{ Hz}$.

The power dissipated in the resistor is

$$P_R(t) = I_R(t)V_R(t) = I_R^2(t)R \tag{12.2.9}$$

from which the average over one period is obtained as:

$$\langle P_R(t)\rangle = \langle I_R^2(t)R\rangle = \frac{1}{2}I_{R0}^2R = I_{\text{rms}}^2R = I_{\text{rms}}V_{\text{rms}} = \frac{V_{\text{rms}}^2}{R} \tag{12.2.10}$$

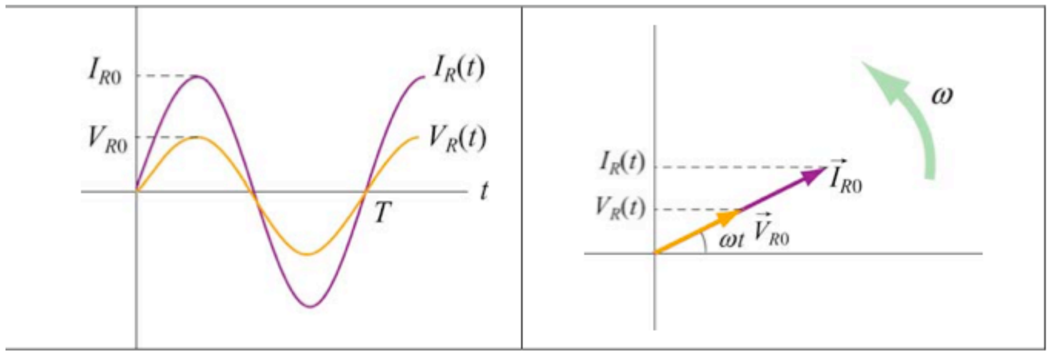
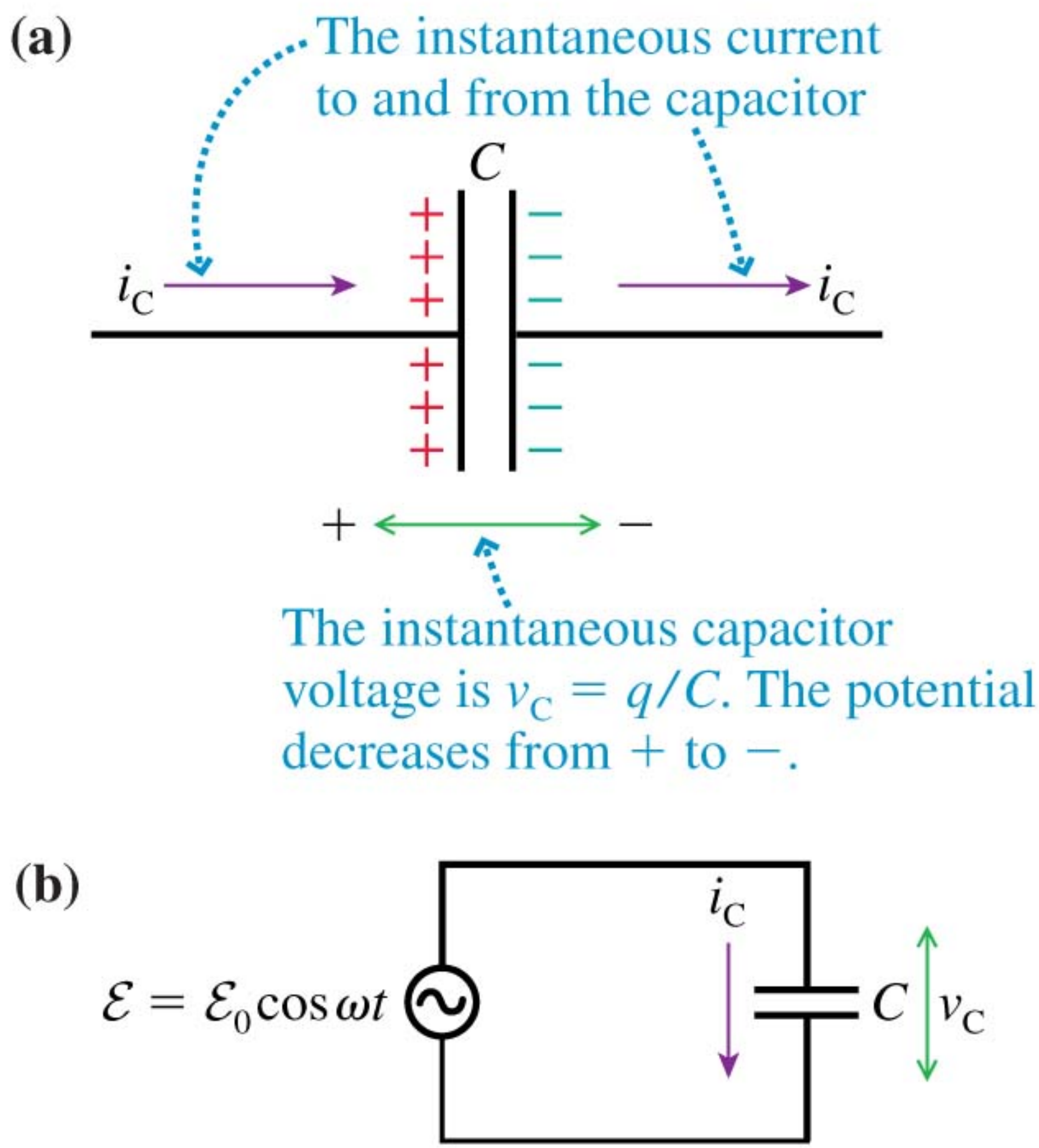

I_{R0}
$I_R(t)$
V_{R0}
$V_R(t)$
t
T
ω
$I_R(t)$
$\vec{I}_{R0}$
$V_R(t)$
ωt
$\vec{V}_{R0}$

FIGURE 36.7 An AC capacitor circuit.



$$v_c = V_c \cos(\omega t)$$

$$q = C v_c = C V_c \cos(\omega t)$$

$$i_c = \frac{dq}{dt} = -\omega C V_c \sin(\omega t)$$

$$i_c = \omega C V_c \cos(\omega t + \frac{\pi}{2}) \equiv$$

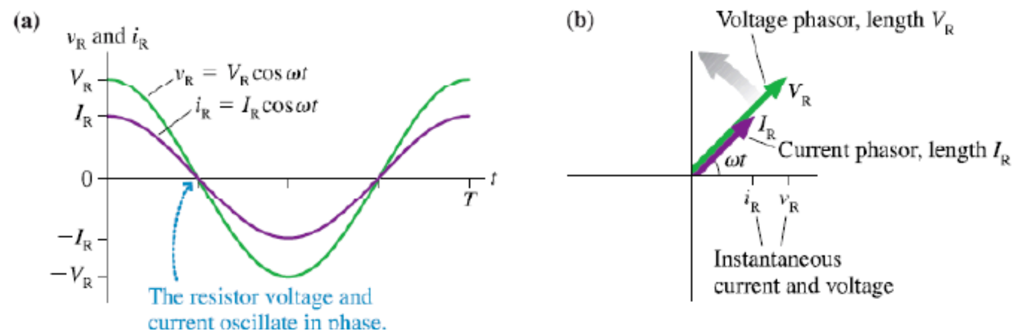
$$\equiv I_C \cos(\omega t + \frac{\pi}{2})$$

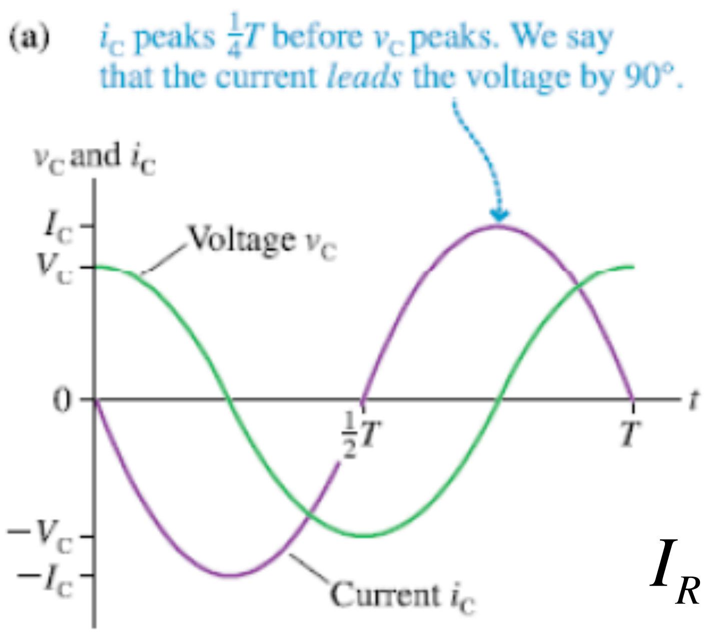
$$v_c = V_c \cos(\omega t)$$

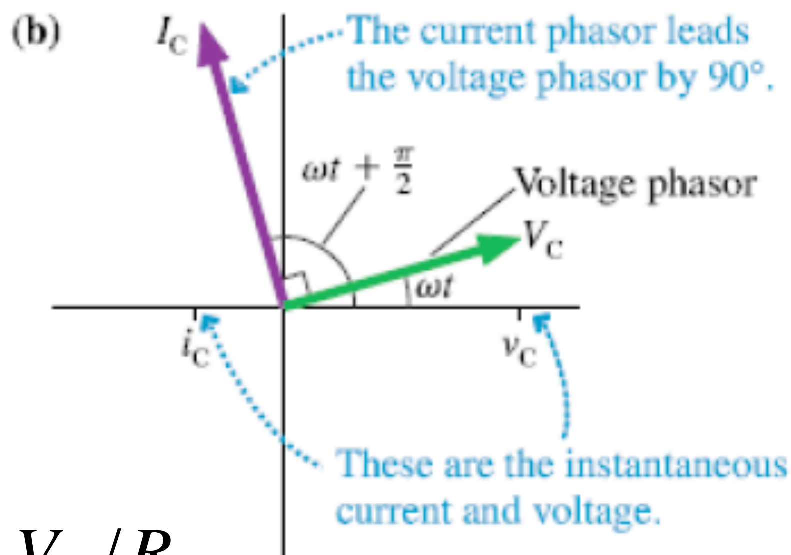
$$i_c = I_C \cos(\omega t + \frac{\pi}{2})$$

$$I_c \equiv \omega C V_c$$

FIGURE 36.5 Graph and phasor diagram of the resistor current and voltage. The current and voltage are in phase.







$$I_R = V_R / R$$

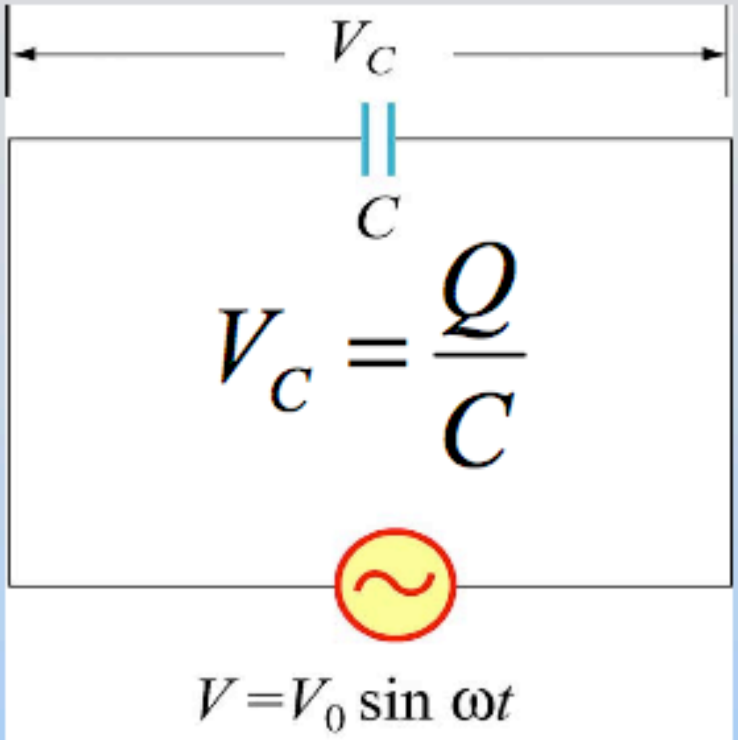
$$I_C = V_c / X_C$$

$$X_C \equiv 1/\omega C$$

Lead if in the cycle peak occurs first or is ahead in the phasor ccw rotation

Capacitive Reactance - ohms

AC Circuit: Capacitors



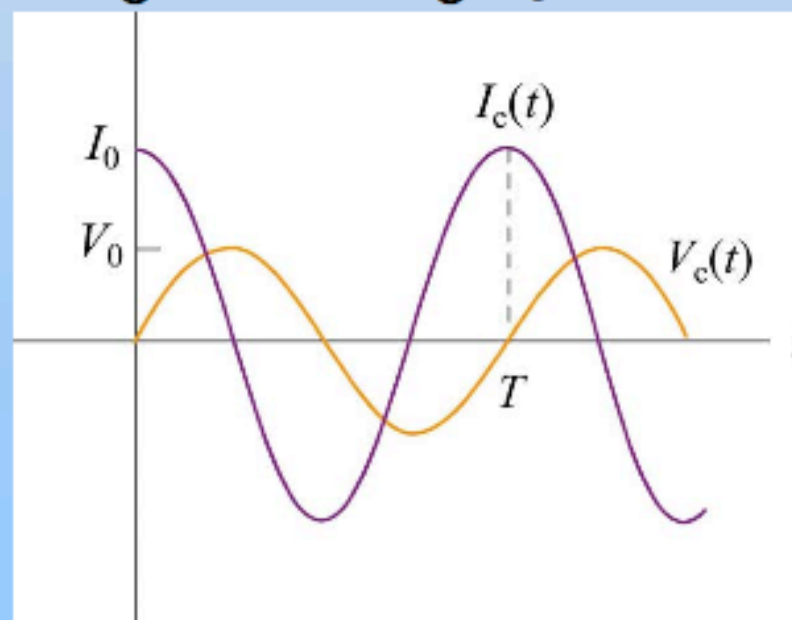
$$V_C = \frac{Q}{C}$$

$$I_C(t) = \frac{dQ}{dt}$$
$$= \omega C V_0 \cos \omega t$$
$$= I_0 \sin(\omega t - {}^{-\pi}\!/_2)$$

$$I_0 = \omega C V_0$$
$$\varphi = -\pi/2$$

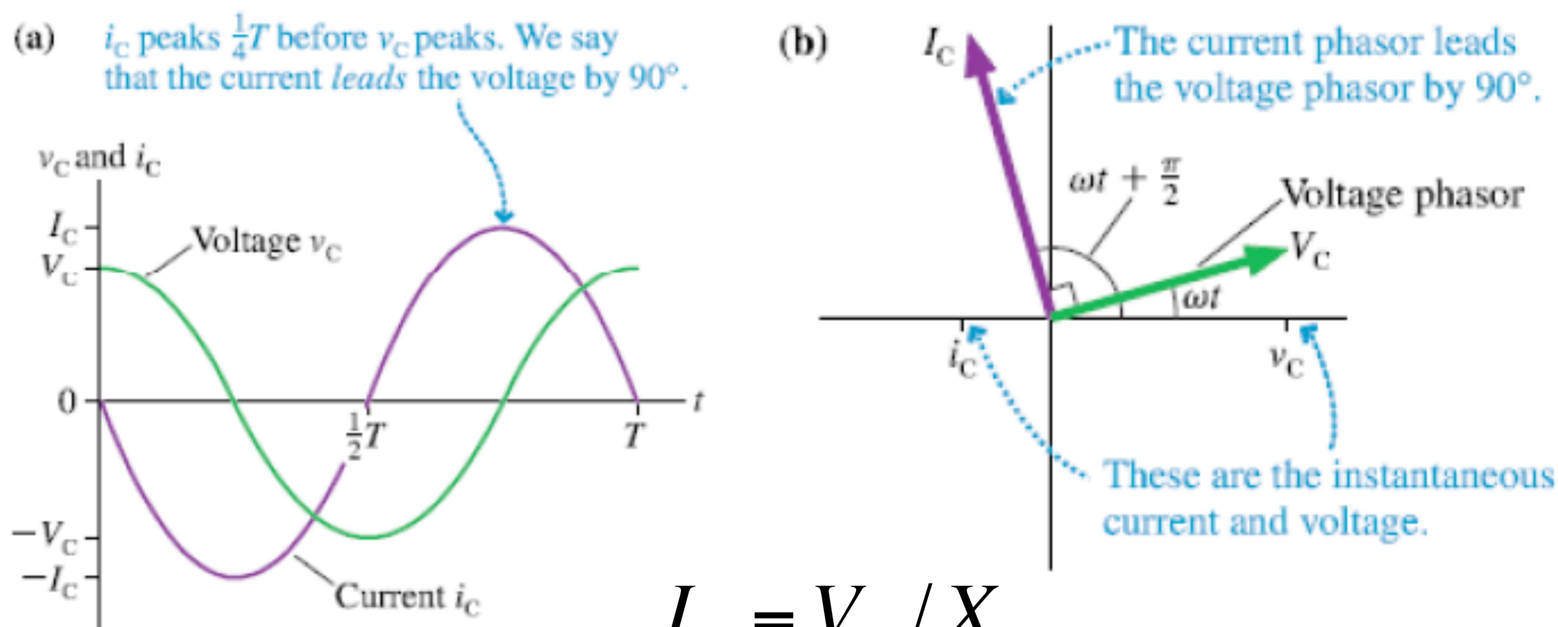
$$Q(t) = CV_C = CV_0 \sin \omega t$$

I_C leads V_C by π/2



AC Circuits - Capacitors

ASSESS Using reactance is just like using Ohm's law, but don't forget it applies to only the *peak* current and voltage, not the instantaneous values.



$$I_C = V_C / X_C$$

$$X_C = 1/\omega C$$

Capacitor Circuits

The instantaneous voltage across a single capacitor in a basic capacitor circuit is equal to the instantaneous emf:

$$v_C = V_C \cos \omega t$$

Where V_C is the maximum voltage across the capacitor, also equal to the maximum emf. The instantaneous current in the circuit is

$$i_C = \omega C V_C \cos\left(\omega t + \frac{\pi}{2}\right)$$

The AC current to and from a capacitor *leads* the capacitor voltage by π/2 rad, or 90°.

FIGURE 36.10 The capacitive reactance as a function of frequency.