HW #2-Solutions

I. Conceptual Questions

33.7. **(a)** The force on a charge moving in a magnetic field is

$$\vec{F}_{\text{on } q} = q\vec{v} \times \vec{B} = (qvB\sin\alpha, \text{ direction of right-hand rule})$$

A *positive* charge moving to the right with $\vec{B}$ down gives a force *into the page*.
(b) The direction of the force on a negative charge is opposite the direction determined by the right-hand rule. Any charge moving parallel to the field has *no* force and *no* deflection.

33.8. **(a)** The force on a charge moving in a magnetic field is

$$\vec{F}_{\text{on } q} = q\vec{v} \times \vec{B} = (qvB\sin\alpha, \text{ direction of right-hand rule})$$

The magnetic field must be in a plane perpendicular to both the $\vec{v}$ and $\vec{F}$ vectors. Using the right-hand rule for a *positive* charge moving to the right, the $\vec{B}$ field must be *out* of the page.
(b) The direction of the force on a negative charge is opposite the direction determined by the right-hand rule. The force $\vec{F}$ on the *negative* charge is into the page. Since the velocity is to the right, the magnetic field $\vec{B}$ must be *up*.

33.12. **(a)** The force is down. The magnetic field at the wire points from left to right, and the current in the wire can be thought of as positive charges moving into the page. The right-hand rule gives the magnetic force downwards.
(b) The two north poles together cause zero magnetic field at their midpoint, so there is no force on the wire.

33.13. The magnet is repelled. The current loop produces a magnetic dipole with the north pole on the left and the south pole on the right. The approaching south pole of the magnet is repelled by the south pole of the current loop.

II. Exercises and Problems

33.46. Model: Use the Biot-Savart law for a current carrying segment.
Visualize: Please refer to Figure P33.46.
Solve: **(a)** The Biot-Savart law (Equation 33.6) for the magnetic field of a current segment $\Delta\vec{s}$ is

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s}\times\hat{r}}{r^2}$$

where the unit vector $\hat{r}$ points from current segment Δs to the point, a distance r away, at which we want to evaluate the field. For the two linear segments of the wire, $\Delta\vec{s}$ is in the same direction as $\hat{r}$, so $\Delta\vec{s}\times\hat{r} = 0$. For the curved segment, $\Delta\vec{s}$ and $\hat{r}$ are always perpendicular, so $\Delta\vec{s}\times\hat{r} = \Delta s$. Thus

$$B = \frac{\mu_0}{4\pi}\frac{I\Delta s}{r^2}$$

Now we are ready to sum the magnetic field of all the segments at point P. For all segments on the arc, the distance to point P is $r = R$. The superposition of the fields is

$$B = \frac{\mu_0}{4\pi}\frac{I}{R^2}\int_{\text{arc}} ds = \frac{\mu_0}{4\pi}\frac{IL}{R^2} = \frac{\mu_0 I\theta}{4\pi R}$$

where $L = R\theta$ is the length of the arc.
(b) Substituting $\theta = 2\pi$ in the above expression,

$$B_{\text{loop center}} = \frac{\mu_0 I 2\pi}{4\pi R} = \frac{\mu_0}{2}\frac{I}{R}$$

This is Equation 33.7, which is the magnetic field at the center of a 1-turn coil.

33.47. Model: Use the Biot-Savart law for a current carrying segment.

Visualize: Please refer to Figure P33.47. The distance from P to the inner arc is r_1 and the distance from P to the outer arc is r_2.

Solve: As given in Equation 33.6, the Biot-Savart law for a current carrying small segment $\Delta\vec{s}$ is

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s}\times\hat{r}}{r^2}$$

For the linear segments of the loop, $B_{\Delta s} = 0$ T because $\Delta\vec{s}\times\hat{r} = 0$. Consider a segment $\Delta\vec{s}$ on length on the inner arc. Because $\Delta\vec{s}$ is perpendicular to the $\hat{r}$ vector, we have

$$B = \frac{\mu_0}{4\pi}\frac{I\Delta s}{r_1^2} = \frac{\mu_0}{4\pi}\frac{Ir_1\Delta\theta}{r_1^2} = \frac{\mu_0}{4\pi}\frac{I\Delta\theta}{r_1} \Rightarrow B_{\text{arc 1}} = \int_{-\pi/2}^{\pi/2}\frac{\mu_0 I d\theta}{4\pi r_1} = \frac{\mu_0 I}{4\pi r_1}\pi = \frac{\mu_0 I}{4r_1}$$

A similar expression applies for $B_{\text{arc 2}}$. The right-hand rule indicates an out-of-page direction for $B_{\text{arc 2}}$ and an into-page direction for $B_{\text{arc 1}}$. Thus,

$$\vec{B} = \left(\frac{\mu_0 I}{4r_1}, \text{ into page}\right) + \left(\frac{\mu_0 I}{4r_2}, \text{ out of page}\right) = \left[\frac{\mu_0 I}{4}\left(\frac{1}{r_1} - \frac{1}{r_2}\right), \text{ into page}\right]$$

The field strength is

$$B = \frac{(4\pi\times10^{-7}\text{ T m/A})(5.0\text{ A})}{4}\left(\frac{1}{0.010\text{ m}} - \frac{1}{0.020\text{ m}}\right) = 7.9\times10^{-5}\text{ T}$$

Thus $\vec{B} = (7.9\times10^{-5}\text{ T, into page})$.

33.48. Model: Assume that the wire is infinitely long.

Visualize: Please refer to Figure P33.48. The wire, looped as it is, consists of a circular part and a linear part.

Solve: Using Equation 33.7 and Example 33.3, the magnetic field at P is

$$B_P = B_{\text{loop center}} + B_{\text{wire}} = \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi R}$$

$$= \frac{4\pi(10^{-7}\text{ T m/A})(5.0\text{ A})}{2(0.010\text{ m})} + \frac{4\pi(10^{-7}\text{ T m/A})(5.0\text{ A})}{2\pi(0.010\text{ m})} = 4.1\times10^{-4}\text{ T}$$

33.61. Model: Electric and magnetic fields exert forces on a moving charge. The fields are uniform throughout the region.

Visualize: Please refer to Figure P33.61.

Solve: **(a)** We will first find the net force on the antiproton, and then find the net acceleration using Newton's second law. The magnitudes of the electric and magnetic forces are

$$F_E = eE = \left(1.60\times10^{-19}\ \text{C}\right)\left(1000\ \text{V/m}\right) = 1.60\times10^{-16}\ \text{N}$$

$$F_B = evB = \left(1.60\times10^{-19}\ \text{C}\right)\left(500\ \text{m/s}\right)\left(2.5\ \text{T}\right) = 2.00\times10^{-16}\ \text{N}$$

The directions of these two forces on the antiproton are opposite. $\vec{F}_E$ points *up* whereas, using the right-hand rule, $\vec{F}_B$ points *down*. Hence,

$$\vec{F}_{net} = \left(2.0\times10^{-16}\ \text{N} - 1.60\times10^{-16}\ \text{N, down}\right) \Rightarrow F_{net} = 0.40\times10^{-16}\ \text{N} = ma$$

$$\Rightarrow \vec{a} = \left(\frac{0.40\times10^{-16}\ \text{N}}{1.67\times10^{-27}\ \text{kg}} = 2.4\times10^{10}\ \text{m/s}^2\text{, down}\right)$$

(b) If $\vec{v}$ were reversed, both $\vec{F}_E$ and $\vec{F}_B$ will point *up*. Thus,

$$\vec{F}_{net} = \left(1.6\times10^{-16}\ \text{N} + 2.0\times10^{-16}\ \text{N, up}\right) \Rightarrow \vec{a} = \left(\frac{3.6\times10^{-16}\ \text{N}}{1.67\times10^{-27}\ \text{kg}} = 2.2\times10^{11}\ \text{m/s}^2\text{, up}\right)$$

33.64. Model: Charged particles moving perpendicular to a uniform magnetic field undergo circular motion at constant speed.

Visualize: Please refer to Figure P33.64.

Solve: The potential difference causes an ion of mass m to accelerate from rest to a speed v. Upon entering the magnetic field, the ion follows a circular trajectory with cyclotron radius $r = mv/eB$. To be detected, an ion's trajectory must have radius $d = 2r = 8$ cm. This means the ion needs the speed

$$v = \frac{eBr}{m} = \frac{eBd}{2m}$$

This speed was acquired by accelerating from potential V to potential 0. We can use the conservation of energy equation to find the voltage that will accelerate the ion:

$$K_1 + U_1 = K_2 + U_2 \Rightarrow 0\text{ J} + e\Delta V = \tfrac{1}{2}mv^2 + 0\text{ J} \Rightarrow \Delta V = \frac{mv^2}{2e}$$

Using the above expression for v, the voltage that causes an ion to be detected is

$$\Delta V = \frac{mv^2}{2e} = \frac{m}{2e}\left(\frac{eBd}{2m}\right)^2 = \frac{eB^2d^2}{8m}$$

An ion's mass is the sum of the masses of the two atoms *minus* the mass of the missing electron. For example, the mass of N_2^+ is

$$m = m_N + m_N - m_{elec} = 2(14.0031\text{ u})(1.661\times10^{-27}\text{ kg/u}) - 9.11\times10^{-31}\text{ kg} = 4.65174\times10^{-26}\text{ kg}$$

Note that we're given the atomic masses very accurately in Exercise 28. We need to retain this accuracy to tell the difference between N_2^+ and CO^+. The voltage for N_2^+ is

$$\Delta V = \frac{(1.6\times10^{-19}\text{ C})(0.200\text{ T})^2(0.08\text{ m})^2}{8(4.65174\times10^{-26}\text{ kg})} = 110.07\text{ V}$$

Ion	Mass (kg)	Accelerating voltage (V)
N_2^+	4.65174×10^{-26}	110.07
O_2^+	5.31341×10^{-26}	96.36
CO^+	4.64986×10^{-26}	110.11

Assess: The difference between N_2^+ and CO^+ is not large but is easily detectable.

33.82. Model: The magnetic field is that of a coaxial cable consisting of a solid inner conductor surrounded by a hollow outer conductor of essentially zero thickness.

Visualize: Please refer to Figure CP33.82. The solid inner conductor and the hollow outer conductor carry equal currents but in opposite directions. The coaxial cable has perfect cylindrical symmetry. So the magnetic field must be tangent to circles that are concentric with the wire.

Solve: **(a)** Ampere's law is

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{through}} = B\int ds = B(2\pi r) \Rightarrow B = \frac{\mu_0 I_{\text{through}}}{2\pi r}$$

For $r < R_1$,

$$I_{\text{through}} = \left(\frac{I}{\pi R_1^2}\right)\pi r^2 = \frac{Ir^2}{R_1^2} \Rightarrow B = \frac{\mu_0}{2\pi}\left(\frac{Ir}{R_1^2}\right)$$

For $R_1 < r < R_2$, $I_{\text{through}} = I$. Hence, $B = \mu_0 I/2\pi r$. For $r > R_2$, $I_{\text{through}} = 0$ A and $B = 0$ T.

(b) In the region $r < R_1$, B is linearly proportional to r; in the region $R_1 < r < R_2$, B is inversely proportional to r; and in the region for $r > 2R_2$, $B = 0$ T. A B versus r graph in the range $r = 0$ m to $r = 2R_2$ is shown below.