HW # 5: Solutions

35.51. Model: The earth is a complete absorber of sunlight. An object gains momentum when it absorbs electromagnetic waves.
Solve: The radiation pressure on an object that absorbs all the light is

$$p_{rad} = \frac{I}{c} = \frac{1360\ \text{W/m}^2}{3.0\times10^8\ \text{m/s}} = 4.533\times10^{-6}\ \text{Pa}$$

Seen from the sun, the earth is a circle of radius R_{earth} and area $A = \pi R_{earth}^2$. The pressure exerts a force on this area

$$F_{rad} = p_{rad}A = p_{rad}\left(\pi R_{earth}^2\right) = \left(4.533\times10^{-6}\ \text{Pa}\right)\pi\left(6.37\times10^6\ \text{m}\right)^2 = 5.78\times10^8\ \text{N}$$

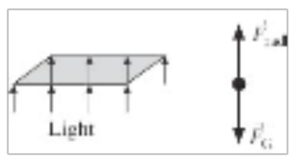
The sun's gravitational force on the earth is

$$F_{grav} = \frac{GM_{sun}M_{earth}}{R_{sun\text{-}earth}^2} = \frac{\left(6.67\times10^{-11}\ \text{N m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\ \text{kg}\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left(1.50\times10^{11}\ \text{m}\right)^2} = 3.53\times10^{22}\ \text{N}$$

$$\Rightarrow \frac{F_{rad}}{F_{grav}} = \frac{5.78\times10^8\ \text{N}}{3.53\times10^{22}\ \text{N}} = 1.64\times10^{-14}$$

That is, F_{rad} is 1.64×10^{-12} % of F_{grav}.

35.53. Model: Assume that the black paper absorbs the light completely. Use the particle model for the paper.
Visualize:



For the black paper to be suspended, the radiation-pressure force must be equal to the gravitational force on the paper.
Solve: From Equation 35.39, $F_{rad} = p_{rad}A = IA/c$. Hence,

$$I = \frac{c}{A}F_{rad} = \frac{c}{A}F_G = \frac{\left(3.0\times10^8\ \text{m/s}\right)\left(1.0\times10^{-3}\ \text{kg}\right)\left(9.8\ \text{m/s}^2\right)}{\left(8.5\ \text{inch}\times11\ \text{inch}\right)\left(2.54\times10^{-2}\ \text{m/inch}\right)^2} = 4.9\times10^7\ \text{W/m}^2$$

35.55. Model: Use the particle model for the astronaut.

Solve: According to Newton's third law, the force of the radiation on the astronaut is equal to the momentum delivered by the radiation. For this force we have

$$F = p_{rad}A = \frac{P}{c} = \frac{1000\ \text{W}}{3.0 \times 10^8\ \text{m/s}} = 3.333 \times 10^{-6}\ \text{N}$$

Using Newton's second law, the acceleration of the astronaut is

$$a = \frac{3.333 \times 10^{-6}\ \text{N}}{80\ \text{kg}} = 4.167 \times 10^{-8}\ \text{m/s}^2$$

Using $v_f = v_i + a(t_f - t_i)$ and a time equal to the lifetime of the batteries,

$$v_f = 0\ \text{m/s} + (4.167 \times 10^{-8}\ \text{m/s}^2)(3600\ \text{s}) = 1.500 \times 10^{-4}\ \text{m/s}$$

The distance traveled in the first hour is calculated as follows:

$$v_f^2 - v_i^2 = 2a(\Delta s)_{\text{first hour}}$$

$$\Rightarrow (1.500 \times 10^{-4}\ \text{m/s})^2 - (0\ \text{m/s})^2 = 2(4.167 \times 10^{-8}\ \text{m/s}^2)(\Delta s)_{\text{first hour}} \Rightarrow (\Delta s)_{\text{first hour}} = 0.270\ \text{m}$$

This means the astronaut must cover a distance of 5.0 m – 0.27 m = 4.73 m in a time of 9 hours. The acceleration is zero during this time. The time it will take the astronaut to reach the space capsule is

$$\Delta t = \frac{4.73\ \text{m}}{1.500 \times 10^{-4}\ \text{m/s}} = 31{,}533\ \text{s} = 8.76\ \text{hours}$$

Because this time is less than 9 hours, the astronaut is able to make it safely to the space capsule.

35.62. Model: Use Malus's law for the polarized light.

Solve: For unpolarized light, the electric field vector varies randomly through all possible values of θ. Because the average value of $\cos^2\theta$ is $\frac{1}{2}$, the intensity transmitted by the first polarizing filter is $I_1 = \frac{1}{2}I_0$. For polarized light, $I_{\text{transmitted}}$ = $I_0 \cos^2\theta$. For the second filter the transmitted intensity is $I_2 = I_1 \cos^2\theta = \frac{1}{2}I_0 \cos^2\theta$. Similarly, $I_3 = I_2 \cos^2\theta = \frac{1}{2}I_0 (\cos^2\theta)^2$, and so on. Thus,

$$I_7 = I_1\left(\cos^2\theta\right)^{7-1} = \tfrac{1}{2}I_0\left(\cos^2\theta\right)^6 = \tfrac{1}{2}I_0\left(\cos^2 15^\circ\right)^6 = 0.33I_0$$

Problem 5.1

Suppose the electric field of a plane electromagnetic wave is given by

$$\vec{\mathbf{E}}(z,t) = E_0 \cos(kz - \omega t)\hat{\mathbf{i}}$$

Find the following quantities:

(a) The direction of wave propagation.

(b) The corresponding magnetic field $\vec{\mathbf{B}}$.

Solution:

(a) By writing the argument of the cosine function as $kz - \omega t = k(z - ct)$ where $\omega = ck$, we see that the wave is traveling in the $+z$ direction.

(b) The direction of propagation of the electromagnetic waves coincides with the direction of the Poynting vector which is given by $\vec{\mathbf{S}} = \vec{\mathbf{E}} \times \vec{\mathbf{B}} / \mu_0$. In addition, $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other. Therefore, if $\vec{\mathbf{E}} = E(z,t)\hat{\mathbf{i}}$ and $\vec{\mathbf{S}} = S\hat{\mathbf{k}}$, then $\vec{\mathbf{B}} = B(z,t)\hat{\mathbf{j}}$. That is, $\vec{\mathbf{B}}$ points in the $+y$-direction. Since $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are in phase with each other, one may write

$$\vec{\mathbf{B}}(z,t) = B_0 \cos(kz - \omega t)\hat{\mathbf{j}} \tag{13.12.2}$$

To find the magnitude of $\vec{\mathbf{B}}$, we make use of Faraday's law:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt} \tag{13.12.3}$$

which implies

$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t} \tag{13.12.4}$$

From the above equations, we obtain

$$-E_0 k\sin(kz-\omega t) = -B_0\omega\sin(kz-\omega t) \tag{13.12.5}$$

or

$$\frac{E_0}{B_0} = \frac{\omega}{k} = c \tag{13.12.6}$$

Thus, the magnetic field is given by

$$\vec{\mathbf{B}}(z,t) = (E_0/c)\cos(kz-\omega t)\,\hat{\mathbf{j}} \tag{13.12.7}$$

Problem 5.2

Verify that, for $\omega = kc$,

$$E(x,t) = E_0\cos(kx-\omega t)$$
$$B(x,t) = B_0\cos(kx-\omega t)$$

satisfy the one-dimensional wave equation:

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\begin{Bmatrix} E(x,t) \\ B(x,t) \end{Bmatrix} = 0$$

Solution:

Differentiating $E = E_0\cos(kx-\omega t)$ with respect to x gives

$$\frac{\partial E}{\partial x} = -kE_0\sin(kx-\omega t), \quad \frac{\partial^2 E}{\partial x^2} = -k^2 E_0\cos(kx-\omega t) \tag{13.12.10}$$

Similarly, differentiating E with respect to t yields

$$\frac{\partial E}{\partial t} = \omega E_0\sin(kx-\omega t), \quad \frac{\partial^2 E}{\partial t^2} = -\omega^2 E_0\cos(kx-\omega t) \tag{13.12.11}$$

Thus,

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \left(-k^2 + \frac{\omega^2}{c^2}\right)E_0 \cos(kx - \omega t) = 0 \qquad (13.12.12)$$

where we have made used of the relation $\omega = kc$. One may follow a similar procedure to verify the magnetic field.

Problem 5.3

A parallel-plate capacitor with circular plates of radius R and separated by a distance h is charged through a straight wire carrying current I, as shown in the Figure 13.12.1:

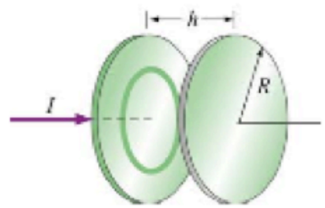


Figure 13.12.1 Parallel plate capacitor

(a) Show that as the capacitor is being charged, the Poynting vector $\vec{\mathbf{S}}$ points radially inward toward the center of the capacitor.

(b) By integrating $\vec{\mathbf{S}}$ over the cylindrical boundary, show that the rate at which energy enters the capacitor is equal to the rate at which electrostatic energy is being stored in the electric field.

Hint: The Electric field E of a capacitor is given by

$E=Q/(\varepsilon_o \pi R^2)$

(a) Let the axis of the circular plates be the z-axis, with current flowing in the $+z$-direction. Suppose at some instant the amount of charge accumulated on the positive plate is $+Q$. The electric field is

$$\vec{\mathbf{E}} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{k}} = \frac{Q}{\pi R^2 \varepsilon_0}\hat{\mathbf{k}} \tag{13.12.13}$$

According to the Ampere-Maxwell's equation, a magnetic field is induced by changing electric flux:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d}{dt} \iint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

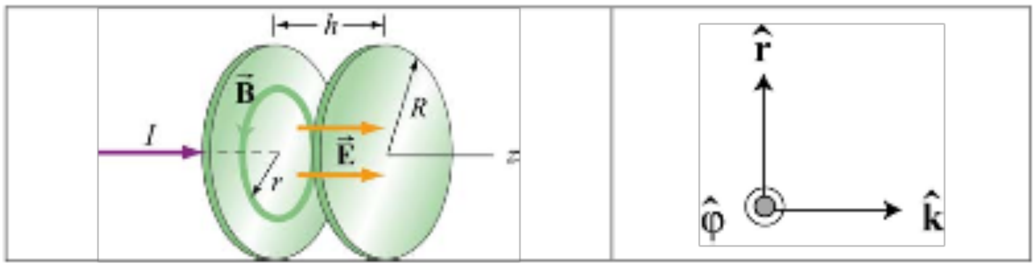


Figure 13.12.2

From the cylindrical symmetry of the system, we see that the magnetic field will be circular, centered on the z-axis, i.e., $\vec{\mathbf{B}} = B\hat{\boldsymbol{\varphi}}$ (see Figure 13.12.2.)

Consider a circular path of radius $r < R$ between the plates. Using the above formula, we obtain

$$B(2\pi r) = 0 + \mu_0 \varepsilon_0 \frac{d}{dt}\left(\frac{Q}{\pi R^2 \varepsilon_0}\pi r^2\right) = \frac{\mu_0 r^2}{R^2}\frac{dQ}{dt} \tag{13.12.14}$$

or

$$\vec{\mathbf{B}} = \frac{\mu_0 r}{2\pi R^2}\frac{dQ}{dt}\hat{\boldsymbol{\varphi}} \tag{13.12.15}$$

The Poynting $\vec{\mathbf{S}}$ vector can then be written as

$$\begin{aligned}\vec{\mathbf{S}} &= \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}} = \frac{1}{\mu_0}\left(\frac{Q}{\pi R^2\varepsilon_0}\hat{\mathbf{k}}\right)\times\left(\frac{\mu_0 r}{2\pi R^2}\frac{dQ}{dt}\hat{\boldsymbol{\varphi}}\right)\\ &= -\left(\frac{Qr}{2\pi^2 R^4\varepsilon_0}\right)\left(\frac{dQ}{dt}\right)\hat{\mathbf{r}}\end{aligned} \tag{13.12.16}$$

Note that for $dQ/dt > 0$ $\vec{\mathbf{S}}$ points in the $-\hat{\mathbf{r}}$ direction, or radially inward toward the center of the capacitor.

(b) The energy per unit volume carried by the electric field is $u_E = \varepsilon_0 E^2/2$. The total energy stored in the electric field then becomes

$$U_E = u_E V = \frac{\varepsilon_0}{2}E^2\left(\pi R^2 h\right) = \frac{1}{2}\varepsilon_0\left(\frac{Q}{\pi R^2\varepsilon_0}\right)^2\pi R^2 h = \frac{Q^2 h}{2\pi R^2\varepsilon_0} \tag{13.12.17}$$

Differentiating the above expression with respect to t, we obtain the rate at which this energy is being stored:

$$\frac{dU_E}{dt} = \frac{d}{dt}\left(\frac{Q^2 h}{2\pi R^2\varepsilon_0}\right) = \frac{Qh}{\pi R^2\varepsilon_0}\left(\frac{dQ}{dt}\right) \tag{13.12.18}$$

$$\oint\vec{\mathbf{S}}\cdot d\vec{\mathbf{A}} = SA_R = \left(\frac{Qr}{2\pi^2\varepsilon_0 R^4}\frac{dQ}{dt}\right)(2\pi Rh) = \frac{Qh}{\varepsilon_0\pi R^2}\left(\frac{dQ}{dt}\right) \tag{13.12.19}$$

which is equal to the rate at which energy stored in the electric field is changing.

Problem 5.4 :

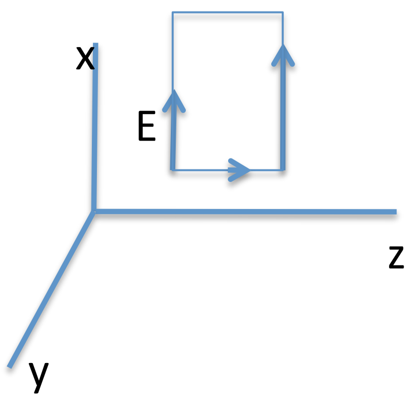
Can parallel electric and magnetic fields make up an electromagnetic wave in vacuum?

Problem 5.5:

4. Explain why the reception for cellular phones often becomes poor when used inside a steel-framed building.

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \iint \vec{B} \cdot d\vec{A}$$

Problem 5.4



$$E(z+\Delta z)\Delta x - E(z)\Delta x = \frac{E(z+\Delta z) - E(z)}{\Delta z}\Delta z \Delta x = \frac{\partial E}{\partial z}\Delta z \Delta x =$$

$$= -\frac{d}{dt} \iint BA(e_x \cdot e_y) = 0$$

$$\frac{\partial E}{\partial z} = 0$$

No propagation - no em wave if B is in x - direction

Problem 5.5 EM wave reflected as well as absorbed by the metal in the wall reaches room with reduced amplitude