Solutions Set # 3

I. Conceptual Questions

34.4 Order: $\Phi_3 > \Phi_1 > \Phi_2 > \Phi_4$

Explanation:

$$\Phi = B \cdot A \cdot \cos\theta = B(\pi r^2)\cos\theta$$
$$\Phi_1 = B\pi(1)^2 \cos 0^\circ = \pi B$$
$$\Phi_2 = B\pi(1)^2 \cos 45^\circ = 0.707\ \Phi_1$$
$$\Phi_3 = B\pi(4)\cos 45^\circ = 2.83\ \Phi_1$$
$$\Phi_4 = B\pi(4)\cos 90^\circ = 0$$

34.7 a. Clockwise from above. The increasing flux through the loop is due to the upward-pointing field of the magnet, so the induced magnetic field is downward.
b. Yes, up. The induced downward field is like a magnet with its north pole on the bottom, which is repulsed by the bar magnet.
c. Yes, down. By Newton's third law, the bar magnet is pushed down.

34.10 a. Right to left. Just after the switch is closed an increasing solenoidal magnetic field in the left coil pointing left to right increases the flux through the right-hand coil. To oppose the increase in flux, a magnetic field pointing right to left is induced, caused by a current flowing right to left through the meter on the right-hand coil.
b. Zero. The field in the left-hand coil is constant, so the flux through the right-hand coil is not changing.
c. Left to right. When the switch is reopened, the flux decreases. A left-to-right magnetic field caused by a left-to-right current through the meter of the right-hand coil is induced in the right-hand coil to oppose the change.

II. Exercises and Problems

34.10. Model: Assume the plane of the loop is perpendicular to the field direction.
Visualize: Please refer to Figure Ex34.10. The flux is due to the field through the area of the triangle. Only the left half gives a contribution as the field strength is zero on the right half.
Solve: (a) The flux is $\Phi = \vec{A}\cdot\vec{B}$. Take $\vec{A}$ to be into the page, perpendicular to the triangle, and thus parallel to $\vec{B}$. In this case $\Phi = AB$ where A is the area of half of the triangle. This smaller triangle has a base of 10 cm and height $20\sin 60^\circ$ cm $= 17.32$ cm. Thus,

$$\Phi = AB = \frac{1}{2}(0.10 \text{ m})(0.1732 \text{ m})\times 0.10 \text{ T} = 8.7\times 10^{-4} \text{ Wb}$$

(b) The flux is directed into the loop. According to Lenz's law, the induced current will try to *prevent the decrease* of flux. To do this, the field of the induced current will have to point into this loop. This requires the induced current to flow *clockwise*.

34.12. Model: Assume the field is uniform.
Visualize: Please refer to Figure Ex34.12. The motion of the loop changes the flux through it. This results in an induced emf and current.
Solve: The induced emf is $\mathcal{E} = |d\Phi/dt|$ and the induced current is $I = \mathcal{E}/R$. The area A is changing, but the field B is not. Take $\vec{A}$ as being out of the page and parallel to $\vec{B}$, so $\Phi = AB$ and $d\Phi/dt = B(dA/dt)$. The flux is through that portion of the loop where there is a field, that is, $A = lx$. The emf and current are

$$\mathcal{E} = B\left|\frac{dA}{dt}\right| = B\left|\frac{d(lx)}{dt}\right| = Blv = \left|(0.20 \text{ T})(0.050 \text{ m})(50 \text{ m/s})\right| = 0.50 \text{ V}$$

$$I = \frac{\mathcal{E}}{R} = \frac{0.50 \text{ V}}{0.10\Omega} = 5.0 \text{ A}$$

The field is out of the page. As the loop moves the flux increases because more of the loop area has field through it. To *prevent the increase*, the induced field needs to point into the page. Thus, the induced current flows *clockwise*.
Assess: This seems reasonable since there is rapid motion of the loop.

34.43. Model: Assume the magnetic field is uniform over the plane of the loop.

Visualize: The oscillating magnetic field strength produces a changing flux through the loop and an induced emf in the loop.

Solve: **(a)** The normal to the surface of the loop is in the same direction as the magnetic field so that $\Phi = \vec{A}\cdot\vec{B} = BA$. The induced emf is

$$\mathcal{E} = \left|\frac{d\Phi}{dt}\right| = A\left|\frac{dB}{dt}\right| = \pi r^2\left|\frac{dB}{dt}\right| = \pi r^2\omega B_0|\cos\omega t|$$

The cosine will oscillate between +1 and −1 so the maximum emf is

$$\mathcal{E}_{max} = \pi r^2\omega B_0 = \pi r^2(2\pi f)B_0 = 2\pi^2(0.125)^2(150\times10^6\text{ Hz})(20\times10^{-9}\text{ T}) = 0.93\text{ V}$$

(b) If the loop is rotated so that the plane is perpendicular to the electric field, then the normal to the surface will be parallel to the magnetic field. There is no magnetic flux through the loop and no induced emf.

34.52. Model: Assume the magnetic field is uniform over the region where the bar is sliding and that friction between the bar and the rails is zero.

Visualize: Please refer to Figure P34.52. The battery will produce a current in the rails and bar and the bar will experience a force. With the battery connected as shown in the figure, the current in the bar will be down and by the right-hand rule the force on the bar will be to the right. The motion of the bar will change the flux through the loop and there will be an induced emf that opposes the change.

Solve: **(a)** As the bar speeds up the induced emf will get larger until finally it equals the battery emf. At that point, the current will go to zero and the bar will continue to move at a constant velocity. We have

$$\mathcal{E} = Blv_{term} = \mathcal{E}_{bat} \Rightarrow v_{term} = \frac{\mathcal{E}_{bat}}{Bl}$$

(b) The terminal speed is

$$v_{term} = \frac{1.0\text{ V}}{(0.50\text{ T})(0.060\text{ m})} = 33\text{ m/s}$$

Assess: This is pretty fast, about 70 mph.

34.47. Model: Assume the field changes abruptly at the boundary and is uniform.

Visualize: Please refer to Figure P34.47. As the loop enters the field region the amount of flux will change as more area has field penetrating it. This change in flux will create an induced emf and corresponding current. While the loop is moving at constant speed, the rate of change of the area is not constant because of the orientation of the loop. The loop is moving along the x-axis.

Solve: **(a)** If the edge of the loop enters the field region at $t = 0$ s, then the leading corner has moved a distance $x = v_0 t$ at time t. The area of the loop with flux through it is

$$A = 2\left(\tfrac{1}{2}\right)yx = x^2 = \left(v_0 t\right)^2$$

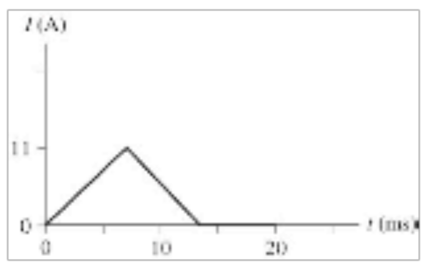
where we have used the fact that $y = x$ since the sides of the loop are oriented at $45°$ to the horizontal. Take the surface normal of the loop to be into the page so that $\Phi = \vec{A}\cdot\vec{B} = BA$. The current in the loop is

$$I = \frac{\mathcal{E}}{R} = \frac{1}{R}\left|\frac{d\Phi}{dt}\right| = \frac{1}{R}B\left|\frac{dA}{dt}\right| = \frac{1}{R}B\left|\frac{d}{dt}\left(v_0 t\right)^2\right| = \frac{1}{R}B\left(2\right)v_0^2 t = \left(\frac{2\left(0.80\text{ T}\right)\left(10\text{ m/s}\right)^2}{0.10\Omega}\right)t = \left(1.6\times 10^3\text{ A}\right)t$$

The current is increasing at a constant rate. This expression is good until the loop is halfway into the field region. The time for the loop to be halfway is found as follows:

$$\frac{10\text{ cm}}{\sqrt{2}} = v_0 t = \left(10\text{ m/s}\right)t \quad\Rightarrow\quad t = 7.1\times 10^{-3}\text{ s} = 7.1\text{ ms}$$

At this time the current is 11 A. While the second half of the loop is moving into the field, the flux continues to increase, but at a slower rate. Therefore, the current will decrease at the same rate as it increased before, until the loop is completely in the field at $t = 14$ ms. After that the flux will not change and the current will be zero.



(b) The maximum current of 11 A occurs when the flux is changing the fastest and this occurs when the loop is halfway into the region of the field.