Problem Set #4 - Solutions

Conceptual Questions:

34.12 a. No. Since $\Delta V = -L\frac{dI}{dt}$ and ΔV and L are both known, we can only find $\frac{dI}{dt}$, not I.

b. Yes, through the right-hand inductor, since $\left|\frac{dI}{dt}\right| = \frac{\Delta V}{L}$.

c. Yes, it is possible to tell: The current is decreasing. If $\frac{dI}{dt} < 0$ then $\Delta V_L > 0$ and the input side is more negative, and the potential increases in the direction of the current.

35.1. **(a)** Yes, up. Andre sees the same magnetic field as the laboratory field, which points up.
(b) Andre sees an electric field $\vec{E}' = \vec{V} \times \vec{B}$, which by the right-hand rule points into the page.

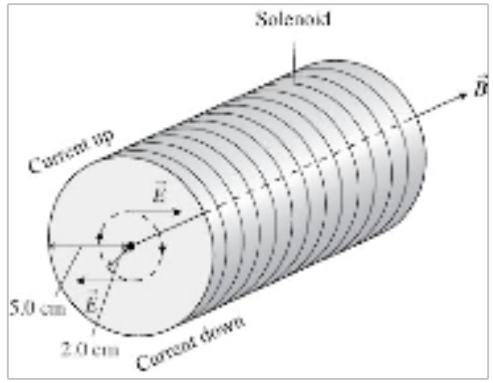
35.7. **(a)** The right-hand rule has $\vec{E} \times \vec{B}$ point out of the page.
(b) $\vec{E} \times \vec{B}$ points up.

35.8. **(a)** Since $I \propto E_0^2$, the new intensity is 40 W/m^2.
(b) For an electromagnetic wave $E_0 \propto B_0$, so 40 W/m^2.
(c) For electromagnetic waves, doubling one amplitude requires that the other is doubled, so 40 W/m^2.
(d) Changing the frequency does not change the amplitudes and hence the intensity is unchanged at 10 W/m^2.

35.9. A loop antenna works by taking advantage of the current produced by magnetic induction in Faraday's law as an electromagnetic wave passes through.

34.16. Model: A changing magnetic field creates an electric field.

Visualize:



Solve: **(a)** Apply Equation 34.26. For a point on the axis, $r = 0$ m, so $E = 0$ V/m.

(b) For a point 2.0 cm from the axis,

$$E = \frac{r}{2}\left|\frac{dB}{dt}\right| = \left(\frac{0.020 \text{ m}}{2}\right)(4.0 \text{ T/s}) = 0.040 \text{ V/m}$$

34.78. Model: Assume an ideal inductor and an ideal (resistanceless) battery.

Visualize: Please refer to Figure P34.78. The current through the battery is the sum of the currents through the left and right branches of the circuit.

Solve: **(a)** Because the switch has been open a long time, no current is flowing the instant before the switch is closed. A basic property of an ideal inductor is that the current through it cannot change instantaneously. This is because the potential difference $\Delta V_L = -L(dI/dt)$ would become infinite for an instantaneous change of current, and that is not physically possible. Because the current through the inductor was zero before the switch was closed, it must still be zero (or very close to it) immediately after the switch is closed. Conservation of current requires the current through the entire right branch to be the same as that through the inductor, so it is also zero immediately after the switch is closed. The only current is through the left 20 Ω resistor, which sees the full battery potential of the battery. Thus $I_{bat} = I_{left} = \Delta V_{bat}/R = (10 \text{ V})/(20\ \Omega) = 0.50$ A.

(b) The current through the inductor increases as time passes. Once the current I_{right} reaches a steady value and is no longer changing, the potential difference across the inductor is $\Delta V_L = -L(dI/dt) = 0$ V. An ideal inductor has no resistance, so the inductor simply acts like a wire and has no effect on the circuit. The circuit is that of two 20 Ω resistors in parallel. The equivalent resistance is 10 Ω and the battery current is $I_{bat} = (10 \text{ V})/(10\ \Omega) = 1.0$ A.

34.82. Model: Assume the RC circuit has wires with zero resistance. Assume that only the bottom horizontal wire in the RC circuit closest to the metal loop contributes to the magnetic flux through the loop. Assume that the wire appears infinite in length compared to the metal loop.

Visualize: See Figure CP34.82. The current in the RC circuit decreases as the capacitor is discharged. There is a changing magnetic flux through the metal loop due to the changing current through the RC circuit, inducing an emf through the loop.

Solve: **(a)** Current $I(t) = I_0 e^{-t/RC}$ flows from right to left through the wire at the bottom of the RC circuit, causing a magnetic field out of the paper through the metal loop. Since the RC circuit current is decreasing, the induced current through the loop is counterclockwise in order to increase the magnetic flux.

(b) Using the result of Example 34.5, the flux through the metal loop due to the wire is

$$\Phi_m = \frac{\mu_0 I(t)(2.0\text{ cm})}{2\pi}\ln\left(\frac{0.5\text{ cm}+1.0\text{ cm}}{0.5\text{ cm}}\right) = \left(4.4\times10^{-9}\right)I_0 e^{-t/RC}$$

The magnitude of the induced emf in the metal loop is

$$\mathcal{E} = \left|\frac{d\Phi}{dt}\right| = \frac{\left(4.4\times10^{-9}\right)I_0 e^{-t/RC}}{RC} = \left(4.4\times10^{-4}\right)I_0 e^{-t/RC}$$

The initial current through the RC circuit $I_0 = 20\text{ V}/2.0\ \Omega = 10$ A. At $t = 5.0\ \mu$s, $\mathcal{E} = 0.267$ mV. The current through the loop $I_{loop} = \mathcal{E}/R = 0.267\text{ mV}/0.050\Omega = 5.3\text{ mA}$.

Assess: As the current through the RC circuit decreases to a constant value of zero at large times, the induced emf also approaches zero.

34.83. Visualize: Please refer to Figure CP34.83. The area within in the loop is changing so the flux will change. This will produce an induced emf and corresponding current. The area of the loop is the area of a square.
Solve: **(a)** The field is out of the page, so the flux is increasing outward as the loop expands. According to Lenz's law, the induced current tries to *prevent the increase* of outward flux. It does so by generating a field *into* the page. This requires a *clockwise* current flow.
(b) Take $\vec{A}$ parallel to $\vec{B}$, so $\Phi = AB$ and $\mathcal{E} = |d\Phi/dt| = B(dA/dt)$. The field is constant but the loop area is changing. Let the length of each edge of the loop be x at time t. This length is increasing linearly with time as the corner of the loop moves outward with speed $v = 10$ m/s. We have

$$x = x_0 + v_x t = 0 + [(10\text{ m/s})\cos 45°]t = 7.07t\text{ m}$$

The loop's area at time t is

$$A = x^2 = (7.07t\text{ m})^2 = 50t^2\text{ m}^2 \Rightarrow \frac{dA}{dt} = 100t\text{ m}^2/\text{s}$$

Consequently, the induced emf at time t is

$$\mathcal{E} = B\frac{dA}{dt} = (0.10\text{ T})(100t\text{ m}^2/\text{s}) = 10t\text{ V}$$

The induced current is $I = \mathcal{E}/R$, where R is the resistance of the loop. The wire has resistance 0.010 Ω *per meter*, so
$R = (0.010\ \Omega/\text{m})l$ where $l = 4x$ is the perimeter of the square at time t. That is,

$$R = 4(0.010\ \Omega/\text{m})(7.07t\text{ m}) = 0.283t\ \Omega$$

Thus, the induced current is

$$I = \frac{\mathcal{E}}{R} = \frac{10t\text{ V}}{0.283t\ \Omega} = 35\text{ A}$$

(c) At $t = 0.1$ s, $\mathcal{E} = 1.0$ V and $I = 35$ A.
Assess: The induced emf depends on time, but so does the resistance. This means the induced current is constant.

35.5. Model: Use the Galilean transformation of fields.

Visualize: Please refer to Figure EX35.5. We are given $\vec{V} = 1.0\times10^6\hat{i}$ m/s, $\vec{B} = 0.50\hat{k}$ T, and $\vec{E} = \left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right)\times10^6$ V/m.

Solve: Equation 35.11 gives the Galilean transformation equation for the electric field in the S and S' frames: $\vec{E}' = \vec{E} + \vec{V}\times\vec{B}$. The electric field from the moving rocket is

$$\vec{E}' = \left(\hat{i}+\hat{j}\right)0.707\times10^6\text{ V/m} + \left(1.0\times10^5\hat{i}\text{ m/s}\right)\times\left(0.50\hat{k}\text{ T}\right) = \left(0.707\times10^6\hat{i} + 0.207\times10^6\hat{j}\right)\text{ V/m}$$

$$\theta = \tan^{-1}\left(\frac{0.207\times10^5\text{ V/m}}{0.707\times10^5\text{ V/m}}\right) = 16.3°\ \text{above the } x'\text{-axis}$$

35.10. Model: The electric field inside a parallel-plate capacitor is uniform. As the capacitor is charged, the changing electric field induces a magnetic field.

Visualize: The induced magnetic field lines are circles concentric with the capacitor. Please refer to Figure 35.18.

Solve: **(a)** The Ampere-Maxwell law is

$$\oint \vec{B} \cdot d\vec{s} = \varepsilon_0 \mu_0 \frac{d\Phi_e}{dt} = \varepsilon_0 \mu_0 A \frac{dE}{dt}$$

where EA is the electric flux through the circle of radius r. The magnetic field is everywhere tangent to the circle of radius r, so the integral of $\vec{B} \cdot d\vec{s}$ around the circle is simply $B(2\pi r)$. The Ampere-Maxwell law becomes

$$2\pi r B = \varepsilon_0 \mu_0 \left(\pi r^2\right) \frac{dE}{dt} \Rightarrow B = \varepsilon_0 \mu_0 \frac{r}{2} \frac{dE}{dt}$$

On the axis, $r = 0$ m, so $B = 0$ T.

(b) At $r = 3.0$ cm,

$$B = \frac{1}{\left(3.0 \times 10^8 \text{ m/s}\right)^2} \left(\frac{0.030 \text{ m}}{2}\right) \left(1.0 \times 10^6 \text{ V/m s}\right) = 1.67 \times 10^{-13} \text{ T}$$

(c) For $r > 5.0$ cm, the electric flux Φ_e is the flux through a 10-cm-diameter circle because $E = 0$ V/m outside the capacitor plates. The Ampere-Maxwell law is

$$B\left(2\pi r\right) = \varepsilon_0 \mu_0 \pi R^2 \frac{dE}{dt}$$

$$\Rightarrow B = \varepsilon_0 \mu_0 \frac{R^2}{2r} \frac{dE}{dt} = \frac{1}{\left(3.0 \times 10^8 \text{ m/s}\right)^2} \frac{\left(0.050 \text{ m}\right)^2}{2\left(0.07 \text{ m}\right)} \left(1.0 \times 10^6 \text{ V/m s}\right) = 1.98 \times 10^{-13} \text{ T}$$

35.34. Model: Use the Galilean transformation of fields. Assume that the wire is infinite.
Visualize: Please refer to Figure P35.34. The laboratory frame is frame S and the circular loop's frame is frame S'.
Solve: **(a)** From Equation 27.15, the electric field $\vec{E}$ due to a wire at rest is

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r} = \left(\frac{\lambda}{2\pi\varepsilon_0 r}, \text{ away from wire}\right)$$

The magnetic field $\vec{B}$ at a point on the loop is zero because there are no moving charges. These are the fields measured in frame S.
(b) In frame S', $\vec{E}' = \vec{E} + \vec{v} \times \vec{B} = \vec{E}$ because $\vec{B} = 0$ T. The magnetic field in frame S' is

$$\vec{B}' = \vec{B} - \frac{1}{c^2}\vec{v} \times \vec{E} = 0\text{ T} - \frac{1}{c^2}\vec{v} \times \hat{r}\ \frac{\lambda}{2\pi\varepsilon_0 r} = \left(\frac{1}{c^2\varepsilon_0}\frac{v\lambda}{2\pi r}, \text{ into the page at the top}\right)$$

(c) Consider a segment of the wire of length Δx with charge $\Delta Q = \lambda \Delta x$. The experimenter in the loop's frame sees a charge on this segment passing by him/her with a velocity v to the left. Thus,

$$I = \frac{\Delta Q}{\Delta t} = \frac{\lambda \Delta x}{\Delta t} = \lambda v$$

(d) The magnetic field in the loop's frame is due to current I:

$$B' = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 \lambda v}{2\pi r} = \frac{\mu_0 \lambda v}{2\pi r} \times \frac{\varepsilon_0}{\varepsilon_0} = \frac{1}{\varepsilon_0 c^2}\frac{\lambda v}{2\pi r}$$

Since the current I is moving to the left, the right-hand rule states the direction of B' is into the page on the top. The electric field $\vec{E}'$ which is due to the charge on the wire would be equal to $\vec{E}$.
(e) As we see, the results in parts (b) and (d) are the same.
(f) There will not be any induced current if the loop is made of a conducting material. This is because (i) the field is perpendicular to the area of the loop and (ii) the flux is not changing.

35.48. Model: Radio waves are electromagnetic waves. Assume that the transmitter unit radiates in all directions.
Solve: The transmitting unit radiates energy in all directions at the rate of 250 mJ per second. From Equation 35.37, the signal intensity at a distance of 42 m is

$$I = \frac{P}{A} = \frac{P}{4\pi r^2} = \frac{250 \times 10^{-3}\text{ W}}{4\pi(42\text{ m})^2} = 1.13 \times 10^{-5}\text{ W/m}^2$$

Using Equation 35.37 again,

$$I = \frac{c\varepsilon_0}{2}E_0^2 \Rightarrow E_0 = \sqrt{\frac{2I}{c\varepsilon_0}} = \sqrt{\frac{2(1.13 \times 10^{-5}\text{ W/m}^2)}{(3.0 \times 10^8\text{ m/s})(8.85 \times 10^{-12}\text{ C}^2/\text{N m}^2)}} = 0.092\ \frac{\text{V}}{\text{m}}$$

A few steps before 42 m, the field strength was 0.100 V/m and the door opened. The manufacturer's claims are correct.