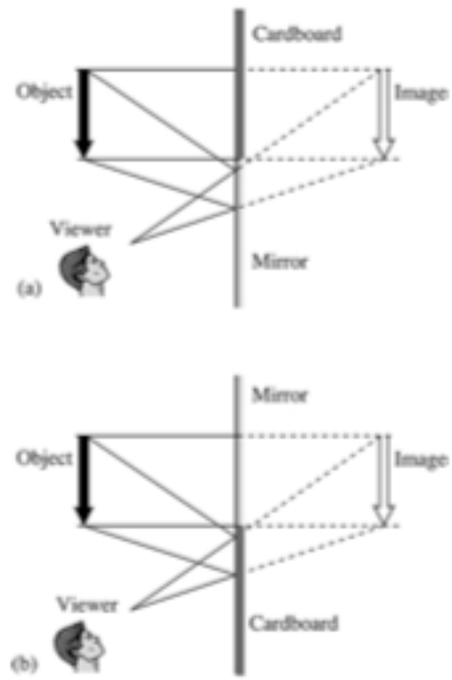
HW #9 Solutions

Conceptual Questions

23.3. Light is scattered off all points of the pencil and into all directions of space. If light directed toward the mirror is reflected into your eye, you see the image of the pencil. **(a)** As part (a) of the figure shows, if the top half of the mirror is covered, light scattered from the pencil and reflected off the mirror can enter your eye and you will see the image of the pencil. **(b)** As part (b) of the figure shows, if the bottom half of the mirror is covered, light scattered from the pencil cannot be reflected off the mirror in such a matter that it enters your eye. You cannot see the image of the pencil.



23.4. The speed of light in a medium is a function of the index of refraction of that medium $(v = c/n)$. If you use optic fiber at the center (where the light would have a shorter travel distance) with a larger index of refraction, you will slow the light down and it will get to the end of the fiber closer to the time the other light gets there.

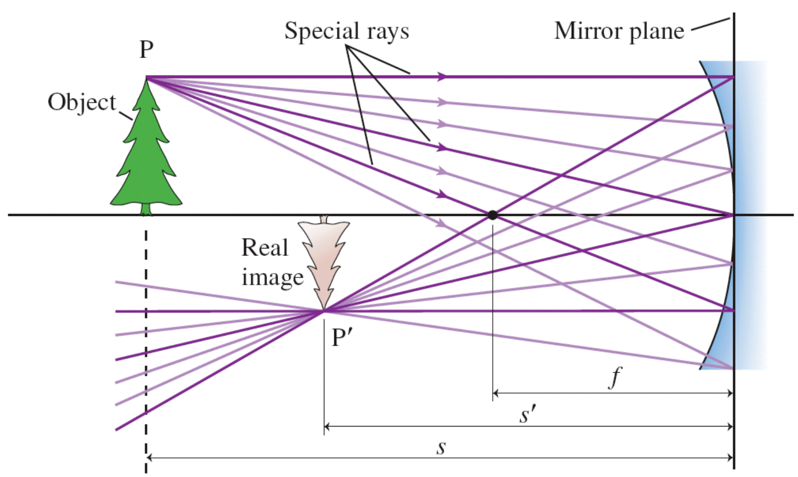
23.5. Section 1: Blue; all other colors are absorbed. Section 2: Black; light essentially does not get through. Section 3: Red; all other colors are absorbed.

23.6. The card is red because it reflects red light and absorbs the other colors. When it is illuminated by red light the red light reflects off the card into your eyes and you see the red card as red. If the card is illuminated with blue light the light is all absorbed. No light is reflected, so the card looks black. If you illuminate the card with white light and look at it through a blue filter it will again look black because the red light reflected by the card is not passed by the blue filter.

23.7. Because scattering increases as wavelength decreases, it is better to use infrared light to reduce the impact of scattering by hydrogen gas.

23.10. While the law of refraction depends on the index of the media, the law of reflection does not. Under water the angle of incidence will still be equal to the angle of reflection. There is no reason for a ray to travel a different path in water than in air in this case. Hence, the sun's rays will be focused the same distance from the mirror. Of course the preceding analysis is not true for light passing through a lens that might be immersed in either water or air; in that case the index of refraction matters. But with a mirror the water doesn't change anything.

23.11. The spoon acts like a concave mirror and the image is inverted when $s > f$. A careful ray tracing diagram will convince you of this. It should be noted that this is only true for $s > f$; when the object is closer to the spoon than the focal distance, $s < f$, then the image is upright. Magnifying mirrors, such as make-up mirrors are concave like this and have a large focal length so that your face is within the focal length of the mirror; the image is virtual and behind the mirror. If you put the spoon very close to your eye you will notice this.



21.84. Model: The overlap of the waves causes interference.
Solve: **(a)** The waves traveling to the left are

$$D_1 = a\sin\left[2\pi\left(-\frac{x}{\lambda}-\frac{t}{T}\right)\right] \qquad D_2 = a\sin\left[2\pi\left(-\frac{(x-L)}{\lambda}-\frac{t}{T}\right)+\phi_{20}\right]$$

The phase difference between the waves on the left side of the antenna is thus

$$\Delta\phi_L = \phi_2 - \phi_1 = 2\pi\left(-\frac{(x-L)}{\lambda}-\frac{t}{T}\right)+\phi_{20} - 2\pi\left(-\frac{x}{\lambda}-\frac{t}{T}\right) = 2\pi\frac{L}{\lambda}+\phi_{20}$$

On the right side of the antennas, where $x_1 = x_2 + L$, the two waves are

$$D_1 = a\sin\left[2\pi\left(\frac{x}{\lambda}-\frac{t}{T}\right)\right] \qquad D_2 = a\sin\left[2\pi\left(\frac{(x-L)}{\lambda}-\frac{t}{T}\right)+\phi_{20}\right]$$

Thus, the phase difference between the waves on the right is

$$\Delta\phi_R = \phi_2 - \phi_1 = 2\pi\left(\frac{(x-L)}{\lambda}-\frac{t}{T}\right)+\phi_{20} - 2\pi\left(\frac{x}{\lambda}-\frac{t}{T}\right) = -\frac{2\pi L}{\lambda}+\phi_{20}$$

We want to have destructive interference on the country side or on the left and constructive interference on the right. This requires

$$\Delta\phi_L = \phi_{20} + \frac{2\pi L}{\lambda} = 2\pi\left(m+\tfrac{1}{2}\right) \quad \Delta\phi_R = \phi_{20} - \frac{2\pi L}{\lambda} = 2\pi n$$

These are two simultaneous equations, and we can satisfy them both if L and ϕ_{20} are properly chosen. Subtracting the second equation from the first to eliminate ϕ_{20},

$$\frac{4\pi L}{\lambda} = 2\pi\left(m+\tfrac{1}{2}-n\right) \Rightarrow L = \left(m+\tfrac{1}{2}-n\right)\frac{\lambda}{2}$$

The smallest value of L that works is for $n = m$, in which case $L = \frac{1}{4}\lambda$.

(b) From the $\Delta\phi_R$ equation,

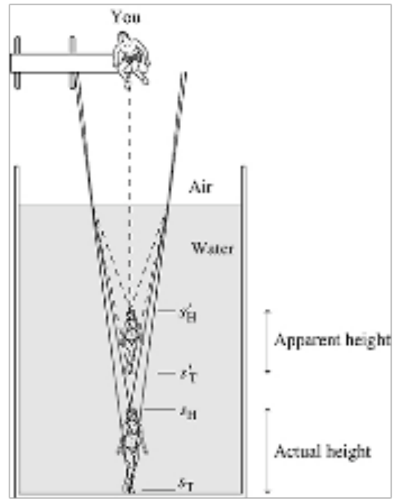
$$\left[\phi_{20} - \frac{2\pi L}{\lambda} = \phi_{20} - \frac{2\pi\left(\frac{1}{4}\lambda\right)}{\lambda} = \phi_{20} - \frac{\pi}{2}\right] = 2\pi n \Rightarrow \phi_{20} = \frac{\pi}{2} + 2\pi n \text{ rad}$$

Adding integer multiples of 2π to the phase constant doesn't really change the wave, so the physically significant phase constant is for $n = 0$, namely $\phi_{20} = \frac{1}{2}\pi$ rad.

(c) We have $\phi_{20} = \frac{1}{2}\pi \text{ rad} = \frac{1}{4}(2\pi)$. If the wave from antenna 2 was delayed by one full period T, it would shift the wave by one full cycle. We would describe this by a phase constant of 2π rad. So a phase constant of $\frac{1}{4}(2\pi)$ rad can be achieved by delaying the wave by $\Delta t = \frac{1}{4}T$.

(d) A wave with frequency $f = 1000 \text{ kHz} = 1.00\times10^6 \text{ Hz}$ has a period $T = 1.00\times10^{-6} \text{ s} = 1.00\ \mu\text{s}$ and wavelength $\lambda = c/f = 300$ m. So this broadcast scheme will work if the antennas are spaced $L = 75$ m apart and if the broadcast from antenna 2 is delayed by $\Delta t = 0.25\ \mu\text{s} = 250$ ns.

23.20. Model: Represent the diver's head and toes as point sources. Use the ray model of light.
Visualize:



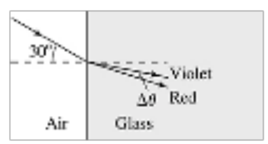
Paraxial rays from the head and the toes of the diver refract into the air and then enter into your eyes. When these refracted rays are extended into the water, the head and the toes appear elevated toward you.
Solve: Using Equation 23.13,

$$s'_T = \frac{n_2}{n_1} s_T = \frac{n_{air}}{n_{water}} s_T \qquad\qquad s'_H = \frac{n_{air}}{n_{water}} s_H$$

Subtracting the two equations, her apparent height is

$$s'_H - s'_T = \frac{n_{air}}{n_{water}}(s_H - s_T) = \frac{1.0}{1.33}(150\text{ cm}) = 113\text{ cm}$$

23.22. Model: Use the ray model of light.
Visualize:



Solve: Using Snell's law,

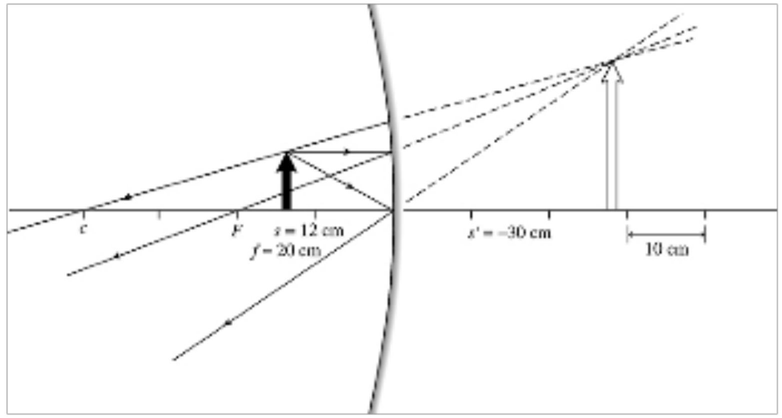
$$n_{air}\sin 30° = n_{red}\sin\theta_{red} \;\Rightarrow\; \theta_{red} = \sin^{-1}\left(\frac{\sin 30°}{1.52}\right) = 19.2°$$

$$n_{air}\sin 30° = n_{violet}\sin\theta_{violet} \;\Rightarrow\; \theta_{violet} = \sin^{-1}\left(\frac{\sin 30°}{1.55}\right) = 18.8°$$

Thus the angular spread is

$$\Delta\theta = \theta_{red} - \theta_{violet} = 19.2° - 18.8° = 0.4°$$

23.38. Solve: The image is at -30 cm as seen in the figure. It is upright.



Assess: When the object is within the focal length we get a magnified upright image.

23.40. Model: The speed of light in a material is determined by the refractive index as $v = c/n$.

Solve: To acquire data from memory, a total time of only 2.0 ns is allowed. This time includes 0.5 ns that the memory unit takes to process a request. Thus, the *travel time* for an infrared light pulse from the central processing unit to the memory unit and back is 1.5 ns. Let d be the distance between the central processing unit and the memory unit. The refractive index of silicon for infrared light is $n_{Si} = 3.5$. Then,

$$1.5\text{ ns} = \frac{2d}{v_{Si}} = \frac{2d}{c/n_{Si}} = \frac{2dn_{Si}}{c} \Rightarrow d = \frac{(1.5\text{ ns})c}{2n_{Si}} = \frac{(1.5\times10^{-9}\text{ s})(3.0\times10^{8}\text{ m/s})}{2(3.5)} \Rightarrow d = 6.4\text{ cm}$$

23.41. Model: Treat the red ball as a point source and use the ray model of light.

Solve: **(a)** Using the law of reflection, we can obtain 3 images of the red ball.

(b) The images of the ball are located at B, C, and D. Relative to the intersection point of the two mirrors, the coordinates of B, C, and D are: B(+1 m, −2 m), C(−1 m, +2 m), and D(+1 m, +2 m).

(c)