

HW # 8 Solutions

Conceptual Questions

1. $\Delta y \propto \frac{\lambda L}{d}$ (a) Decreases; (b) Decrease; (c) Increases
2. Nothing happens in the central band ($m=0$). Colors spread in the remaining bands with red on the outside and blue and violet on the inside. (see Fig. 22.8)
3. Two reasons. First the sources are not coherent since each headlight generates its own random phase. (Reason for using the same source and splitting it in two slit experiments. Second, even if we had coherent sources since $\Delta y = \lambda L/d$ and λ/d is less than 10^{-7} the only way to see a pattern will be to go at least 10^5 meters away.
4. $w \propto 1/D$ it decreases
5. $Iw = \text{const.}$, $I \propto D$ it decreases
6. No. For N slits the intensity is N^2 that of a single slit. For two it is four times larger. Coherent addition.
- 7.

22.4. (a) The equation for gratings does not contain the number of slits, so increasing the number of slits can't affect the angles at which the bright fringes appear as long as d is the same. So the number of fringes on the screen stays the same. (b) The number of slits does not appear in the equation for the fringe spacing, so the spacing stays the same. (c) Decreases; the fringes become narrower. (d) The equation for intensity does contain the number of slits, so each fringe becomes brighter: $I_{\text{max}} = N^2 I_1$.

22.30. Model: Two closely spaced slits produce a double-slit interference pattern with the intensity graph looking like Figure 22.3(b). The intensity pattern due to a single slit diffraction looks like Figure 22.14. Both the spectra consist of a central maximum flanked by a series of secondary maxima and dark fringes.

Solve: (a) The light intensity shown in Figure P22.30 corresponds to a double-slit aperture. This is because the fringes are equally spaced and the decrease in intensity with increasing fringe order occurs slowly.

(b) From Figure P22.30, the fringe spacing is $\Delta y = 1.0 \text{ cm} = 1.0 \times 10^{-2} \text{ m}$. Therefore,

$$\Delta y = \frac{\lambda L}{d}$$
$$\Rightarrow d = \frac{\lambda L}{\Delta y} = \frac{(6.00 \times 10^{-9} \text{ m})(2.5 \text{ m})}{0.010 \text{ m}} = 0.15 \text{ mm}$$

22.48. Visualize: The relationship between the diffraction grating spacing d , the angle at which a particular order of constructive interference occurs θ_m , the wavelength of the light, and the order of the constructive interference m is $d \sin \theta_m = m\lambda$. Also note $N = 1/d$.

Solve: The first order diffraction angle for green light is

$$\theta_1 = \sin^{-1}(\lambda/d) = \sin^{-1}(5.5 \times 10^{-7} \text{ m}/2.0 \times 10^{-6} \text{ m}) = \sin^{-1}(0.275) = 0.278 \text{ rad} = 16^\circ$$

Assess: This is a reasonable angle for a first order maximum.

22.68. Model: A diffraction grating produces an interference pattern like the one shown in Figure 22.8. We also assume that the small-angle approximation is valid for this grating.

Solve: (a) The general condition for constructive-interference fringes is

$$d \sin \theta_m = m\lambda \quad m = 0, 1, 2, 3, \dots$$

When this happens, we say that the light is diffracted at an angle θ_m . Since it is usually easier to measure distances rather than angles, we will consider the distance y_m from the center to the m th maximum. This distance is $y_m = L \tan \theta_m$. In the small-angle approximation, $\sin \theta_m \approx \tan \theta_m$, so we can write the condition for constructive interference as

$$d \frac{y_m}{L} = m\lambda \Rightarrow y_m = \frac{m\lambda L}{d}$$

The fringe separation is

$$y_{m+1} - y_m = \Delta y = \frac{\lambda L}{d}$$

(b) Now the laser light falls on a film that has a series of "slits" (i.e., bright and dark stripes), with spacing

$$d' = \frac{\lambda L}{d}$$

Applying once again the condition for constructive interference:

$$d' \sin \theta'_m = m\lambda \Rightarrow d' \frac{y'_m}{L} = m\lambda \Rightarrow y'_m = \frac{m\lambda L}{d'} = \frac{m\lambda L}{\lambda L/d} = md$$

The fringe separation is $y'_{m+1} - y'_m = \Delta y' = d$.

That is, using the film as a diffraction grating produces a diffraction pattern whose fringe spacing is d , the spacing of the original slits.

8.1 Solution

On the viewing screen, light intensity is a maximum when the two waves interfere constructively. This occurs when

$$d \sin \theta = m\lambda, \quad m=0, \pm 1, \pm 2, \dots \quad (14.11.1)$$

where λ is the wavelength of the light. At $\theta = 45.0^\circ$, $d = 3.20 \times 10^{-4}$ m and $\lambda = 500 \times 10^{-9}$ m, we get

$$m = \frac{d \sin \theta}{\lambda} = 452.5 \quad (14.11.2)$$

Thus, there are 452 maxima in the range $0 < \theta < 45.0^\circ$. By symmetry, there are also 452 maxima in the range $-45.0^\circ < \theta < 0$. Including the one for $m = 0$ straight ahead, the total number of maxima is

$$N = 452 + 452 + 1 = 905 \quad (14.11.3)$$

8.2 Solution

(a) The phase difference ϕ between the two wavefronts is given by

$$\phi = \frac{2\pi}{\lambda} \delta = \frac{2\pi}{\lambda} d \sin \theta \quad (14.11.4)$$

With $\theta = 0.800^\circ$, we have

$$\phi = \frac{2\pi}{(5.00 \times 10^{-7} \text{ m})} (1.00 \times 10^{-4} \text{ m}) \sin(0.800^\circ) = 17.5 \text{ rad} \quad (14.11.5)$$

(b) When θ is small, we make use of the approximation $\sin\theta \approx \tan\theta = y/L$. Thus, the phase difference becomes

$$\phi \approx \frac{2\pi}{\lambda} d \left(\frac{y}{L} \right) \quad (14.11.6)$$

For $y = 4.00 \text{ mm}$, we have

$$\phi = \frac{2\pi}{(5.00 \times 10^{-7} \text{ m})} (1.00 \times 10^{-4} \text{ m}) \left(\frac{4.00 \times 10^{-3} \text{ m}}{1.00 \text{ m}} \right) = 5.03 \text{ rad} \quad (14.11.7)$$

(c) For $\phi = 1/3 \text{ rad}$, we have

$$\frac{1}{3} \text{ rad} = \frac{2\pi}{\lambda} d \sin\theta = \frac{2\pi}{(5.00 \times 10^{-7} \text{ m})} (1.00 \times 10^{-4} \text{ m}) \sin\theta \quad (14.11.8)$$

which gives

$$\theta = 0.0152^\circ \quad (14.11.9)$$

(d) For $\delta = d \sin\theta = \lambda/4$, we have

$$\theta = \sin^{-1} \left(\frac{\lambda}{4d} \right) = \sin^{-1} \left[\frac{5.00 \times 10^{-7} \text{ m}}{4(1.00 \times 10^{-4} \text{ m})} \right] = 0.0716^\circ \quad (14.11.10)$$

8.3 Solution

The path difference between the two rays is

$$\delta = d \sin \theta_1 - d \sin \theta_2 \quad (14.11.11)$$

The condition for constructive interference is $\delta = m\lambda$, where $m = 0, \pm 1, \pm 2, \dots$ is the order number. Thus, we have

$$d(\sin \theta_1 - \sin \theta_2) = m\lambda \quad (14.11.12)$$

8.4 Solution

(a) The average intensity is given by

$$I = I_0 \cos^2 \left(\frac{\phi}{2} \right) \quad (14.11.13)$$

where I_0 is the maximum light intensity. Thus,

$$0.60 = \cos^2 \left(\frac{\phi}{2} \right) \quad (14.11.14)$$

which yields

$$\phi = 2 \cos^{-1} \left(\sqrt{\frac{I}{I_0}} \right) = 2 \cos^{-1} (\sqrt{0.60}) = 78.5^\circ = 1.37 \text{ rad} \quad (14.11.15)$$

(b) The phase difference ϕ is related to the path difference δ and the wavelength λ by

$$\delta = \frac{\lambda \phi}{2\pi} = \frac{(500 \text{ nm})(1.37 \text{ rad})}{2\pi} = 109 \text{ nm} \quad (14.11.16)$$