Problem Set #7: Solutions

Conceptual questions:

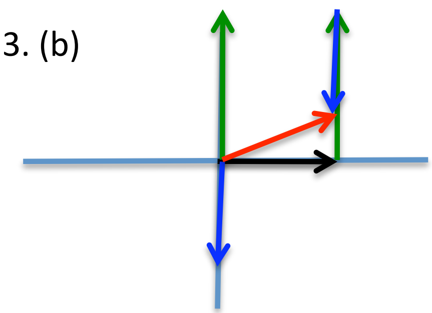
1(a) $X_c = 1/\omega C$; It is halved when frequency is doubled and doubled when frequency is halved

1(b) Yes during the part of the cycle that the current flows towards the battery. See Fig. 36.23 during cycles labeled as "energy from field to circuit". First and third quarter.

2. If voltage leads the current, then current lags the voltage and response is inductive-> $X_L > X_c$ and $\omega > \omega_o$

3. (a) $IX_L > IX_c$-> as above $\omega > \omega_o$

3. (b)



V_{Ro} black, V_{Lo} blue, V_{Co} green, V_o Red

3. (c) V_{ro} about twice as big as V_{co}-V_{lo} $\tan\theta = 1/2$ the voltage leads the current by 26 degrees,

4. $$\cos\phi = R/Z = 1/\sqrt{1+(\omega L - 1/\omega C)^2/R^2}$$

Increases with increasing R, decreases with increasing L and C

36.58. Solve: **(a)** The peak current in a series *RLC* circuit is

$$I = \frac{\mathcal{E}_0}{Z} = \left(\frac{\mathcal{E}_0}{R}\right)\left(\frac{R}{Z}\right)$$

The maximum current is $I_{max} = \mathcal{E}_0/R$ and it occurs only at resonance because this is when the impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$ is the smallest. Using this expression for Z,

$$\frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{1}{\sqrt{1 + \left(\frac{X_L - X_C}{R}\right)^2}} = \frac{1}{\sqrt{1 + \tan^2\phi}} = \frac{1}{\sqrt{\sec^2\phi}} = \cos\phi$$

Thus, $I = I_{max}\cos\phi$.

(b) From Equation 36.44, the average power is

$$P_{avg} = \tfrac{1}{2}I\mathcal{E}_0\cos\phi = \tfrac{1}{2}(I_{max}\cos\phi)\mathcal{E}_0\cos\phi = \left(\tfrac{1}{2}I_{max}\mathcal{E}_0\right)\cos^2\phi = P_{max}\cos^2\phi$$

where $P_{max} = \frac{1}{2}I_{max}\mathcal{E}_0$ is the maximum power the source can deliver to the circuit.

36.59. Solve: **(a)** The resonance frequency is

$$\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f_0 \Rightarrow C = \frac{1}{4\pi^2 f_0^2 L} = \frac{1}{4\pi^2(104.3\times10^6\ \text{Hz})^2(0.200\ \mu\text{H})} = 11.64\ \text{pF}$$

(b) The current produced by the out-of-tune radio station is 0.10% of the resonance current. Therefore,

$$\frac{\mathcal{E}_0}{\sqrt{R^2 + (X_L - X_C)^2}} = (10^{-3})\frac{\mathcal{E}_0}{R} \Rightarrow R^2 + (X_L - X_C)^2 = 10^6 R^2 \cong (X_L - X_C)^2 \Rightarrow |X_L - X_C| = 10^3 R$$

$$\Rightarrow R = \left|10^{-3}\left(\omega L - \frac{1}{\omega C}\right)\right| = \left|10^{-3}\left[2\pi(103.9\ \text{MHz})(0.20\ \mu\text{H}) - \frac{1}{2\pi(103.9\ \text{MHz})(11.6\ \text{pF})}\right]\right| = 1.49\times10^{-3}\ \Omega$$

Assess: The impedance at resonance is $Z = R$ because $X_L = X_C$.

36.60. Solve: **(a)** The resonance frequency is

$$f_0 = 57\ \text{MHz} = \frac{1}{2\pi\sqrt{LC}} \Rightarrow L = \frac{1}{4\pi^2 f_0^2 C} = \frac{1}{4\pi^2 \left(57\times10^6\ \text{Hz}\right)^2 \left(16\times10^{-12}\ \text{F}\right)} = 0.487\ \mu\text{H} \approx 0.49\ \mu\text{H}$$

(b) We have $I_{\text{end frequency}} = \frac{1}{2} I_{\text{resonance}}$. Therefore,

$$\frac{\mathcal{E}_0}{\sqrt{R^2 + \left(X_L - X_C\right)^2}} = \frac{1}{2}\frac{\mathcal{E}_0}{R} \Rightarrow R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 4R^2 \Rightarrow \omega L - \frac{1}{\omega C} = \sqrt{3}R$$

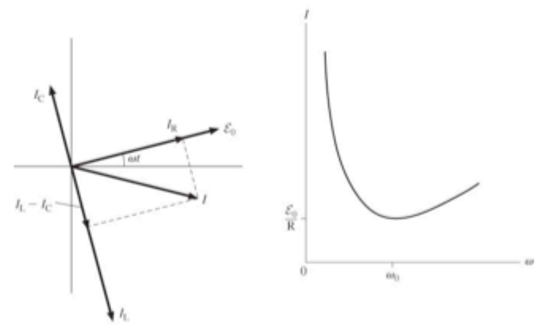
For $f = 60$ MHz,

$$R = \frac{1}{\sqrt{3}}\left[2\pi\left(60\times10^6\ \text{Hz}\right)\left(0.487\ \mu\text{H}\right) - \frac{1}{2\pi\left(60\times10^6\ \text{Hz}\right)\left(16\times10^{-12}\ \text{F}\right)}\right]$$

$$= \frac{1}{\sqrt{3}}\left[183.70\ \Omega\ -\ 165.79\ \Omega\right] = \ 10.3\ \Omega$$

For $f = 54$ MHz, $R = 10.9\ \Omega$. The minimum possible value of the circuit resistance is thus $10.9\ \Omega$.

36.71. Visualize: Voltage is the same for circuit elements in parallel, so start with the $\mathcal{E}_0$ phasor. Please refer to Figure CP36.71.



Solve: **(a)** We see from the above phasor diagram that

$$I^2 = I_R^2 + (I_L - I_C)^2 \Rightarrow \left(\frac{\mathcal{E}_0}{Z}\right)^2 = \left(\frac{\mathcal{E}_0}{R}\right)^2 + \left(\frac{\mathcal{E}_0}{\omega L} - \frac{\mathcal{E}_0}{1/\omega C}\right)^2 \Rightarrow Z = \left[\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2\right]^{-1/2}$$

Since $I = \mathcal{E}_0 / Z$, the current is

$$I = \mathcal{E}_0 \sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2}$$

(b) As $\omega \to 0$ rad/s, $\omega L \to 0\ \Omega$, so $I \to \infty$. That is, the inductor becomes a short for $\mathcal{E}_0$. On the other hand, as $\omega \to \infty$, $I \to \infty$ because now the capacitor becomes a short for $\mathcal{E}_0$.

(c) To find the frequency for which I is minimum, we set $dI/d\omega = 0$. We get

$$\frac{dI}{d\omega} = 0 = \frac{\mathcal{E}_0\left(\frac{1}{2}\right)2\left(\frac{1}{\omega L} - \omega C\right)\left(-\frac{1}{\omega^2 L} - C\right)}{2\sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2}} \Rightarrow \left(\frac{1}{\omega L} - \omega C\right)\left(-\frac{1}{\omega^2 L} - C\right) = 0$$

$$\Rightarrow \frac{1}{\omega L} - \omega C = 0 \Rightarrow \omega^2 = \frac{1}{LC} = \omega_0^2 \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

The resonance frequency is the same as a series RLC circuit.

(d) We know the current is a minimum at $\omega = \omega_0$ and diverges as $\omega \to 0$ or $\omega \to \infty$. Thus the current graph must look more or less as sketched above.

7.1 (a) Current leads by 45 degrees
(b) Current leads voltage circuit capacitive (see Fig. 12.8.1 in problem tips
(c) No current/voltage not in phase
(d) cos45=1/sqrt(2)=.71