Conc. Qs

HW # 12 Solutions

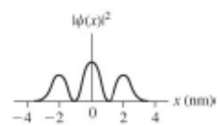
37.8. **(a)** No, you measured the left end first.
(b) Yes, experimenters in S' are at rest relative to the meter stick so they are measuring the proper length and the proper time.

37.9. Yes, the experimenters on the ground will measure the train as length contracted, and if it is going fast enough $L < 80$ m.

40.1. **(a)** At $x \approx 1$ because the probability density graph is higher there.
(b) From Equation 40.14 we see that $\text{Prob}(\text{in } \delta x \text{ at } x) = P(x)\delta x$. If 1,000,000 photons are detected then the number expected in a 1-mm-wide interval is $1{,}000{,}000P(x)\delta x$ where $\delta x = 1.0$ mm. For $x = 0.25$ m, $P(x) = 0.5\ \text{m}^{-1}$ and the number expected is $1{,}000{,}000(0.5\ \text{m}^{-1})(0.0001\ \text{m}) = 500$. For $x = 0.75$ m, $P(x) = 1.5\ \text{m}^{-1}$ and the number expected is $1{,}000{,}000(1.5\ \text{m}^{-1}) \cdot (0.0001\ \text{m}) = 1500$.

40.2. The relationship between probability and probability density is similar to the relationship between mass m and mass density ρ. Regions of higher mass density tell us where mass is concentrated. The mass itself is a more tangible quantity that depends both on the mass density and on the size of a specific piece of material. Similarly, probability density tells us regions in which a particle is more likely, or less likely, to be found. The probability is a definite number between 0 and 1. Probability depends both on the probability density and on the size of the specific region we are considering.

40.3. The probability of finding a particle at position x is determined by $|\psi(x)|^2$.



The electron is most likely to be found at the point or points where $|\psi(x)|^2$ is a maximum. The graph given in the problem shows ψ(x). The figure here shows $|\psi(x)|^2$. Notice that $|\psi(x=0\ \text{nm})|^2 > |\psi(x=\pm 2\ \text{nm})|^2$, even though ψ (x = 0 nm) < 0 in the original graph. So, the electron is most likely to be found at x = 0 nm. The electron is least likely to be found where $|\psi(x)|^2$ is a minimum. From the figure, $|\psi(x)|^2 = 0$ at x = ±1 nm. Thus, the electron is *least* likely to be found at x = ±1 nm.

40.4. **(a)** The probability density is maximum at $x \pm 2$ mm.
(b) We cannot tell where the wave function is most positive; it could be at either $x = 2$ mm or $x = -2$ mm. It will be positive at one and negative at the other.

40.5. The area under the probability density curve must be one. That is, $\int_{-\infty}^{\infty} P(x)dx = 1$. For that to be true for Figure Q40.4, a must be 2 mm^{-1}, because the area of a triangle is half the base times the height.

40.6. Particle 1 because it has a less definite Δx and therefore a more definite $\Delta p = \Delta mv$.

Exercises and Problems

37.39. Model: The hamburger is a classical particle whose rest energy is $E_0 = mc^2$.
Solve: (a) We have

$$E_0 = mc^2 = (200 \times 10^{-3}\ \text{kg})(3.0 \times 10^8\ \text{m/s})^2 = 1.8 \times 10^{16}\ \text{J}$$

(b) The ratio of the energy equivalent to the food energy is

$$\frac{1.8 \times 10^{16}\ \text{J}}{2 \times 10^6\ \text{J}} = 9.0 \times 10^9$$

37.71. Model: Mass and energy are equivalent and given by Equation 37.43.
Solve: (a) The power plant running at full capacity for 80% of the year runs for

$$(0.80)(365 \times 24 \times 3600)\ \text{s} = 2.52 \times 10^7\ \text{s}$$

The amount of thermal energy generated per year is

$$3 \times (1000 \times 10^6\ \text{J/s}) \times (2.52 \times 10^7\ \text{s}) = 7.56 \times 10^{16}\ \text{J} \approx 7.6 \times 10^{16}\ \text{J}$$

(b) Since $E_0 = mc^2$, the mass of uranium transformed into thermal energy is

$$m = \frac{E_0}{c^2} = \frac{7.56 \times 10^{16}\ \text{J}}{(3.0 \times 10^8\ \text{m/s})^2} = 0.84\ \text{kg}$$

37.72. Model: Mass and energy are equivalent and given by Equation 37.43.
Solve: (a) The sun radiates energy for 3.154×10^7 s per year. The amount of energy radiated per year is

$$(3.8 \times 10^{26}\ \text{J/s})(3.154 \times 10^7\ \text{s}) = 1.198 \times 10^{34}\ \text{J/y}$$

Since $E_0 = mc^2$, the amount of mass lost is

$$m = \frac{E_0}{c^2} = \frac{1.198 \times 10^{34}\ \text{J}}{(3.0 \times 10^8\ \text{m/s})^2} = 1.33 \times 10^{17}\ \text{kg} \approx 1.3 \times 10^{17}\ \text{kg}$$

(b) Since the mass of the sun is 2.0×10^{30} kg, the sun loses 6.7×10^{-12} % of its mass every year.
(c) The lifetime of the sun can be estimated to be

$$T = \frac{2.0 \times 10^{30}\ \text{kg}}{1.33 \times 10^{17}\ \text{kg/y}} = 1.5 \times 10^{13}\ \text{years}$$

The sun will not really last this long in its current state because fusion only takes place in the core and it will become a red giant when the core hydrogen is all fused.

39.20. Model: To conserve energy, the emission and the absorption photons must have exactly the energy lost or gained by the atom in the appropriate quantum jumps.
Visualize: The energy of a light quantum is $E = hf = hc/\lambda$.
Solve: **(a)** The wavelength of the emission photon from the $n = 2$ to $n = 1$ transition is

$$\lambda = \frac{hc}{E_2 - E_1} = \frac{(4.14 \times 10^{-15}\ \text{eV s})(3.0 \times 10^{8}\ \text{m/s})}{1.5\ \text{eV}} = 828\ \text{nm}$$

Likewise, $\lambda = 497$ nm for the $3 \rightarrow 2$ transition with $\Delta E = 2.5$ eV, and $\lambda = 311$ nm for the $3 \rightarrow 1$ transition with $\Delta E =$ 4.0 eV.
(b) Because the atom in the ground state is in the $n = 1$ state, the absorption lines correspond to the $1 \rightarrow 2$ and $1 \rightarrow 3$ transitions. The absorption wavelengths are 828 nm and 311 nm. The $2 \rightarrow 3$ transition is not seen in absorption.

39.30. Solve: Photons emitted from the $n = 4$ state start in energy level $n = 4$ and undergo a quantum jump to a lower energy level with $m < 4$. The possibilities are $4 \rightarrow 1$, $4 \rightarrow 2$, and $4 \rightarrow 3$. According to Equation 39.36, the transition $4 \rightarrow m$ emits a photon of wavelength.

$$\lambda = \frac{\lambda_0}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)} = \frac{91.18\ \text{nm}}{\left(\frac{1}{m^2} - \frac{1}{16}\right)}$$

These values are given in the table below.

Transition	Wavelength
$4 \rightarrow 1$	97.3 nm
$4 \rightarrow 2$	486 nm
$4 \rightarrow 3$	1876 nm

40.10. Solve: $|\psi(x)|^2\delta x$ is a probability, which is dimensionless. The units of δx are m, so the units of $|\psi(x)|^2$ are m^{-1} and thus the units of ψ are $\text{m}^{-1/2}$.

40.6. Model: The probability density of finding a photon is directly proportional to the square of the light-wave amplitude $|A(x)|^2$.

Solve: The probability of finding a photon within a narrow region of width δx at position x is

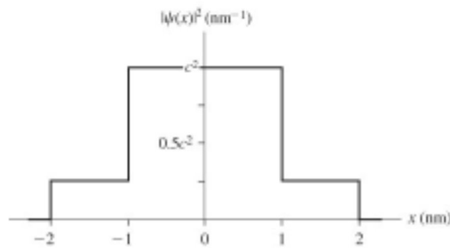
$$\text{Prob(in } \delta x \text{ at } x) \propto |A(x)|^2\,\delta x \Rightarrow \frac{\text{Prob}(\text{in } \delta x_1 \text{ at } x_1)}{\text{Prob}(\text{in } \delta x_2 \text{ at } x_2)} = \frac{|A(x_1)^2|\delta x}{|A(x_2)^2|\delta x}$$

Let N be the total number of photons and N_2 the number of photons detected at x_2 in a width δx. The above equation simplifies to

$$\frac{2000/\text{N}}{N_2/N} = \frac{(10\text{ V/m})^2(0.10\text{ mm})}{(30\text{ V/m})^2(0.10\text{ mm})} \Rightarrow N_2 = \frac{(2000)(30\text{ V/m})^2}{(10\text{ V/m})^2} = 18{,}000$$

40.16. Model: The probability of finding the particle is determined by the probability density $P(x) = |\psi(x)|^2$.

Solve: **(a)** According to Equation 40.18, $\int_{-\infty}^{\infty}|\psi(x)|^2\,dx = 1$. We calculate this integral by drawing the graph of $|\psi(x)|^2$ and finding the area under the curve.



The area of the $|\psi(x)|^2$ versus x graph is

$$\int_{-2\text{nm}}^{-1\text{ nm}}|\psi(x)|^2\,dx + \int_{-1\text{ nm}}^{1\text{ nm}}|\psi(x)|^2\,dx + \int_{1\text{ nm}}^{2\text{ nm}}|\psi(x)|^2\,dx = (0.25c^2)(1\text{ nm}) + (c^2)(2\text{ nm}) + (0.25c^2)(1\text{ nm}) = 2.5c^2$$

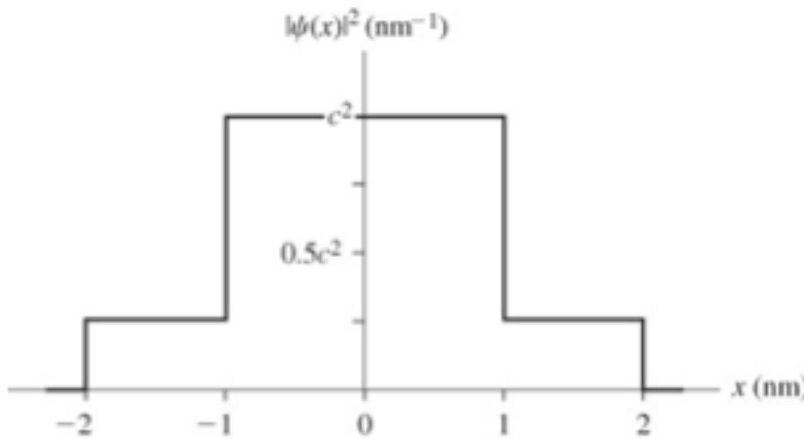
$$\Rightarrow 2.5c^2\text{ nm} = 1 \Rightarrow c = \frac{1}{\sqrt{2.5}}\text{ nm}^{-1/2} = 0.632\text{ nm}^{-1/2}$$

(b) The graph is shown in part (a).
(c) The probability is

$$\text{Prob}(-1.0\text{ nm} \le x \le 1.0\text{ nm}) = \text{area} = (c^2)(2\text{ nm}) = \left(\frac{1}{2.5}\text{ nm}^{-1}\right)(2\text{ nm}) = 0.80$$

40.16. Model: The probability of finding the particle is determined by the probability density $P(x)=|\psi(x)|^2$.

Solve: **(a)** According to Equation 40.18, $\int_{-\infty}^{\infty}|\psi(x)|^2\,dx=1$. We calculate this integral by drawing the graph of $|\psi(x)|^2$ and finding the area under the curve.



The area of the $|\psi(x)|^2$ versus x graph is

$$\int_{-2\text{nm}}^{-1\text{ nm}}|\psi(x)|^2\,dx+\int_{-1\text{ nm}}^{1\text{ nm}}|\psi(x)|^2\,dx+\int_{1\text{ nm}}^{2\text{ nm}}|\psi(x)|^2\,dx=(0.25c^2)(1\text{ nm})+(c^2)(2\text{ nm})+(0.25c^2)(1\text{ nm})=2.5c^2$$

$$\Rightarrow 2.5c^2\text{ nm}=1\Rightarrow c=\frac{1}{\sqrt{2.5}}\text{ nm}^{-1/2}=\ 0.632\text{ nm}^{-1/2}$$

(b) The graph is shown in part (a).
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$$\text{Prob}(-1.0\text{ nm}\le x\le 1.0\text{ nm})=\ \text{area}=(c^2)(2\text{ nm})=\left(\frac{1}{2.5}\text{ nm}^{-1}\right)(2\text{ nm})=0.80$$

40.25. Model: Protons are subject to the Heisenberg uncertainty principle.

Solve: We know the proton is somewhere within the nucleus, so the uncertainty in our knowledge of its position is at most $\Delta x = L = 4.0$ fm. With a finite Δx, the uncertainty Δp_x is given by the uncertainty principle:

$$\Delta p_x = m\Delta v_x = \frac{h/2}{\Delta x} \Rightarrow \Delta v_x = \frac{h}{2mL} = \frac{6.63\times10^{-34}\ \text{J s}}{2\left(1.67\times10^{-27}\ \text{kg}\right)\left(4.0\times10^{-15}\ \text{m}\right)} = 5.0\times10^{7}\ \text{m/s}$$

Because the average velocity is zero, the best we can say is that the proton's velocity is somewhere in the range -2.5×10^{7} m/s to 2.5×10^{7} m/s. Thus the smallest range of speeds is 0 to 2.5×10^{7} m/s.

Problem 1

1. **For the light cone to be located at 45 degrees the units of distance in Jim's frame should be lightseconds. The same is true for the units of Mary's frame.**

$$\tan\theta = \beta$$

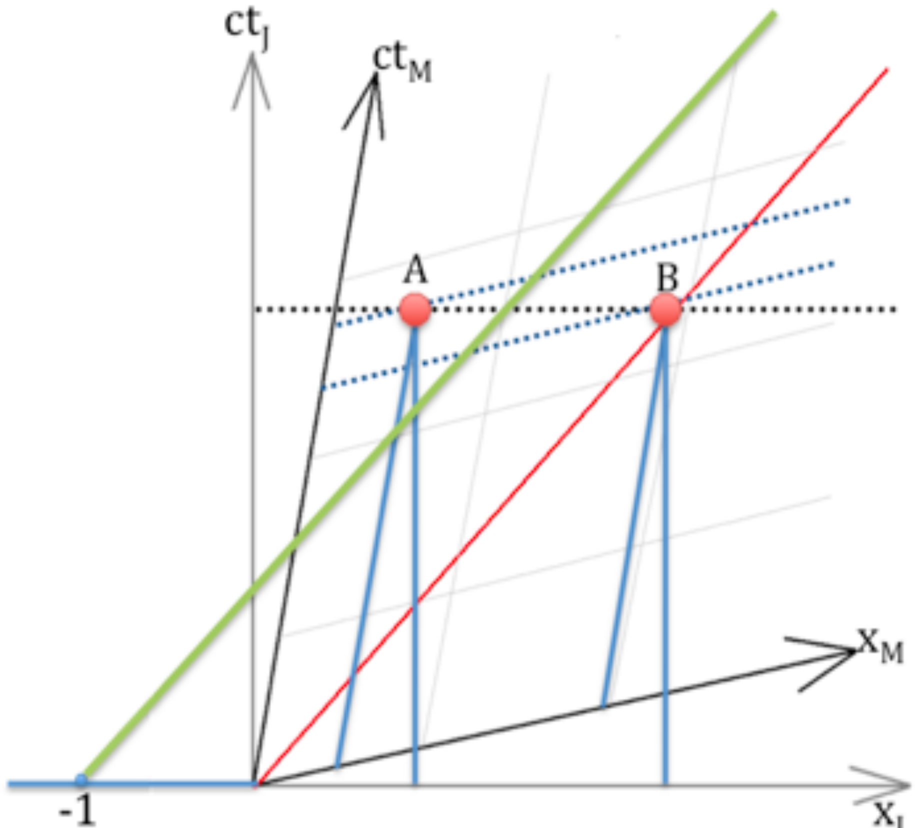
2. $$\theta = \tan^{-1}(\sqrt{3}/2)$$

   $$\theta = 41 \text{ degrees}$$

   $\phi=\theta$, because the light cone should make equal angles with the Mary's time and space axis to preserve the invariance of the speed of light in all frames.

3. **Since $\gamma=2$, Mary's coordinates will shrink by a factor of two. So the scale will be one cm.**
4. **Events A and B are shown in the Figure below. The dotted axis going through A and B is Jim's simultaneity axis. Both A and B are on it and are therefore simultaneous in his frame. The other two dotted axes are the locus of events simultaneous with A (upper line) and the locus of events simultaneous with B (lower line) in Mary's frame. It is clear that event B happens before event A in her frame.**

5. **Mary traveled a distance $x_J=\beta ct_J=2.6$ lightseconds. In Mary's frame it took is $3/\gamma=1.5$ seconds**
6. **The green line represents the light cone from location ($ct_{oJ}=0, x_J=-1$). It will intersect Jim's time axis in $ct_J=1$, i.e after 1 second. When it intersects Mary's time axis Mary would have moved in Jim's frame a distance $x_J=\beta t_J=.86$ lightseconds.**



ct_J
ct_M
A
B
x_M
-1
x_J

Problems not assigned

40.25. Model: Protons are subject to the Heisenberg uncertainty principle.
Solve: We know the proton is somewhere within the nucleus, so the uncertainty in our knowledge of its position is at most $\Delta x = L = 4.0$ fm. With a finite Δx, the uncertainty Δp_x is given by the uncertainty principle:

$$\Delta p_x = m\Delta v_x = \frac{h/2}{\Delta x} \Rightarrow \Delta v_x = \frac{h}{2mL} = \frac{6.63\times10^{-34}\text{ J s}}{2(1.67\times10^{-27}\text{ kg})(4.0\times10^{-15}\text{ m})} = 5.0\times10^7\text{ m/s}$$

Because the average velocity is zero, the best we can say is that the proton's velocity is somewhere in the range -2.5×10^7 m/s to 2.5×10^7 m/s. Thus the smallest range of speeds is 0 to 2.5×10^7 m/s.

40.42. Model: A pulse is a wave packet, hence it must satisfy the relation $\Delta f\,\Delta t \approx 1$.
Solve: **(a)** The wavelength of 600 nm corresponds to a center frequency of

$$f_0 = \frac{c}{\lambda_0} = \frac{3\times10^8\text{ m/s}}{600\times10^{-9}\text{ m}} = 5.0\times10^{14}\text{ Hz}$$

(b) The pulse duration is 6.0 fs, that is, $\Delta t = 6.0 \times 10^{-15}$ s. Because the time period of the center frequency is $T = f_0^{-1} = 2.0\times10^{-15}$ s, the number of cycles in the pulse is

$$\frac{\Delta t}{T} = \frac{6.0\times10^{-15}\text{ s}}{2.0\times10^{-15}\text{ s}} = 3$$

(c) The frequency bandwidth for a 6.0-fs-long pulse is

$$\Delta f = \frac{1}{\Delta t} = \frac{1}{6.0\times10^{-15}\text{ s}} = 1.67\times10^{14}\text{ Hz}$$

This bandwidth is centered on $f_0 = 5.00 \times 10^{14}$ Hz, so the necessary range of frequencies from $f_0 - \frac{1}{2}\Delta f$ to $f_0 + \frac{1}{2}\Delta f$ is from 4.17×10^{14} Hz to 5.83×10^{14} Hz.
(d) The pulse travels at speed c, so the length is $\Delta x = c\Delta t = (3.0 \times 10^8\text{ m/s})(6.0 \times 10^{-15}\text{ s}) = 1.8 \times 10^{-6}\text{ m} = 1.8$ μm. This is 3λ, in agreement with the finding that there are 3 cycles in the pulse.
(e) The graph has three oscillations spanning $1.8\ \mu\text{m} = 3\lambda$.