HW # 10 Solutions

Conceptual Questions

24.5. Equation 24.13 gives the width of the spot as $w_{min} = \dfrac{2.44\lambda f}{D}$. The minimum spot width is achieved for incoming parallel rays when the lens-to-screen distance is f. (a) Decreasing λ decreases w. (b) Decreasing D increases w. (c) Decreasing f decreases w. (d) Decreasing the lens-to-screen distance means the screen is no longer in the focal plane and the spot is larger than its minimum in the focal plane.

24.7. Equation 24.14 gives the angular resolution of a telescope as $\theta_{min} = \dfrac{1.22\lambda}{D}$. Two objects are marginally resolvable if $\alpha = \theta_{min}$ (our original condition) and are resolvable if $\alpha > \theta_{min}$. (a) Yes, using a shorter wavelength decreases θ_{min} which makes $\alpha > \theta_{min}$, so the resolution is improved. (b) Changing the focal length does not affect the resolution because f is not in the formula for resolution. (c) Yes, using a larger lens increases the resolution; increasing D decreases θ_{min} which makes $\alpha > \theta_{min}$. (d) Changing magnification does not affect the resolution.

25.4. Model: The angles of incidence for which diffraction from parallel planes occurs satisfy the Bragg condition.

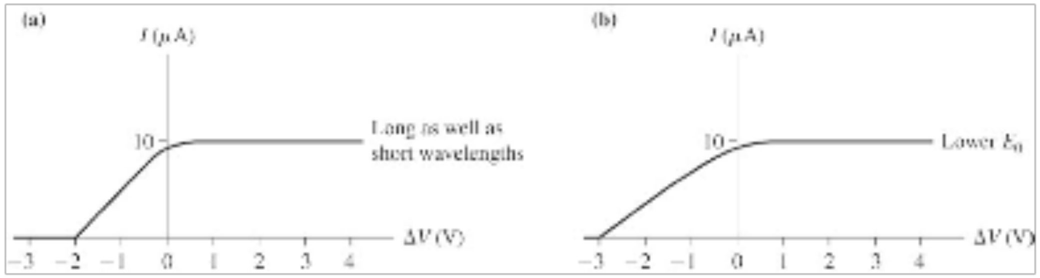
Solve: The Bragg condition is $2d\cos\theta_m = m\lambda$, where m = 1, 2, 3, … For first and second order diffraction,

$$2d\cos\theta_1 = (1)\lambda \qquad 2d\cos\theta_2 = (2)\lambda$$

Dividing these two equations,

$$\frac{\cos\theta_2}{\cos\theta_1} = 2 \Rightarrow \theta_2 = \cos^{-1}(2\cos\theta_1) = \cos^{-1}(2\cos 68^\circ) = 41^\circ$$

39.5. The photoelectric current depends on the potential difference ΔV between the two electrodes, the nature of the cathode metal, and the intensity of the light.

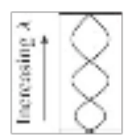


Solve: **(a)** According to classical physics, there is no dependence on the light's wavelength. If the light intensity remains constant (same amount of energy falling on the metal cathode), the photocurrent will be unchanged.

(b) The maximum kinetic energy of the electrons emitted from a cathode is $K_{max} = E_{elec} - E_0$. If E_0 is smaller for a different metal, the emitted electrons will have a higher kinetic energy and thus the stopping potential will be larger.

39.6. Lower speed. Since $\lambda_1 = \lambda_2$ then $f_1 = f_2$. And $(E_{elec})_1 = hf_1$ is equal to $(E_{elec})_2 = hf_2$. But $(E_0)_1 > (E_0)_2$; $(K_{max})_1 = E_{elec} - (E_0)_1$ and $(K_{max})_2 = E_{elec} - (E_0)_2$, so $(K_{max})_1 < (K_{max})_2$ and $v_1 < v_2$.

39.9. The particle has to set up a standing wave inside the box. An $n = 3$ wave has three antinodes. In this case, however, the particle's wavelength is not constant. The particle's de Broglie wavelength is longer near the top of the box, where its velocity is slower, and shorter near the bottom of the box, where its velocity is faster. The spacing between two adjacent nodes on a standing wave is $\lambda/2$, so the nodes will be further apart near the top of the box and closer together near the bottom. This leads to a standing wave like the one shown in the figure.



Exercises and Problems

23.80. Model: Use the ray model of light.

Solve: **(a)** The time (t) is the time to travel from A to the interface (t_1) and from the interface to B (t_2). That is,

$$t = t_1 + t_2 = \frac{d_1}{v_1} + \frac{d_2}{v_2} = \frac{d_1}{c/n_1} + \frac{d_2}{c/n_2} = \frac{n_1 d_1}{c} + \frac{n_2 d_2}{c} = \frac{n_1}{c}\sqrt{x^2 + a^2} + \frac{n_2}{c}\sqrt{(w-x)^2 + b^2}$$

(b) Because t depends on x and there is only one value of x for which the light travels from A to B in the least possible amount of time, we have

$$\frac{dt}{dx} = 0 = \frac{n_1 x}{c\sqrt{x^2 + a^2}} - \frac{n_2 (w-x)}{c\sqrt{(w-x)^2 + b^2}}$$

The solution (hard to do!) would give x_{min}.

(c) From the geometry of the figure,

$$\frac{x}{\sqrt{x^2 + a^2}} = \frac{x}{d_1} = \sin\theta_1 \qquad \frac{w-x}{\sqrt{(w-x)^2 + b^2}} = \frac{w-x}{d_2} = \sin\theta_2$$

Thus, the condition of part (b) becomes

$$\frac{n_1}{c}\sin\theta_1 - \frac{n_2}{c}\sin\theta_2 = 0 \Rightarrow n_1 \sin\theta_1 = n_2 \sin\theta_2$$

24.43. Model: The width of the central maximum that accounts for a significant amount of diffracted light intensity is inversely proportional to the size of the aperture. The lens is an aperture that focuses light.

Solve: To focus a laser beam, which consists of parallel rays from $s = \infty$, the focal length needs to match the distance to the target: $f = L = 5.0$ cm. The minimum spot size to which a lens can focus is

$$w = \frac{2.44\lambda f}{D} \Rightarrow 5.0\times10^{-6}\text{ m} = \frac{2.44(1.06\times10^{-5}\text{ m})(5.0\times10^{-2}\text{ m})}{D} \Rightarrow D = 2.6\text{ cm}.$$

25.42. Model: This is an integrated problem that uses concepts from Chapter 22. There are two L's in the problem: L in Chapter 22 refers to the screen distance from the slits, and the L we want here is the length of the box. The wavelength of the neutron determined by the two-slit pattern is the same as the wavelength in the confined box.

Visualize: The figure shows $L_{box} = 2\lambda$.

We also need Equation 22.6: $y_m = \dfrac{m\lambda L_{screen}}{d}$. Also from the figure we see that $y_2 = 0.20\times10^{-3}\text{m}$. We are given $L_{screen} = 2.0\text{ m}$ and $d = 15\times10^{-6}\text{m}$.

Solve: Solve Equation 22.6 for λ.

$$\lambda = \frac{dy_m}{mL_{screen}}$$

$$L_{box} = 2\lambda = 2\frac{dy_m}{mL_{screen}} = 2\frac{(15\times10^{-6}\text{ m})(0.20\times10^{-3}\text{ m})}{(2)(2.0\text{ m})} = 1.5\text{ nm}$$

Assess: The two pieces of this problem fit together and make sense together.

25.43. Model: Electrons have a de Broglie wavelength given by $\lambda = h/p$. Trapped electrons in the confinement layer behave like a de Broglie wave in a closed-closed tube or like a string fixed at both ends.

Solve: **(a)** The four longest standing-wave wavelengths in the layer are $\lambda = 2L$, L, $\frac{2}{3}L$, and $\frac{1}{2}L$. This follows from the general relation for closed-closed tubes: $\lambda = 2L/n$. Thus, λ = 10.0 nm, 5.00 nm, 3.33 nm, and 2.50 nm.

(b) We have

$$p = mv = \frac{h}{\lambda} \Rightarrow v = \frac{h}{m\lambda} = \frac{6.63\times10^{-34}\text{ Js}}{(9.11\times10^{-31}\text{ kg})\lambda} = \frac{0.7278\times10^{-3}\text{ m}^2\text{/s}}{\lambda}$$

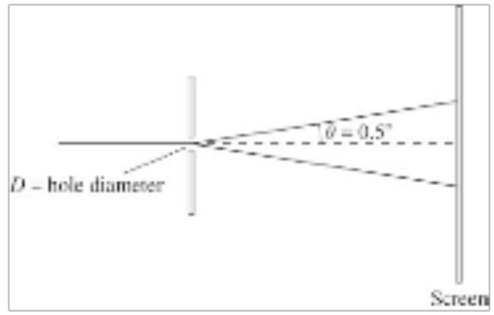
Using the above four longest values of λ we get the four smallest values of v. Thus,

$$v_1 = \frac{0.7278\times10^{-3}\text{ m}^2\text{/s}}{10.0\times10^{-9}\text{ m}} = 7.28\times10^4\text{ m/s}$$

$v_2 = 1.46\times10^5\text{ m/s}$, $v_3 = 2.18\times10^5\text{ m/s}$, and $v_4 = 2.91\times10^5\text{ m/s}$.

39.46. Model: Electrons have both particle-like and wave-like particles.

Visualize:



Solve: The kinetic energy of the electrons is

$$K_f = \tfrac{1}{2}mv^2 = K_i + e\Delta V = 0\text{ J} + e(250\text{ V}) = (1.60\times10^{-19}\text{ C})(250\text{ V}) = 4.00\times10^{-17}\text{ J}$$

$$\Rightarrow\ v = \sqrt{\frac{2(4.00\times10^{-17}\text{ J})}{9.11\times10^{-31}\text{ kg}}} = 9.37\times10^{6}\text{ m/s}$$

The de Broglie wavelength at this speed is

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}\text{ J s}}{(9.11\times10^{-31}\text{ kg})(9.37\times10^{6}\text{ m/s})} = 7.77\times10^{-11}\text{ m}$$

Circular-aperture diffraction produces the first minimum, defining the edge of the central maximum, at $\theta = 1.22\lambda/D$ for small angles, as is the case here. The diameter of the opening is

$$D = \frac{1.22\,\lambda}{\theta} = \frac{1.22(7.77\times10^{-11}\text{ m})}{(0.5^\circ/180^\circ)\pi\text{ rad}} = 1.09\times10^{-8}\text{ m}$$

Assess: The diameter of the hole that diffracts the electron beam is small indeed, and understandably so.