Problem Set # 5

See also strategy for solving problems at the end of the Set

Conceptual Questions 35.10, Exercises and Problems 35.51, 35.53, 35.55, 35. 62

Problem 5.1

Suppose the electric field of a plane electromagnetic wave is given by

$$\vec{\mathbf{E}}(z,t) = E_0 \cos(kz - \omega t)\,\hat{\mathbf{i}}$$

Find the following quantities:

(a) The direction of wave propagation.

(b) The corresponding magnetic field $\vec{\mathbf{B}}$.

Problem 5.2

Verify that, for $\omega = kc$,

$$E(x,t) = E_0 \cos(kx - \omega t)$$
$$B(x,t) = B_0 \cos(kx - \omega t)$$

satisfy the one-dimensional wave equation:

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\begin{Bmatrix} E(x,t) \\ B(x,t) \end{Bmatrix} = 0$$

Problem 5.3

A parallel-plate capacitor with circular plates of radius R and separated by a distance h is charged through a straight wire carrying current I, as shown in the Figure 13.12.1:

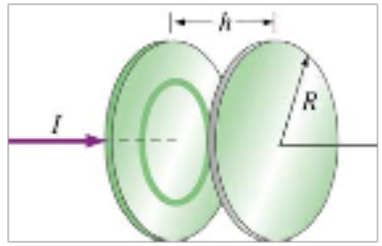

Figure 13.12.1 Parallel plate capacitor

(a) Show that as the capacitor is being charged, the Poynting vector $\vec{\mathbf{S}}$ points radially inward toward the center of the capacitor.

(b) By integrating $\vec{\mathbf{S}}$ over the cylindrical boundary, show that the rate at which energy enters the capacitor is equal to the rate at which electrostatic energy is being stored in the electric field.

Hint: The Electric field E of a capacitor is given by

$$E=Q/(\varepsilon_o \pi R^2)$$

Problem 5.4 :

Can parallel electric and magnetic fields make up an electromagnetic wave in vacuum?

Problem 5.5:

4. Explain why the reception for cellular phones often becomes poor when used inside a steel-framed building.

Problem-Solving Strategy: Traveling Electromagnetic Waves

This chapter explores various properties of the electromagnetic waves. The electric and the magnetic fields of the wave obey the wave equation. Once the functional form of either one of the fields is given, the other can be determined from Maxwell's equations. As an example, let's consider a sinusoidal electromagnetic wave with

$$\vec{\mathbf{E}}(z,t) = E_0 \sin(kz - \omega t)\hat{\mathbf{i}}$$

The equation above contains the complete information about the electromagnetic wave:

1. Direction of wave propagation: The argument of the sine form in the electric field can be rewritten as $(kz - \omega t) = k(z - vt)$, which indicates that the wave is propagating in the $+z$-direction.

2. Wavelength: The wavelength λ is related to the wave number k by $\lambda = 2\pi / k$.

3. Frequency: The frequency of the wave, f, is related to the angular frequency ω by $f = \omega / 2\pi$.

4. Speed of propagation: The speed of the wave is given by

$$v = \lambda f = \frac{2\pi}{k} \cdot \frac{\omega}{2\pi} = \frac{\omega}{k}$$

   In vacuum, the speed of the electromagnetic wave is equal to the speed of light, c.

5. Magnetic field $\vec{\mathbf{B}}$: The magnetic field $\vec{\mathbf{B}}$ is perpendicular to both $\vec{\mathbf{E}}$ which points in the $+x$-direction, and $+\hat{\mathbf{k}}$, the unit vector along the $+z$-axis, which is the direction of propagation, as we have found. In addition, since the wave propagates in the same direction as the cross product $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$, we conclude that $\vec{\mathbf{B}}$ must point in the $+y$-direction (since $\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$).

Since $\vec{\mathbf{B}}$ is always in phase with $\vec{\mathbf{E}}$, the two fields have the same functional form. Thus, we may write the magnetic field as

$$\vec{\mathbf{B}}(z,t) = B_0 \sin(kz - \omega t)\hat{\mathbf{j}}$$

where B_0 is the amplitude. Using Maxwell's equations one may show that $B_0 = E_0(k/\omega) = E_0/c$ in vacuum.

6. The Poytning vector: Using Eq. (13.6.5), the Poynting vector can be obtained as

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}} = \frac{1}{\mu_0}\left[E_0 \sin(kz - \omega t)\hat{\mathbf{i}}\right] \times \left[B_0 \sin(kz - \omega t)\hat{\mathbf{j}}\right] = \frac{E_0 B_0 \sin^2(kz - \omega t)}{\mu_0}\hat{\mathbf{k}}$$

7. Intensity: The intensity of the wave is equal to the average of S:

$$I = \langle S \rangle = \frac{E_0 B_0}{\mu_0}\langle \sin^2(kz - \omega t)\rangle = \frac{E_0 B_0}{2\mu_0} = \frac{E_0^2}{2c\mu_0} = \frac{cB_0^2}{2\mu_0}$$

8. Radiation pressure: If the electromagnetic wave is normally incident on a surface and the radiation is completely *reflected*, the radiation pressure is

$$P = \frac{2I}{c} = \frac{E_0 B_0}{c\mu_0} = \frac{E_0^2}{c^2 \mu_0} = \frac{B_0^2}{\mu_0}$$