## Problem 1

Read below the story of Mr. Sceptic visiting Einsteinville and answer the questions below.
Assume that in Einsteinville.

- The speed of light is 20 miles $/ \mathrm{hr}$
- All cars are boxes with height and width of 1 meter and length 3 meters
- The runner is running with speed of 10 miles $/ \mathrm{hr}$.


## Answer the following:

1. Mr. Sceptic's car in his real world reaches $60 \mathrm{miles} / \mathrm{hr}$ in 10 seconds. What is the reason he is stuck between 17 and 18 miles $/ \mathrm{hr}$. (See paragraph marked in the text by 1.$)$
2. What is the aspect ratio (aspect ratio is the ratio of the length of the height of the car) of a parked car he sees when he passes it while moving at 18 miles $/ \mathrm{hr}$ (see paragraph marked in the text by 2 ).
3. Paragraph marked 3 states that Mr. Sceptic sees a car (assume that it is standing on a red light while Mr. Sceptic moves at 18 miles/hr) "Stretched out as ...rubber". Explain the statement. What are the dimensions of the car that Mr. Sceptic sees?
4. Paragraph marked 4 states that when Mr. Sceptic slows down (assume that he stops) everything appears normal except for a car passing him that "appears squashed". If the car is moving at 12 miles $/ \mathrm{hr}$ what is the car aspect ratio that Mr. Sceptic sees.
5. Paragraph marked 5 states that when Mr. Sceptic drives at the same speed with the runner, the runner looks normal, despite moving at $1 / 2$ the speed of light. Explain why.
6. Paragraph marked 6 states that after driving for 20 minutes, the church clock shows 4:50, while Mr. Sceptic's watch 4:36. What can you say about the speed of Mr. Sceptic's car. (Assume that it was moving at constant speed).

Again we want to illustrate this with a little story, only this time it cannot be drawn from everyday life. This is simply because it is difficult for us to imagine a motor-car travelling down a road at about 170,000 miles $/ \mathrm{sec}$., whilst at the same time the driver is making contemplative observations about his surroundings! We leave everything as it is, with one exception: we slow down the velocity of light considerably so that instead of its familiar 186,000 miles $/ \mathrm{sec}$., it now only amounts to 20 miles $/ \mathrm{hr}$.

We begin our story by introducing Mr Tom Sceptic who, having read our book up to this point, finds the ideas of the relativity theory 'absolutely incredible'. At the moment he is on a business trip, and somewhere along the main road he pulls up to have a short rest. He even drops off to sleep for a while. Sud-
denly he awakens from his slumber. 'Four o'clock already!' he discovers after a quick businesslike glance at his watch. 'But I have to reach town C today.'

Mr Sceptic starts his car and moves off. Since the scenery is monotonous and flat, his attention is concentrated on the road. Whilst travelling in top gear at full throttle he glances at the speedometer: the needle dances 1 between 17 and 18 miles $/ \mathrm{hr}$ ! 'But that can't be correct, the speedometer must be broken. I must call at a garage tomorrow and have it fixed.' He is a little annoyed, since the car had only been serviced the previous week.

While he is still grumbling to himself, another car comes towards him. He had already been surprised at the unusually sparse amount of traffic on what was usually a very busy highway. However, now he suffered a considerable shock and had to look several times in order to believe what he saw. The car approaching him seemed to have suffered a severe accident. The whole car appeared to be completely squashed. Sceptic was surprised that it could still keep going. As it passed him he had an even greater shock, the driver of the car appeared to be squashed flat, just like the 'knave of clubs' in a pack of cards. The stranger didn't even look at him as he passed.

Before Sceptic had recovered from this shock, more vehicles passed him, including a truck. All appeared to be squashed together like accordions. Even a few pedestrians he passed appeared to be as flat as flounders! Sceptic thought that everything appeared as one sees things in a distorting mirror in the fairgrounds. Next he sees a long distance runner in a track suit who also appears to be flat. Even the sprint which he is starting does* not seem to do him much good since he moves only slightly faster. In addition, he becomes even flatter than before.
Finally Sceptic comes to town although he cannot find a signpost. He can see, however, that the towers and houses of the town

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appear to be very high. They, too, appear as if seen through a distorting mirror. A small side road leads into a very broad main street. In this narrow street on the left Sceptic sees a car which is not flattened. On the contrary, it is stretched out as if it were made of rubber. Just in front of Sceptic it turns into the broad main street. He can hardly believe his eyes; as the car turns it swells to its normal width and now approaches him on the opposite side of the road with a squashed front! Tom Sceptic cannot contain his amazement any longer.
Then he thinks that he can see a signpost at the second road junction ahead. Feeling relieved, he puts his foot on the brake and, whilst slowing down, his surroundings again change. The human 'flounders' become normal, the crossroads become broader, and the walls of the houses expand. Everything, in fact, regains a completely 'normal' appearance, with the exception of a car that is just passing and still appears to be a bit squashed. Now even the pedestrians around him look like 'real' people. The signpost only says 'Lorentz Street' but Tom Sceptic is thinking of anything but the Lorentz-transformation. The clock on the church tower indicates that it is already half-past four. 'That's funny, my wrist watch must be slow.' Sceptic sighs and puts it right.

He decides to continue with his journey. Just before he starts off again, the sprinter whom he had seen before overtakes him. Again he appears to be a little flattened, but not nearly so much as before. Sceptic decides to catch him up and ask him a few questions. As he speeds along in pursuit of the runner, people at the edge of the street once again appear flattened; the trees, houses and neighbouring streets appear to shrink in the direction he is moving and to expand sideways. 'It's started again, a completely crazy world!' sighs the motorist in despair.

Meanwhile he has caught up with the runner in the track suit. He starts the con-
versation politely. 'Good day, my name is Sceptic.' The runner smiles amiably at him. 'Pleased to meet you, my name is Miller. Can I help you?'
Sceptic, driving at the same speed at which Miller is running now notices that the man with the ordinary name no longer looks out of the ordinary; he doesn't appear to be flattened or drawn out; just a normal man like anybody else. Sceptic thinks quickly and replies: 'Yes, please, can you tell me what this peculiar town is?'
'This "peculiar town" is called Einstein town, and we are all very proud of it,' Miller replies factually and a little coolly.
'Oh, don't misunderstand me, I didn't mean any offence,' stutters Sceptic, feeling somewhat embarrassed. 'But you know everything here seems to me to be peculiarly flattened,' and adds timidly, 'I even saw you flattened before, although now you appear to be quite normal.'
'Whatever makes you think that this should be strange? You seem to be an intelligent person. Are you trying to tell me that you've never heard of the Lorentz contraction? Or are you from one of those countries where it is sometimes referred to as the Fitzgerald contraction? It really is elementary knowledge of physics.'

Tom Sceptic's head is spinning; he has only understood that 'contraction' means shrinking. However, in order not to disappoint Miller he nods in agreement but feels somewhat embarrassed.
'Perhaps you'd like to ride with me in the car for a while? You must feel tired running all the time.' Miller grins and jumps in, adding: 'It's quite all right for you to go a bit faster now.'

Sceptic accelerates away but he notices that he cannot go faster than 18 miles $/ \mathrm{hr}$. Miller smiles: 'Yes, if it were not for the speed of light.' Also, glancing at the practically paper-thin passers-by he explains: 'A result of the Lorentz transformation ...'

At last the penny had dropped for Sceptic. 'Might I ask then, what is the velocity of light here?'

Miller looks at him somewhat astonished. 'Why? "c" equals 20 miles $/ \mathrm{hr}$, as it does everywhere. Any child knows that.'
(At this point let us not forget that Sceptic has been removed to a world in which light propagates at such a low velocity. In our world, of course, the value of c remains at the value of about 186,000 miles $/ \mathrm{sec}$.)
'So that is the reason why I can only drive so slowly. I'm being limited by the speed of light!' Sceptic suddenly feels very proud for thinking of this explanation.
Miller from Einstein town laughs heartily and says, 'Well, you really are a clown. You behave as if you had just invented the multiplication table.' And he thinks to himself: 'This Sceptic must surely live behind the moon.'

In the meantime, with Sceptic's car, they arrive at the middle of the town. The town is much larger than at first appeared. They had been travelling for twenty minutes. 'For goodness' sake,' the driver calls out, as he looks at the clock on the church tower which is at ten minutes to five, although his own watch shows four thirty-four.
'Damn, now which of the two is correct?' Sceptic growls angrily. Again Miller laughs: 'Why, both, of course! Or did we by any chance not move relative to the church tower?'

Tom Sceptic swallows twice, again feeling embarrassed. 'Yes, but ...'
'What do you want then! Think of time dilation! While we were travelling, our watches ran slowly. Look, my watch gives the same time as yours.'

Suddenly he understands the Lorentz-contraction, which, in a reference system moving at about $90 \%$ of the velocity of light, allows objects to shrink to about half their size. He


Tom Sceptic has been transferred to a world in which the velocity of light, c, amounts to only $20 \mathrm{mis} /$ hour. Suddenly he passes in the road a car which appears to be completely compressed. At the steering wheel sits a driver who appears to be squashed flat.


This was the scene that presented itself to Sceptic as he drove at a speed of approximately $17 \mathrm{mls} /$ hour through Einstein-town.
is now no longer surprised by the fact that the people in Einstein town appear to be flat when seen from a moving motor-car; he even realizes that to them even he must also appear to look flat. He knows now that at velocities comparable to $c$, time passes much more slowly in a moving system than in one that is in a state of rest. A watch travelling at the velocity of light would appear to stop, even though it was still working perfectly. These
effects arise from Einstein's 'time dilation'.
Miller's voice jerks Sceptic from his thoughts. 'Then I will bid you farewell, for my nephew is standing over there.' Sceptic is surprised beyond measure yet again. 'That old man over there is your nephew? But surely he is a good twenty years older than you!'

Miller smiles: 'So what! Look, nothing is simpler; my nephew works in the town
administration. All day long he sits behind his files. I, on the other hand, am constantly moving. While in training, I often run with speeds of 0.8 to 0.9 c . In addition, I drive a fast sports car which has a maximum speed of 19 miles $/ \mathrm{hr}$. Because of this, I age much more slowly than my nephew. Such is life; think of the Lorentz transformation. Goodbye, and safe journey home!'
Now it was Tom Sceptic's turn to grin: 'This Special Theory of Relativity is a wonderful theory, clear, self-evident, but tremendously important. Thank goodness that the velocity of light is unattainably high at home! Thus, nothing of the sort that happens here
in Einstein town can happen to us. Thank you for the interesting discussion Mr Miller.' During the last sentence Sceptic bows in Miller's direction and blows a final 'farewell' on his hooter.
A loud noise from a hooter startles him greatly: during his short sleep behind the wheel he had accidentally leaned on it. Did he dream all this? Of course, but it was nevertheless a sensible and scientifically correct dream. Future generations of space travellers will no doubt experience some of these consequences which can be described by the equations of the Lorentz transformation.

