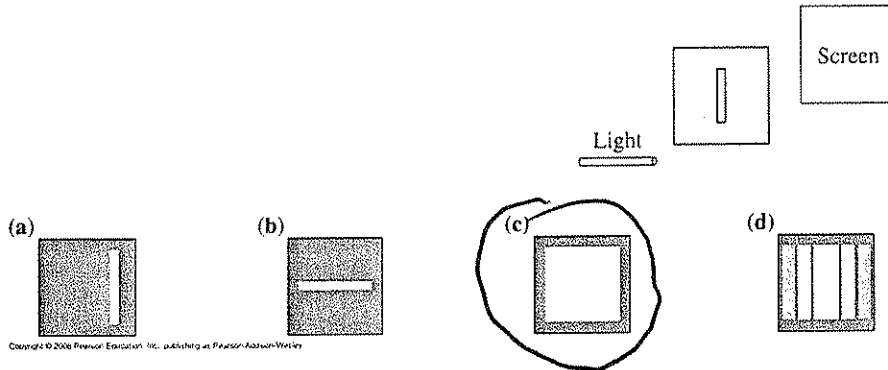


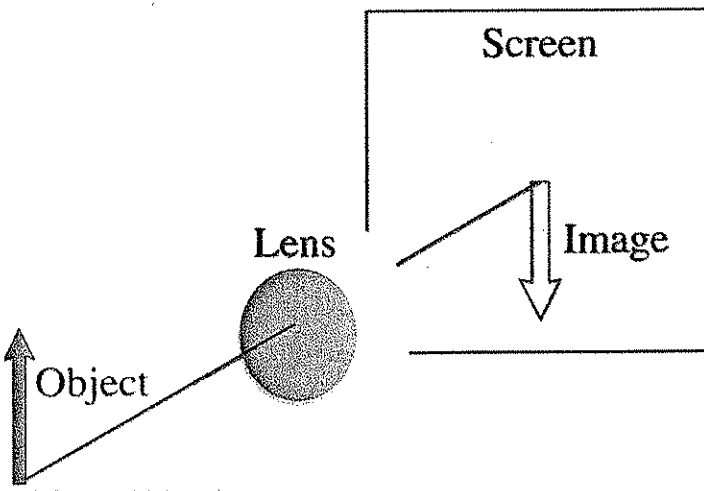
Problem 1: Short and Multiple Choice Questions (Total 55)

1.1 A long thin light bulb illuminates a vertical aperture. Which pattern of light do you see on a viewing screen behind the aperture? (5)



1.2 A lens produces a sharply-focused inverted image on a screen. What will you see on the screen if the lens is removed? (3)

- a. The image will be inverted and blurry.
- b. The image will remain the same but much dimmer.
- c. There will not be image at all.
- d. The image will be right-side-up and sharp.
- e. The image will be right-side-up and blurry.

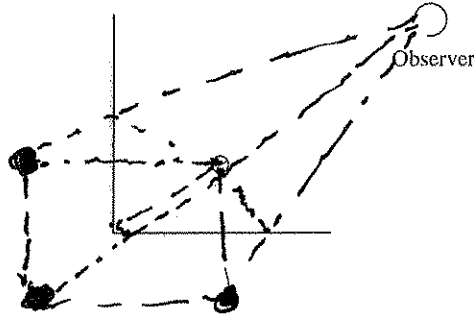


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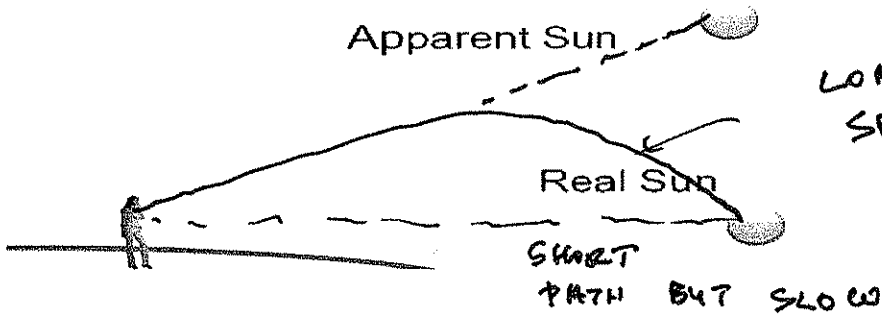
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1.3 Two plane mirrors form a right angle. How many images of the ball you can see in the mirror. Mark their approximate location in the picture. (6)

THREE



1.4 You are looking at the Sun near your horizon but you see it higher (apparent Sun). You know that the thinner the air the faster the speed of light (or equivalently the smaller the index of refraction). Can you explain why you see the Sun higher in the sky than its real position? (Make a sketch showing the ray path). (10)



LONG PATH BUT
SPEED HIGHER AT
THINNER AIR.
FERMAT PRINCIPLE
OF LEAST TIME.

1.5 An astronomer is trying to observe two distant stars. When he looks through a green filter the stars are barely resolved. Should he order a blue or a red filter to improve the resolution of his observations. Explain your answer. (Assume that the his telescope resolution is not affected by the atmosphere) (3)

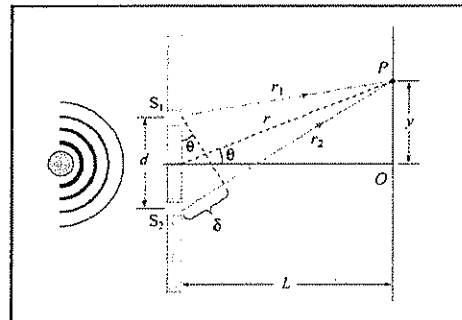
BLUE FILTER - RAYLEIGH CRITERION

$$\theta_{\min} \approx \frac{1.22 \lambda}{D}$$

SHORTER λ (BLUE)
BETTER RESOLUTION

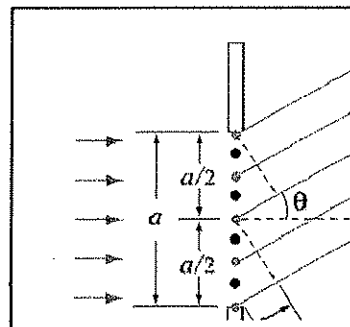
1.6 Coherent monochromatic plane waves impinge on two apertures separated by distance d . An approximate formula for the path length difference between the two rays is (3)

- ① $d \sin \theta$
2. $L \sin \theta$
3. $d \cos \theta$
4. $L \cos \theta$

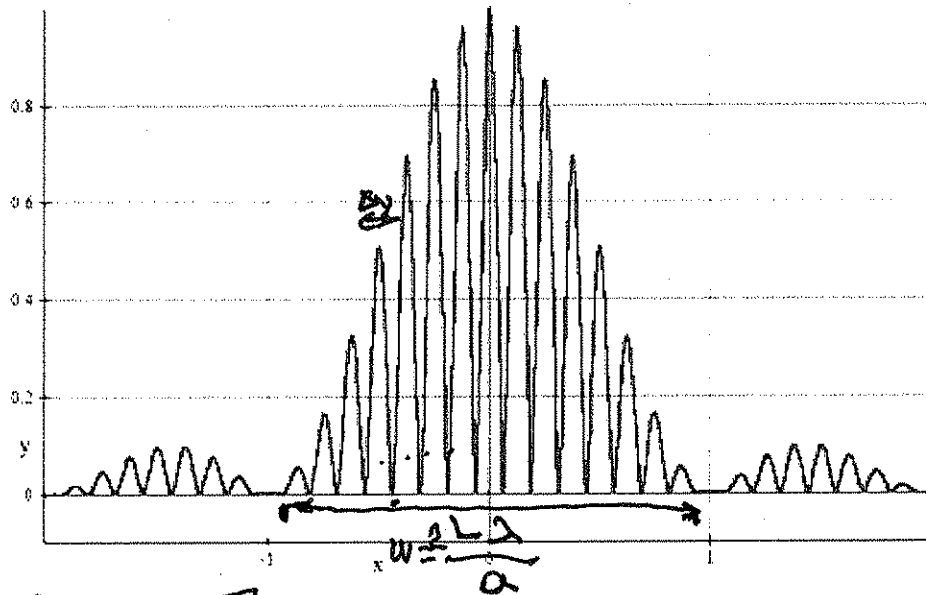


1.7 The light passing through this slit when seen on a screen far from the slit will exhibit *destructive* interference when (3)

1. $(\alpha/2)\sin\theta = \lambda/4$
- ② $(\alpha/2)\sin\theta = \lambda/2$
3. $(\alpha/2)\sin\theta = \lambda$
4. $(\alpha/2)\sin\theta = \lambda/8$



- 1.8 Coherent monochromatic plane waves impinge on two long narrow (width a) apertures separated by a distance d . The resulting pattern on the screen far away is shown below. For this arrangement The value d/a is about:
 (a) $1/8$ (b) $1/4$ (c) 4 (d) 8 (e) cannot be determined. Justify your answer (10)



$$\boxed{\frac{d}{a} \approx 8}$$

$$\cos \theta \approx \frac{x}{L}$$

$$\Delta y = \frac{L\lambda}{d}$$

$$W = \frac{2L\lambda}{a}$$

$$\frac{W}{\Delta y} = \frac{2L\lambda}{a} \cdot \frac{d}{L\lambda} = \frac{2d}{a}$$

$$\frac{d}{a} \approx \frac{1}{2} \frac{W}{\Delta y} \approx 8$$

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1.9 A paraxial ray (3)

- a. moves in a parabolic path.
- b. is a ray that has been reflected from parabolic mirror.
- c. is a ray that moves nearly parallel to the optical axis.
- d. is a ray that moves exactly parallel to the optical axis.

1.10 A virtual image is (3)

- a. the cause of optical illusions.
- b. a point from which rays appear to diverge.
- c. an image that only seems to exist.
- d. the image that is left in space after you remove a viewing screen.

11.1 The focal length of a converging lens is (3)

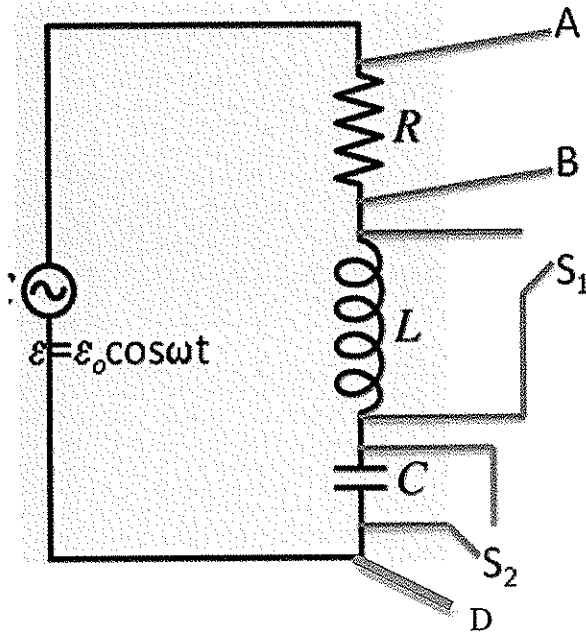
- a. the distance at which an image is formed.
- b. the distance at which an object must be placed to form an image.
- c. the distance at which parallel light rays are focused.
- d. the distance from the front surface to the back surface.

1.12 A red card is illuminated with red light. What color will appear? What if is illuminated with blue light? (explain your answer) (3)

(b) IT WILL APPEAR ~~BLACK~~ + RED CARD ABSORBS BLACK
BLUE.
 (a) RED LIGHT IS REFLECTED BY RED CARD
IT WILL APPEAR RED.

Name _____

Problem 2 (30): Consider an RLC circuit such as shown below with the following parameters: $\epsilon_0=10\text{ V}$, $R=10\Omega$ and $\omega=2\pi f=100\text{ c/sec}$.



2.1: What are the values of the maximum current I , voltage V_{AB} between the terminals A and B and phase of V_{AB} relative to the phase of the applied EMF, when the switches S_1 and S_2 are closed (inductance and capacitance shorted). (5)

2.2: Is there a combination of L and C that will produce the same current and voltage values as in question 2.1 with both switches open? (Explain) (10)

2.3 What is the maximum average power that the circuit can deliver to a motor connected at the

terminals A and B with both switches open? (5)

2.4 What is the maximum average power that the circuit will deliver to the motor connected to the terminals A and D with both switches open and the terminal D disconnected? (Explain) (10)

2.1

$$I = \epsilon_0 / R = 1A$$

$$V_{AB} = IR = 10\text{ Volts}$$

$$\phi = 0$$

2.2

$$I = \epsilon_0 / Z$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

For $X_L = X_C$ (resonance)

$$\omega L = 1/\omega C$$

$$I = 1A, V_{AB} = 10V, \phi = 0$$

$$LC = 1/\omega^2 = 10^4\text{ HenryFarad}$$

2.3

$$P = \varepsilon_{RMS} I_{RMS} = \frac{IV_{AB}}{2} = 50Watts$$

2.4 At resonance the values of V_L and V_C are cancelled out because they are anti-phased. As a result the impedance from a to D is R, similar to A to B. The maximum average power will still be 50 Watts.

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Problem 3 (40):

A laser shines light of wavelength 500 nm head-on toward a diffraction grating containing 100 vertical slits spread evenly over a horizontal distance of 1.00 millimeter. A gray wall is 4.00 meters behind the diffraction grating from the laser. Let P denote the point on the wall where the laser beam would have hit if the diffraction grating weren't there.

- (a) Your roommate says, "The reason there's a bright spot at point P is because the light that doesn't undergo any diffraction or interference reaches P." Do you agree? Disagree? Explain.

A bright spot appears at P not for the reason your roommate says, but rather, because constructive interference occurs there. Point P is approximately the same distance from all the slits; so crests that simultaneously pass through all the slits "meet up" at P. More formally, at P, the path length difference for two adjacent slits is $\Delta L = 0$, which means that point satisfies the constructive interference condition, $\Delta L = n\lambda$ for integer n , namely $n = 0$. You can reach the same conclusion starting with $d\sin \theta = n\lambda$, since P is where $\theta = 0$.

- (b) How far from point P is the center of the next nearest bright spot on the wall? Show your reasoning.

Start with the constructive interference condition $d\sin \theta = n\lambda$ for integer n . Point P is where $n = 0$. So we need to find the distance to the $n = 1$ bright spot (interference maximum). I'll find that distance, x , by calculating the angle θ and then using it to find x . Since 100 slits are spread out over 1.0×10^{-3} meters, the distance between adjacent slits is $d = (1.0 \times 10^{-3} \text{ meters})/100 = 1.0 \times 10^{-5}$ meters. The wavelength is $\lambda = 500 \text{ nm} = 5.0 \times 10^{-7}$ meters. So, for $n = 1$, θ works out to be quite small, which means we can approximate $\sin \theta$ as θ :

$$\theta \approx \sin \theta = n\lambda/d = (1)(5.0 \times 10^{-7} \text{ meters})/(1.0 \times 10^{-5} \text{ meters}) = 0.050 \text{ radians.}$$

For that small angle, the linear distance along the wall approximately equals the distance along the corresponding arc length: $x = R\theta = (4.00 \text{ meters})(0.050) = 0.20 \text{ meters.}$

(c) All the slits are now covered except 2 neighboring slits near the center. Describe in what ways the light pattern on the wall does and does not change, commenting specifically on (i) its overall brightness, (ii) the spacing between the bright spots, and (iii) the width of those bright spots.

- (i) The spots are much less bright, because only 2 slits-worth instead of 100 slits-worth of total light passes through the grating. *By a factor of $50^2 = 2500$.*
- (ii) The spacing stays the same, because having 100 (or 500 or 1000) slits reinforces the regions of constructive interference produced by just 2 slits instead of making a fundamentally different interference pattern.
- (iii) The bright spots get wider (less crisp) because more slits contributing to destructive interference between the bright spots results in wider regions of nearly-perfect destructive interference and hence thinner regions of constructive interference.

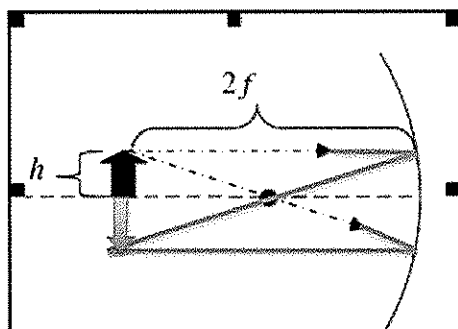
(d) The slits are all uncovered again. But now the laser light is replaced with an equally thin beam of light coming from a regular household light bulb. Does the same pattern of bright spots appear on the wall as appeared in parts (a) and (b)? Explain.

An interference pattern no longer appears. The “white” light from a regular bulb consists of many different wavelengths all mixed together, with no fixed phase differences. So, it’s no longer the case that when a crest reaches slit 1, a crest simultaneously reaches slits 2 through 100. By drawing the usual “overlapping circles” diagrams, you can see that a stable interference pattern forms in the two-slit (or 100-slit) scenario only if the waves reaching the diffraction grating are coherent, consisting of a single wavelength and with a fixed phase difference between the waves reaching adjacent slits.

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Problem 5: An object of height h is placed in front of a convex mirror. The object is placed at distance of twice the focal length. You may assume that the height of the object is much smaller than radius of curvature of the mirror. A diagram of the set up is shown below; the dot represents the focal point, the dashed line in the center is the axis of symmetry and the two dashed arrows represent rays traveling from the top of the object--one traveling toward the focal point and the other parallel to the symmetry axis. The mirror causes an image to be formed. (30)

5.1 On the diagram below continue the trace of the two rays including their reflections from the mirror to find the outgoing rays. Indicate where the outgoing rays meet forming an image (10)



5.2 Algebraically solve for the distance the image is from the mirror using the appropriate formula. Express your answer in terms of f . Is this position consistent with your ray tracing in 5.1? (10)

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$
$$s = 2f$$
$$\frac{1}{2f} - \frac{1}{f} = -\frac{1}{s'}$$
$$s' = 2f$$

Name:

5.3 Indicate whether the following statements are true or false. (You do not need to justify your answers). (10)

- i. The image is virtual.
- ii. The image is inverted.
- iii. The image is on the opposite side of the mirror from object.
- iv. The magnitude of the magnification is less than 2.
- v. The magnification is negative.

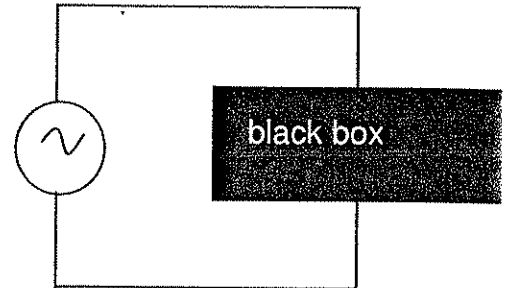
Items ii, iv, and v are true

Problem 4: (30)

Consider a "black box" which contains an arrangement of components which may include resistors, capacitors and inductors, the details of which you do not know. Two wires lead into the box and are hooked up to an AC voltage source with $\mathcal{E} = \mathcal{E}_0 \cos(\omega t)$ as in the figure to the right: The current going into the black box is measured as a function of time and is found to be of the

$$\text{form: } I(t) = \frac{\mathcal{E}_0 \omega_0}{L_0} (A(\omega) \cos(\omega t) + B(\omega) \sin(\omega t))$$

where $A(\omega)$, $B(\omega)$ are measured functions, L_0 is a constant with dimensions of inductance



- a. Show that the average power dissipated in the black box is $\bar{P} = \frac{\mathcal{E}_0^2 \omega_0 A(\omega)}{2L_0}$. Hint: The easiest way to do this does not require computing the phase shift. (10)

$$\begin{aligned} \bar{P} &= \frac{1}{T} \int_0^T I(t) \mathcal{E}(t) dt = \\ &= \frac{\mathcal{E}_0^2 \omega_0}{L_0 T} \int_0^T [A(\omega) \cos^2 \omega t + B(\omega) \cos \omega t \sin \omega t] dt = \\ &= \frac{1}{2} \frac{\mathcal{E}_0^2 \omega_0 A(\omega)}{L_0} \end{aligned}$$

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- b. Although you cannot look into the black box, from the form of the measured current outside one can deduce whether or not there is a resistor in the box hooked into the circuit. Is there a resistor? How do you know? (Hint: think about the result of part a.) (15)

SINCE THE AVERAGE POWER $\bar{P} \neq 0$ IT HAS A RESISTOR. INDUCTORS AND CAPACITORS DO NOT DISSIPATE POWER. SIMPLY INDUCE PHASE SHIFT BETWEEN $\mathcal{E}(t)$ AND $\mathcal{I}(t)$

- c. Find an expression for the phase shift between current and voltage. Recall that if the voltage is written in the form $\mathcal{E} = \mathcal{E}_0 \cos(\omega t)$ and the current as $\mathcal{I}(t) = I_0 \cos(\omega t - \phi)$ then the phase shift is ϕ . Express your answer in terms of $A(\omega)$ and $B(\omega)$. You may wish to use trig identities such as $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$ or $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$. (15)

$$\begin{aligned} \mathcal{I}(t) &= I_0 \cos[\omega t - \phi] \\ &= I_0 [\cos(\omega t) \cos \phi + \sin \omega t \sin \phi] \\ &= \frac{\mathcal{E}_0 \omega_0}{L_0} [A(\omega) \cos \omega t + B(\omega) \sin \omega t] \end{aligned}$$

$$\begin{aligned} \Rightarrow I_0 \cos \phi &= \frac{\mathcal{E}_0 \omega_0}{L_0} A(\omega) \\ I_0 \sin \phi &= \frac{\mathcal{E}_0 \omega_0}{L_0} B(\omega) \end{aligned}$$

$$\tan \phi = \frac{B(\omega)}{A(\omega)}$$