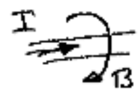
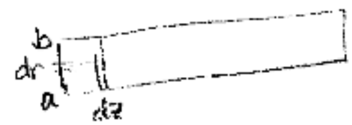
Final Exam Solutions

Problem 1: C,D;a;c;a;a;a;d;a;d;a

2a) 

$|B| = \frac{\mu_0 I}{2\pi r}$ inside

Outside $B \approx 0$

2b) 

$\phi = \int_a^b dr\,dz \frac{\mu_0 I}{2\pi r}$

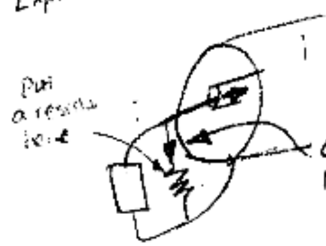
$\Phi = \frac{\mu_0 I L}{2\pi} \ln \frac{b}{a}$

2c) $V_{AB} > 0$ for $dI/dt > 0$

Explanation # 1 This is an inductor &

$V = L\, dI/dt$

Explanation # 2 Lens Law



current will flow through resistor to resist change in flux ∴ V_{AB} must be positive

2d) V_{AB} increases by 4

2e) Power goes to energy stored in magnetic field

Problem 3 Solution

(a) With both switches closed the current goes through the generator and resistor only. The total impedance of the circuit is R and

$$I_R(t) = (V_o / R)\cos\omega t$$

(b) The average power delivered to the circuit is

$$< P(t) >=< I_R(t)V(t) >= (V_o^2 / R) < \cos^2(\omega t) >= V_o^2 / 2R$$

(c) If only S_1 is opened and after a long time the current will pass through the generator, the resistor and the inductor. For this RL circuit the impedance amplitude becomes

$$Z = \frac{1}{\sqrt{R^2 + X_L^2}} = \frac{1}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\phi = \tan^{-1}(\frac{\omega L}{R})$$

The current as a function of time is given by

(d) Switches open, voltage in phase with current

$$\tan\phi = \frac{\omega L - 1/\omega C}{R} = 0$$

$$C = 1/\omega^2 L$$

(e) The circuit is at resonance since $X_L = X_C$. The impedance becomes

Z=R

(f) The energy stored I the capacitor is

$$U_E = \frac{1}{2}CV_C^2 = \frac{1}{2}C(IX_C)^2$$

Maximum occurs when the current is maximum, so

$$U_{c\max} = \frac{1}{2}CI_o^2X_C^2 = \frac{1}{2}C(V_0^2/R^2)(1/\omega C)^2 = \frac{V_o^2 L}{2R^2}$$

Average energy ½ of the above. Solution without ½ acceptable

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Problem 5 Solution:

(a) from standing wave pattern as shown, n=3

(b) $\lambda_e = \frac{2l}{n} = \frac{2 \bullet 6nm}{3} = 4nm$

(c) $\lambda_e = \frac{2l}{n} = \frac{2 \bullet 6nm}{3} = 4nm \Rightarrow K_e = \frac{p^2}{2m} = \frac{h^2}{2m_e\lambda^2} = \frac{(6.63\times10^{-34}Js)^2}{2(9.11\times10^{-31}kg)\left(4\times10^{-9}m\right)^2} = 1.5\times10^{-20}J$

$= 0.093eV$

or ,from energy level of infinite potential well:

$$K_e = E_n = n^2\frac{h^2}{8mL^2} = 9\frac{(6.626\times10^{-34}Js)^2}{8\times(9.11\times10^{-31}kg)\times\left(6\times10^{-9}m\right)^2} = 1.5\times10^{-20}J = 0.093eV$$

(d) $a\left(\frac{y_{m-dark}}{L}\right) = m\lambda_e \Rightarrow L = a\left(\frac{y_{m-dark}}{m\lambda_e}\right) = \left(1\times10^{-6}m\right)\left(\frac{1\times10^{-3}m}{4\times10^{-9}m}\right) = 0.25m$