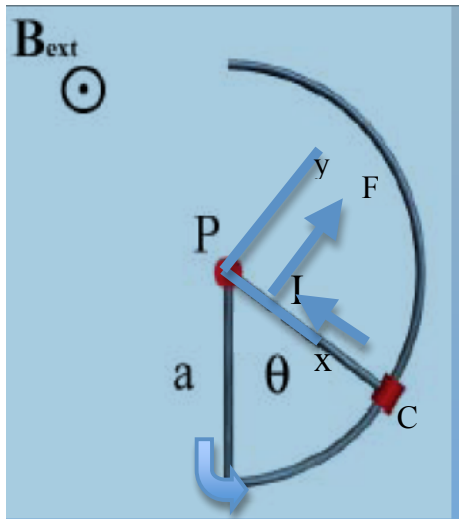


Problem #3: Consider the pie shaped circuit shown in the Figure. The sliding arm PC is free to pivot about the center, P, and has mass m and resistance R. (25)

- If the angle θ decreases with time (the bar is falling), what is the direction of the current? (5)
- If $\theta = \theta(t)$, what is the rate of change of the magnetic flux through the pie-shaped circuit? (10) [Hint: The area A enclosed by the pie shaped circuit is given by $A = \pi a^2 (\theta / 2\pi)$]
- What is the magnetic force on the arm. Give both magnitude and direction. Draw the force direction by a vector in the direction of the force on the arm in the Figure)? (10)
[Express your results in terms of B_{ext} , a, $d\theta/dt$ and resistance R]



a. The direction of the current should be such as to oppose the change of the magnetic flux through the loop. Since the area decreases with time the direction of the current should drive a magnetic field along B_{ext} (out of the page). As a result I is in the ccw direction as shown in the Figure.

$$b. A = \pi a^2 \frac{\theta}{2\pi} = a^2 \frac{\theta}{2}$$

$$\frac{d\Phi}{dt} = \frac{d(BA)}{dt} = \frac{Ba^2}{2} \frac{d\theta}{dt}$$

$$c. d\vec{F} = Id\vec{s} \times \vec{B}$$

Take the x - axis along the PC direction and the axis out of the page

$$Id\vec{s} = -I\hat{e}_x dx$$

$$\vec{B} = B_{ext}\hat{e}_z$$

$$d\vec{F} = Id\vec{s} \times \vec{B} = IdxB_{ext}\hat{e}_y$$

$$\vec{F} = \hat{e}_y IB_{ext} \int_0^a dx = \hat{e}_y IB_{ext} a$$

$$I = \mathcal{E}/R = \frac{B_{ext} a^2}{2R} \frac{d\theta}{dt}$$

$$\vec{F} = \hat{e}_y \frac{B_{ext}^2 a^3}{2R} \frac{d\theta}{dt}$$

