Name ___________________________________

Problem # 2: Consider the currents illustrated in the Figure and assume that the straight wires are infinite long. (20)

a. Use the appropriate law (Faraday or Biot-Savart) to determine the magnetic field at the point P (both magnitude and direction) due to the straight wires 1 and 2. (8)
b. Use the appropriate law (Faraday or Biot-Savart) to determine the magnetic field at the point P (both magnitude and direction) due to the current flowing in the shape 3 (8)
c. What is the combined magnetic field at point P (magnitude and direction) due to the currents flowing in 1,2 and 3 (4)
(Express your answer in terms of I,R,θ,π and μ_o)

For the problem consider a cylindrical coordinate system with its origin at P and the z-axis out of the page.

a. Apply Ampere's law $\oint \vec{B}\cdot d\vec{s} = \mu_o I$ or $2\pi\rho B(\rho) = \mu_o I$ to currents 1 and 2 with $\rho_1 = R/2$ and $\rho_2 = R$ to find

$\vec{B}_1(\text{at P}) = -\hat{e}_z \dfrac{\mu_o I}{\pi R}$ and $\vec{B}_2(\text{at P}) = -\hat{e}_z \dfrac{\mu_o I}{2\pi R}$ so

$$\vec{B}_{1,2}(\text{at P}) = -\hat{e}_z \frac{3\mu_o I}{2\pi R}$$

b. For long arms $d\vec{s}\times(-\hat{e}_r) = 0$ they do not contribute to the B field at P. For the arc cuurrent

$$d\vec{B}_3 = \frac{\mu_o I}{4\pi}\frac{d\vec{s}\times(-\hat{e}_r)}{R^2} = \frac{\mu_o I}{4\pi}\frac{(Rd\theta)\hat{e}_\theta\times(-\hat{e}_r)}{R^2} = -\hat{e}_z\frac{\mu_o I}{4\pi}\frac{Rd\theta}{R^2}$$

$$\vec{B}_3(\text{at P}) = -\hat{e}_z\frac{\mu_o I}{4\pi R}\int_0^\theta d\theta = -\hat{e}_z\frac{\mu_o I}{R}\frac{\theta}{4\pi}$$

c. $\vec{B}_{1,2,3}(\text{at P}) = -\hat{e}_z\dfrac{\mu_o I}{2\pi R}\left(3+\dfrac{\theta(radians)}{2}\right)$